Construction of Dirichlet Forms on Standard Forms of von Neumann Algebras

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Abstract

For a von Neumann algebra $\mathcal{M}$ acting on a Hilbert space $\mathcal{H}$ with a cyclic and seperating vector ξ_0, we give an explicit expression of Dirichlet forms on the natural standard form $(\mathcal{M}, \mathcal{H}, \mathcal{P}, J)$ associated the pair $(\mathcal{M}, \xi_0)$. For any self-adjoint analytic element x of $\mathcal{M}$ and an admissible function f, we construct a (bounded) Dirichlet form which generates a Markovian semigroup on $\mathcal{H}$. We then apply our result to construct translation invariant Markovian semigroups for quantum spin systems with finite range interactions.

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1 Introduction

The purpose of this work is to construct Dirichlet forms on quantum probability spaces. We empoly the theory of Dirichlet forms and Markovian semigroups on standard forms of von Neumann algebras developed by Cipriani [Cip 1,2] to define Dirichlet forms which generate Markovian semigroups. For a von Neumann algebra $\mathcal{M}$ acting on a Hilbert space $\mathcal{H}$ with a cyclic and separating vector ξ_0 for $\mathcal{M}$, let $(\mathcal{M}, \mathcal{H}, \mathcal{P}, J)$ be the natural standard form associated with the pair $(\mathcal{M}, \xi_0)$. For any admissible function f (Definition 3.1) and any analytic element x in $\mathcal{M}$, we construct a bounded Dirichlet form which generates a Markovian semigroup on $\mathcal{H}$.

Applying our result we are able to construct translation invariant Markovian semigroups for quantum spin systems with finite range interactions.

The efforts in construction and study of quantum Markov semigroups are as old as the equilibrium description of quantum systems [Dav]. The study of non-commutative Dirichlet forms was pioneered by Albeverio and Høegh-Krohn [AH-K], Sauvageot [Sau 1,2,3] and extensively developed by Davies and Lindsay [DL], and Guido, Isola and Scarlatt [GIS]. All these authors considered Markovianity of forms and semigroups only with respect to a tracial functional ϕ_0. Recently Goldstein and Lindsay [GL 1,2], and Cipriani [Cip 1,2] have extended the theory to a faithful normal state ϕ_0 in the context of standard forms.

The need to construct continuous Markovian semigroups on von Neumann algebras, which are symmetric with respect to a non-tracial state, is clear for various applications to open quantum systems [Dav], quantum statistical mechanics [BR] and quantum probability [Acc,Par]. Although on an abstract level we have quite well developed theory as stated above, the progress in concrete application is very slow. We would like to mention few recent works in this direction. The completely positive Hamiltonian semigroup in ground state representation has been considered in [Mat,FNW]. A special example of translation invariant stochastic dynamic satisfying a detailed balance condition has been constructed in [GM], where the authors used a representation associated to a classical Gibbs measure at a finite temperature. In [MZ1,2], the authors used the generalized conditional expectations [Acc,AC] to construct generators of spin-flip type dynamics for quantum spin systems. We should mention that in [MZ1,2] the authors also have constructed Markovian semigroups of a diffusion type generators (which have been introduced formally in [SQV]) for quantum systems under the condition of strong asymptotic abelianess, and translation invariant Markovian semigroups for quantum spin systems under the condition of hyper asymptotic abelianess which is not proven yet. The Dirichlet forms we introduced are also diffusion type generators and are reduced to usual (canonical) forms in the case of tracial states. Using the Dirichlet forms, we also construct translation invariant Markov semigroup for quantum systems. However, we do not need any kind of asymptotic abelianess. Translation invariant Markovian semigroups for various quantum systems, such as CAR and CCR algebras with quasi-free states, are under construction.

