Wave-particle duality and the unitary nature of electric charge

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Abstract – It is shown that the *unitary* nature of electric charge is inseparably linked to the mathematical simplicity and unity of the Stoney system of absolute units, which lead, almost unavoidably, to a conceptualization of the elementary charge as a viable wave-particle phenonemon at a *sub*-Planckian length scale. The results so obtained are employed to render a resolution of the vexing question of the origin of the electron's rest mass.

Introduction. – The fact that quantum mechanics was developed in the absence of a fundamental understanding of the *unitary* nature of electric charge must be regarded as one of the most remarkable achievements in the annals of theoretical physics. Nevertheless, the question of why the unit of electric charge e, which is considered to be a *structureless* entity, is observed to be identical on all charged particles, despite wide variations in their other characteristics, remains one of the great unanswered questions of particle physics. Indeed, aside from the fact that electric charge is the source of the electromagnetic interaction, and its conservation is understood, it stubbornly continues to elude our understanding at a fundamental level, which suggests that there is no good reason to maintain the prevailing view that electric charges are structureless entities. The present paper is a natural outgrowth of the *algebraic* relationship between the Stoney system of absolute units and quantum theory, whose interpretation of *sub*-microscopic phenomena has shown itself to be so wonderfully adept at rendering transparent such fundamental problems.

Passage to a *sub*-Planckian length scale. – What quantities one chooses to regard as fundamental depends on the domain one seeks to investigate. For example, if one seeks to set the scale for atomic and molecular sizes, then the Bohr radius 10^{-9} cm appears as the appropriate fundamental unit of length. However, if one's objective is to set the scale for determining the nature of the elementary charge then one finds that the Stoney scale [1] is the appropriate scale since in addition to giving prominence to the electromagnetic interaction it has the advantage of being an order of magnitude *smaller* than the Planck length, which affords the possibility of investigating *sub*-Planckian spacetime. Its base units for mass, length, and time consist of the electric charge (e), the Newtonian gravitational constant (G), and the velocity of light (c). By taking powers and ratios, Stoney was able to construct a system of *absolute* units, which has the form

$$M_0 = \left(\frac{e^2}{G}\right)^{1/2} = 1.86\times10^{-6}\,\mathrm{g} \tag{1}$$

$$L_0 = \left(\frac{e^2G}{c^4}\right)^{1/2} = 1.38\times10^{-34}\,\mathrm{cm} \tag{2}$$

$$T_0 = \left(\frac{e^2G}{c^6}\right)^{1/2} = 4.60\times10^{-45}\,\mathrm{s} \tag{3}$$

where we have employed the modern values (NIST),
$e = 4.803\times10^{-10}\,\mathrm{g}^{1/2}\,\mathrm{cm}^{3/2}\,\mathrm{s}^{-1}$
$G = 6.674\times10^{-8}\,\mathrm{cm}^3\,\mathrm{g}^{-1}\,\mathrm{s}^{-2}$
$c = 2.998\times10^{10}\,\mathrm{cm\ s}^{-1}$.
As we shall see below, these absolute units are especially revealing when applied to quantities such as charge and mass.

Conceptualization. – We shall take eq. (1) as the proper starting point by expressing the *gravitational* mass equivalent of the elementary charge's electrostatic poten-

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tial energy in terms of Einstein's mass-energy relation

$$\begin{aligned} E_0 &= M_0c^2 \\ &= 1.67 \times 10^{15}\,\mathrm{erg} \end{aligned} \tag{4}$$

which is advantageous, since it allows us to substitute the elementary charge's electrostatic potential energy with its gravitational mass *equivalent*, M_0c^2, in the context of the Stoney system of *absolute* units whose mathematical simplicity and unity provides a road map to a quantum-theoretical formulation of the elementary charge that enters its tenets, in a natural manner, as the product of the energy M_0c^2 and the absolute unit of time T_0, which has the dimensions of an "action," and is quantitatively equal to the ratio of two universal constants, namely e^2/c, which is given added significance by the fact that it is inseparably linked to M_0c^2. We are thus as good as forced to consider its implications for the corresponding momentum, M_0c, since energy and momentum are fundamentally linked in the closest possible way. Hence, it is essential that both be considered on a *completely* equal footing, which implies that it should be possible to express the "action" constant e^2/c in terms of the product of the momentum M_0c and the *sub*-Planckian absolute unit of length L_0, in the form

$$(M_0c)\,L_0 = 7.70 \times 10^{-30} \mathit{momentum} \cdot \mathit{length} = \frac{e^2}{c} \tag{5}$$

where $[(M_0c)\,L_0]$ is the emergent *sub*-Planckian length gravitational mass *equivalent* of the electrostatic quantum of "action" e^2/c. The significant aspect of eq. (5) is the *quantization* of the absolute unit of length from the more exact expression

$$L_0 = \frac{e^2/c}{(M_0c)} \tag{6}$$

which is physically meaningful since it sets the scale at which something peculiar happens to the structure of space. More succinctly, it provides a basis for formulating a consistent quantum-theoretical conceptualization of the elementary charge in a natural and unambiguous manner that corresponds to reality.

