

## PRELIMINARY EXAM: ALGEBRAIC TOPOLOGY

**Date:** January 9, 2014

**Instructions:** Do all three problems.

**Time Limit:** 90 minutes

**Problem 1.** Let  $F$  be the closed surface obtained by gluing together two copies of the Möbius band by a homeomorphism between their boundaries.

- (a) Identify the surface  $F$ .
- (b) Let  $A \subset F$  be the image of the boundaries of the Möbius bands. Compute the relative homology  $H_*(F, A)$ .

**Problem 2.** Let  $X$  be the 1-skeleton of a tetrahedron and  $Y$  the 1-skeleton of a cube. Is  $Y$  a covering space of  $X$ ? (If “yes”, exhibit a covering, and if “no”, prove that no such covering exists.)

**Problem 3.**

- (a) Classify the maps  $S^1 \rightarrow \mathbb{R}P^1$  up to homotopy.
- (b) Let  $f : S^1 \rightarrow \mathbb{R}P^1 \subset \mathbb{R}P^3$  be a map and let  $X_f$  be the space obtained by attaching a 2-cell to  $\mathbb{R}P^3$  via  $f$ . Compute  $\pi_1(X_f)$ .