The paper is organized as follows: In section 2, we give a brief review of the theory of Dirichlet forms and Markovian semigroup on standard forms of von Neumann algebra developed by Cipriani [Cip1,2]. Section 3 is devoted to the construction of Dirichlet forms. We first introduced the notion of admissible functions. Then, for any admissible function f and an analytic element $x \in \mathcal{M}$, we construct a Dirichlet form (Theorem 3.3) which generates a Markovian semigroup. In section 4, we give an explicit expression of the generator L defined on $\mathcal{M}$ associated with a Dirichlet operator H defined on a standard form and show directly that L generates a Markovian semigroup on $\mathcal{M}$. In section 5, we use the result in section 3 to construct Dirichlet forms which generate translation invariant Markov semigroups for quantum spin systems with finite range interactions.

2 Review on the Theory of Non-commutative Dirichlet Forms

We give a brief review on the theory of Dirichlet forms and Markov semigroups on standard form of von Neumann algebras developed by Cipriani [Cip1,2]. For the details we refer to [Cip2].

Let $\mathcal{M}$ be a σ-finite von Neumann algebra acting on a complete Hilbert space $\mathcal{H}$. A self-dual con $\mathcal{P}$ in $\mathcal{H}$ is a subset satisfying the property

$$\{\xi \in \mathcal{H} : (\xi, \eta) \geq 0 \quad \forall \eta \in \mathcal{P}\} = \mathcal{P}.$$

$\mathcal{P}$ is then a closed convex cone and $\mathcal{H}$ is the complexification of the real subspace $\mathcal{H}^J =: \{\xi \in \mathcal{H} : (\xi, \eta) \in \mathbb{R} \quad \forall \eta \in \mathcal{P}\}$, which elements are called *J-real*: $\mathcal{H} = \mathcal{H}^J + i\mathcal{H}^J$. Such a $\mathcal{P}$ gives rise to a structure of ordered Hilbert space $\mathcal{H}^J$ (denoted by $\leq$) and to an anti-unitary involution J on $\mathcal{H}$, which preserves $\mathcal{H}^J$: $J(\xi + i\eta) =: \xi - i\eta$ for all $\xi, \eta \in \mathcal{H}^J$. Any J-real element $\xi \in \mathcal{H}^J$ can be decomposed uniquely as a difference, $\xi = \xi_+ - \xi_-$, of two orthogonal, positive elements, called the positive and the negative part of $\xi : \xi_+, \xi_- \in \mathcal{P}, \quad (\xi_+, \xi_-) = 0$.

A standard form $(\mathcal{M}, \mathcal{H}, \mathcal{P}, J)$ of the von Neumann algebra $\mathcal{M}$ acting faithfully on the Hilbert space $\mathcal{H}$ consists of self-dual, closed, convex cone $\mathcal{P}$ in $\mathcal{H}$ and the anti-unitary involution J satisfying the properties:

(a) $J\mathcal{M}J = \mathcal{M}'$

(b) $JxJ = x^*, \quad \forall x \in \mathcal{M} \cap \mathcal{M}'$

(c) $J\xi = \xi, \quad \forall \xi \in \mathcal{P}$

(d) $xJxJ(\mathcal{P}) \subset \mathcal{P}, \quad \forall x \in \mathcal{M}$,

where $\mathcal{M}'$ is the commutant of $\mathcal{M}$.

A bounded operator A on $\mathcal{H}$ is called *J-real* if $AJ = JA$ and *positive preserving* if $A\mathcal{P} \subset \mathcal{P}$. The semigroup $\{T_t\}_{t\geq 0}$ is said to be *J-real* if T_t is J-real for any $t \geq 0$ and it is called *positive preserving* if T_t is positive preserving for any $t \geq 0$.

Let us fix a cyclic and seperating vector ξ_0 in $\mathcal{P}$. A bounded operator $A : \mathcal{H} \to \mathcal{H}$ is called *sub-Markovian* (with respect to ξ_0) if $0 \leq \xi \leq \xi_0$ implies $0 \leq A\xi \leq \xi_0$. A is called *Markovian* if it is sub-Markovian and also $A\xi_0 = \xi_0$. A semigroup $\{T_t\}_{t\geq 0}$ is said to be *sub-Markovian* (with respect to ξ_0) if T_t is sub-Markovian for every $t \geq 0$. The semigroup $\{T_t\}_{t\geq 0}$ is called *Markovian* if T_t is Markovian for every $t \geq 0$.