It is tacitly understood from eq. (5) that the elementary charge's *experimentally* determined value can be expressed as a squared quantity, in the form

$$e^2 = \left(M_0c^2\right) L_0. \tag{7}$$

In order to give physical meaning to this expression it is necessary to consider the nature of electric charge in an entirely new way that is consistent with the general principles of quantum theory, which tend to be simpler the greater their generality. It is therefore quite reasonable to assert, upon close inspection, that eq. (6) clearly suggests that the elementary charge can be characterized as a quantum-theoretical *wave-particle* entity at the *sub*-Planckian length L_0 .

Since in quantum theory the simplest assumption often turns out to be the most natural, we shall assume, on the basis of eq. (6), that at the *sub*-Planckian length, L_0, the *fabric* of spacetime *vibrates* with an *intrinsic* energy equivalent to the gravitational mass equivalent of the elementary charge's electrostatic potential energy, M_0c^2. We are then able to ascribe to the fabric of spacetime a vibrational *frequency*, v_0, that is inversely proportional to the *absolute* unit of time, T_0, and directly proportional to M_0c^2. The vibrational frequency and energy can then be linked through the equation

$$j\nu_0 = M_0c^2 \tag{8}$$

where, for simplicity of presentation, j denotes the *sub*-Planckian scale quantum of "action" $[(M_0c)\,L_0]$, deriving from eq. (5), and $j\nu_0$ is the *quantized* intrinsic vibrational energy of the fabric of spacetime. This is the decisive quantum-theoretical condition that *equates* the intrinsic *vibrational* energy of the fabric of spacetime with the elementary charge's *electrostatic* potential energy. Now, since we know from eq. (8) that a quantum of vibrational energy $j\nu_0$ has a momentum $j\nu_0/c$ denoted by p_0, and a wavelength that can be expressed in terms of momentum as

$$\lambda_0 = \frac{j}{p_0} \tag{9}$$

we can then directly identify eq. (7) with eqs. (8) and (9) as a *wave-particle* phenomenon by demonstrating that the vibrational energy of the fabric of spacetime, *per cycle*, corresponds to the universal constant e as a squared value, in the form

$$\begin{aligned} e^2 &= (j\nu_0)\lambda_0 \\ &= 2.30 \times 10^{-19}\,\mathrm{erg} \cdot \mathrm{cm} \end{aligned} \tag{10}$$

and

$$\begin{aligned} e &= \sqrt{(j\nu_0)\lambda_0} \\ &= 4.80 \times 10^{-10}\,\mathrm{esu} \end{aligned} \tag{11}$$

in quantitative agreement with the experimental value from which it draws its justification.

We have thus achieved an easily interpreted, observationally consistent, *wave-particle* conceptualization of electric charge as a manifestation of the quantization of the intrinsic vibrational energy of the fabric of spacetime at a *sub*-Planckian length scale.

Application. – A simple illustration of the application of this conceptualization of electric charge is afforded by the vexing question of the *origin* of the electron's *rest* mass. If we subscribe to the view that the electron's rest mass, m_e, is *entirely* electromagneic in origin, then it must have a mass equivalent to

$$m_e = \frac{(m_ec^2)}{c^2} \tag{12}$$

that is associated with the momentum

$$m_ec = \frac{j}{r_e} \tag{13}$$

which is required for the electron's localization through

$$m_e c^2 = \left(\frac{j}{r_e}\right) c \tag{14}$$

where r_e is the so-called "classical" electron radius. It then follows, in a natural manner, from eq. (14), that the electron has a *quantized* intrinsic *vibrational* frequency, ν_e, whose value is given by

$$\begin{aligned} \nu_e &= \frac{m_e c^2}{j} \\ &= 1.06 \times 10^{23} s^{-1} \end{aligned} \tag{15}$$

from which it follows that

$$j\nu_e = m_e c^2 \tag{16}$$

which links the electron's intrinsic vibrational *frequency* ν_e to its *mass*-energy $m_e c^2$. Hence,

$$\begin{aligned} m_e &= \frac{j\nu_e}{c^2} \\ &= 9.11 \times 10^{-28}\,\mathrm{g} \end{aligned} \tag{17}$$

which is easily seen to be *equivalent* to our basic equation (12). This result furnishes strong support for both the electromagnetic origin of the electron's rest mass, and the wave-particle conceptualization of electric charge.

Conclusion. – It is possible, in the context of the Stoney system of absolute units, to extend the quantum mechanical concept of wave-particle duality to the elementary charge. The elementary processes imposed by the Stoney system of units on the elementary charge leads, in the simplest possible manner, to a conceptualization of the elementary charge as a manifestation of the quantization of the intrinsic vibrational energy of the fabric of space-time at a *sub*-Planckian length scale; which is particularly satisfying since it provides a basis for an exact evaluation of the electron's intrinsic *vibrational* frequency, whose validity can be subjected to experimental verification.

To conclude, the foregoing considerations demonstrate, in an instructive manner, that no theory of particles that deals only with particles will ever explain particles.

REFERENCES

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