Next, we consider a sesquilinear form on some linear manifold of $\mathcal{H} : \mathcal{E}(\cdot, \cdot) : D(\mathcal{E}) \times D(\mathcal{E}) \to \mathbb{C}$. We also consider the associated quadratic form: $\mathcal{E}[\cdot] : D(\mathcal{E}) \to \mathbb{C}$, $\mathcal{E}[\xi] =: \mathcal{E}(\xi, \xi)$. A real quadratic form $\mathcal{E}[\cdot]$ is said to be *semibounded* if $\inf\{\mathcal{E}[\xi] : \xi \in D(\mathcal{E}),\ ||\xi|| = 1\} = -b > -\infty$. A quadratic form $(\mathcal{E}, D(\mathcal{E}))$ is said to be *J-real* if $JD(\mathcal{E}) \subset D(\mathcal{E})$ and $\mathcal{E}[J\xi] = \overline{\mathcal{E}[\xi]}$ for any $\xi \in D(\mathcal{E})$. For a given semibounded quadratic form $\mathcal{E}$, one consider the inner product given by $(\xi, \eta)_\lambda =: \mathcal{E}(\xi, \eta) + \lambda(\xi, \eta)$, for $\lambda > b$. The form $\mathcal{E}$ is *closed* if $D(\mathcal{E})$ is a Hilbert space for some of the above norms. The form $\mathcal{E}$ is called *closable* if it admits a closed extension.

Associated to a semibounded closed form $\mathcal{E}$, there is a self-adjoint operator $(H, D(H))$ and a strongly continuous, symmetric semigroup $\{T_t\}_{t\geq 0}$. Each of the above objects determines uniquely the others according to well known relations (see [BR, 3.1], [Fuk, 1.3]).

From now on we will consider only J-real, real-valued, densely defined quadratic forms. It is easy to check that these forms satisfy the relation : $\mathcal{E}[\xi+i\eta]=\mathcal{E}[\xi]+\mathcal{E}[\eta]$ for all $\xi+i\eta\in D(\mathcal{E})^J+iD(\mathcal{E})^J=D(\mathcal{E})$.

Let us denoted by $\mathrm{Proj}(\xi,\mathcal{Q})$ the projection of the vector $\xi\in\mathcal{H}^J$ onto the closed, convex cone $\mathcal{Q}\subset\mathcal{P}$. For $\xi,\eta\in\mathcal{H}^J$, define

$$\begin{aligned}\xi\vee\eta &=: \mathrm{Proj}(\xi,\eta+\mathcal{P})\\ \xi\wedge\eta &=: \mathrm{Proj}(\xi,\eta-\mathcal{P})\end{aligned}$$

Definition 2.1. *A J-real, real-valued, densely defined quadratic form $(\mathcal{E},D(\mathcal{E}))$ is called Markovian with respect to $\xi_0\in\mathcal{P}$ if*

$$\xi\in D(\mathcal{E})^J \textit{ implies } \xi\wedge\xi_0\in D(\mathcal{E}) \textit{ and } \mathcal{E}[\xi\wedge\xi_0]\le\mathcal{E}[\xi].$$

A closed Markovian form is called a Dirichlet form.

Next, we state Proposition 4.5 and Theorem 4.7 of [Cip2] which we will use in the next section:

Proposition 2.2 *[Cip2, Prop.4.5(b)]. Let $(\mathcal{E},D(\mathcal{E}))$ be a J-real, real-valued, densely defined quadratic form. The followings are equivalent:*

(a) $\mathcal{E}[\cdot]\ge 0$ and $\xi\in D(\mathcal{E})^J$ implies $\xi_\pm\in D(\mathcal{E})$, $\quad\mathcal{E}(\xi_+,\xi_-)\le 0$.

(b) $\xi\in D(\mathcal{E})^J$ implies $\xi_+\in D(\mathcal{E})$ and $\mathcal{E}[\xi_+]\le\mathcal{E}[\xi]$.

Proposition 2.3 *[Cip2, Theorem 4.7 (b)]. Let $\{T_t\}_{t\ge0}$ be a J-real, strongly continuous, symmetric semigroup on $\mathcal{H}$ and let $(\mathcal{E},D(\mathcal{E}))$ be the associated J-real, real-valued quadratic form. The followings are equivalent:*

(a) $\xi\in D(\mathcal{E})^J$ implies $\xi_+\in D(\mathcal{E})$ and $\mathcal{E}[\xi_+]\le\mathcal{E}[\xi]$.

(b) $\{T_t\}$ is positive preserving and contractive.

The following is one of the Main results in [Cip2] which is the characterization of sub-Markovian semigroups in terms of Dirichlet forms with respect to the fixed vector $\xi_0\in\mathcal{P}$:

Theorem 2.4 *[Cip2, Theorem 4.11]. Let $\{T_t\}$ be a J-real, strongly continuous, symmetric semigroup on $\mathcal{H}$ and let $(\mathcal{E},D(\mathcal{E}))$ be the associated densely defined J-real, real-valued quadratic form. The followings are equivalent :*

(a) $\{T_t\}_{t\ge0}$ is sud-Markovian.

(b) $\mathcal{E}$ is a Dirichlet form.

3 Construction of Dirichlet Forms

In this section we construct Dirichlet forms on the natural standard forms of von Neumann algebras associated with the Tomita-Takesaki theory[BR,Tak]. The forms we construct generate Markovian semigroups.

Let $\mathcal{M}$ be a σ-finite von Neumann algebra acting on a Hilbert space $\mathcal{H}$ and let $\xi_0 \in \mathcal{H}$ be a cyclic and seperating vector for $\mathcal{M}$. We use Δ and J to denote respectively, the modular operator and the modular conjugation associated with the pair $(\mathcal{M}, \xi_0)$ [BR, 2.5]. The associated modular automorphism group is denoted by $\sigma_t : \sigma_t(A) = \Delta^{it} A \Delta^{-it}$, $A \in \mathcal{M}$. Finally, $j : \mathcal{M} \to \mathcal{M}'$ is the antilinear *-isomorphism defined by $j(A) = JAJ, \quad A \in \mathcal{M}$.

The natural positive cone $\mathcal{P}$ associated with $(\mathcal{M}, \xi_0)$ is the closure of the set

$$\{Aj(A)\xi_0; A \in \mathcal{M}\}.$$

By a general result the cone $\mathcal{P}$ can be obtained by the closure of the set

$$\{\Delta^{1/4} AA^* \xi_0; A \in \mathcal{M}\}.$$

The natural cone $\mathcal{P}$ is self-dual [BR, 2.5]. For the detailed properties of $\mathcal{P}$, we refer to [BR, 2.5]. The form $(\mathcal{M}, \mathcal{H}, \mathcal{P}, J)$ is the standard form associated with the pair $(\mathcal{M}, \xi_0)$.

In order to express Dirichlet forms, let us introduce the notion of admissible functions:

Definition 3.1. *An analytic function $f : D \to \mathbb{C}$ on a domain D containing the strip $Im\ z \in [-1/4, 1/4]$ is said to be admissible if the following properties hold;*

(a) $f(t) \geq 0$ for $\forall t \in \mathbb{R}$,

(b) $f(t + i/4) + f(t - i/4) \geq 0$ for $\forall t \in \mathbb{R}$,

(c) there exist $M > 0$ and $p > 1$ such that the bound

$$|f(t + is)| \leq M(1 + |t|)^{-p}$$

holds uniformly in $s \in [-1/4, 1/4]$.

Lemma 3.2. *For any $p > 1$, there exists an admissible function such that the bound in Definition 3.1 (c) holds with p.*

Proof. We will prove the lemma by giving an example. Let $\tilde{g}$ be the function given by

$$\tilde{g}(t) = \frac{2}{\sqrt{2\pi}} \int (e^k + e^{-k})^{-1} e^{ikt}\, dk. \tag{3.1}$$

We first show that $\tilde{g}$ is a non-negative function. Using the residue integration method, it is easy to check that

$$\begin{aligned} \int_{-\infty}^{\infty} (\cosh k)^{-1} e^{ikt}\, dk &= 2\pi e^{-\frac{\pi}{2}|t|}(1 + e^{-\pi|t|})^{-1} \\ &= 2\pi(e^{\frac{\pi}{2}t} + e^{-\frac{\pi}{2}t})^{-1}. \end{aligned} \tag{3.2}$$

This proved that the function given in (3.1) is non-negative.

Let g be the function defined by $g(t) = 4\tilde{g}(4t)$, and define

$$f(t) = \frac{2}{\sqrt{2\pi}} \int (e^{k/4} + e^{-k/4})^{-1} e^{-\frac{1}{2}k^2} e^{-ikt}\, dt. \tag{3.3}$$

Clearly f has an analytic extension to a domain containing the strip. We will show that f satisfies all requirements. Since f is the convolution of g and $\exp(-t^2/2)$, f is non-negative on $\mathbb{R}$. Notice that for any $t \in \mathbb{R}$

$$\begin{aligned} f(t + i/4) + f(t - i/4) &= \frac{2}{\sqrt{2\pi}} \int e^{-\frac{1}{2}k^2} e^{-ikt} dk \\ &\geq 0. \end{aligned}$$

It is easy to check that for any $n \in \mathbb{N}$ there is a positive constant $M(n)$ such that the bound

$$|t^n f(t + is)| \leq M(n)$$

holds uniformly in $s \in [-1/4, 1/4]$. This completes the proof. ■

We are ready to give a construction of Dirichlet forms on the standard form $(\mathcal{M}, \mathcal{H}, \mathcal{P}, J)$ associated with the pair $(\mathcal{M}, \xi_0)$. Denote by $\mathcal{M}_0$ the dense subset of $\mathcal{M}$ consisting of σ_t-analytic elements on a domain containing the strip $I_{1/2} =: \{z; |Im\ z| \leq \frac{1}{2}\}$ [BR]. By Prop. 2.5.21 of [BR], any $A \in \mathcal{M}_0$ is strongly analytic. For given admissible function f and $x = x^* \in \mathcal{M}_0$, define a sesquilinear form $\mathcal{E} : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$ by

$$\mathcal{E}(\eta, \xi) = \int \Big(\big(\sigma_{t-i/4}(x) - j\,(\sigma_{t-i/4}(x))\big)\eta, \big(\sigma_{t-i/4}(x) - j\,(\sigma_{t-i/4}(x))\big)\xi \Big) f(t)\, dt. \tag{3.4}$$

Then the associated quadratic form is given by

$$\mathcal{E}[\xi] = \int ||\, [\sigma_{t-i/4}(x) - j\,(\sigma_{t-i/4}(x))]\, \xi\, ||^2 f(t)\, dt. \tag{3.5}$$

The form given in the above is a bounded form.

Theorem 3.3. *For given admissible function f and self-adjoint element $x = x^* \in \mathcal{M}_0$, let $(\mathcal{E}, \mathcal{H})$ be defined as in (3.5). Let H be the self-adjoint operator associated with $(\mathcal{E}, \mathcal{H})$. Assume that there exists a constant $M > 0$ such that the bound*

$$\sup_{s \in [-1/4, 1/4]} ||\, \sigma_{t+is}(x)\, || \leq M$$

holds uniformly in $t \in \mathbb{R}$. Then the following properties hold:

(a) $H\xi_0 = 0$,

(b) $\mathcal{E}$ *is J-real,*

(c) $\mathcal{E}(\xi_+, \xi_-) \leq 0 \quad$ *for* $\forall \xi \in \mathcal{H}^J$.

Furthermore the form $(\mathcal{E},\mathcal{H})$ is a Dirichlet form.

We will produce the proof of Theorem 3.3 at the end of this section. It may be worth to give some remarks on the assumption in the above theorem:

Remark 3.4. *(a) If $\mathcal{H}$ is a finite dimensional Hilbert space, Δ is strictly positive bounded self-adjoint operator. It is clear that the bound in the theorem holds.*

(b) In section 5, we will show that the bound in the theorem holds for quantum spin systems under an appropriate condition on interactions. See section 5 for the details.

(c) In fact, the bound can be relaxed by

$$\sup_{s\in[-1/4,1/4]} ||\,\sigma_{t+is}(x)\,|| \le M(1+|t|)^{\alpha}$$

for any $\alpha>0$. Then one choose an admissible function f such that the bound in Definition 3.1(c) holds with p greater than $\alpha+1$. See the proof of the part (c) of Theorem 3.3.

The following is a consequence of Theorem 3.3:

Theorem 3.5. *Let $\{T_t\}_{t\ge 0}$ be the semigroup generated by the form $(\mathcal{E},\mathcal{H})$ in Theorem 3.3. Then $\{T_t\}_{t\ge 0}$ is a J-real, strongly continuous, symmetric Markovian semigroup.*

Proof. It follows from Theorem 3.3(a) that $T_t(\xi_0)=\xi_0$ for any $t\ge 0$. Thus the theorem follows from Theorem 2.4. ■

We now produce the proof of Theorem 3.3.

Proof of Theorem 3.3. (a) Notice that $JA\xi_0=\Delta^{1/2}A^*\xi_0$ for any $A\in\mathcal{M}$. Since $x=x^*\in\mathcal{M}_0$,

$$\begin{aligned}\Big(\sigma_{t-i/4}(x)-j\left(\sigma_{t-i/4}(x)\right)\Big)\,\xi_0 &= \Delta^{1/4}(\sigma_t(x))\xi_0 - J\Delta^{1/4}\sigma_t(x)\Delta^{-1/4}\xi_0\\ &= \Delta(\sigma_t(x))\xi_0-\Delta^{1/4}\sigma_t(x)^*\xi_0\\ &= 0.\end{aligned}$$

Thus (a) follows from (3.4) and the above fact.

(b) A direct estimate shows that

$$\begin{aligned}&||\,\Big(\sigma_{t-i/4}(x)-j\left(\sigma_{t-i/4}(x)\right)\Big)\,J\xi\,||^2\\ &=||\,J\,j\left([\sigma_{t-i/4}(x)-j\left(\sigma_{t-i/4}(x)\right)]\right)\xi\,||^2\\ &=||\,\Big(\sigma_{t-i/4}(x)-j\left(\sigma_{t-i/4}(x)\right)\Big)\,\xi\,||^2,\end{aligned}$$

which implies $\mathcal{E}[J\xi]=\mathcal{E}[\xi]$.

(c) By the expression of $\mathcal{E}[\xi]$ in (3.5), $\mathcal{E}(\xi_+,\xi_-)$ can be written as

$$\mathcal{E}(\xi_+,\xi_-)=\mathrm{I}+\ \mathrm{II}, \tag{3.6}$$

where

$$\mathrm{I} = \int \left((\sigma_{t-i/4}(x)\xi_+, \sigma_{t-i/4}(x)\xi_-) + (\sigma_{t-i/4}(x)\xi_-, \sigma_{t-i/4}(x)\xi_+)\right) f(t)\, dt, \tag{3.7}$$
$$\mathrm{II} = -\int \left((\sigma_{t-i/4}(x)\xi_+, j\,(\sigma_{t-i/4}(x))\xi_-) + (j\,(\sigma_{t-i/4}(x))\xi_+, \sigma_{t-i/4}(x)\xi_-)\right) f(t)\, dt.$$

By Theorem 4 of [Ara], $\mathcal{M}\xi_+ \perp \mathcal{M}\xi_-$, which implies $\mathrm{I} = 0$. Next, we consider II. Since $\sigma_{t-i/4}(x)^* = \sigma_{t+i/4}(x)$ and $j\,(\sigma_{t-i/4}(x)) = \sigma_{t+i/4}(j(x))$,

$$\begin{aligned}(\sigma_{t-i/4}(x)\xi_+\,,\, j\,(\sigma_{t-i/4}(x))\xi_-) &= (\sigma_{t-i/4}(x)\xi_+\,,\, (\sigma_{t+i/4}(j(x)))\xi_-) \\ &= (\,\xi_+\,,\, \sigma_{t+i/4}(xj(x))\xi_-)\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}(j\,(\sigma_{t-i/4}(x))\xi_+\,,\, (\sigma_{t-i/4}(x))\xi_-) &= (\sigma_{t+i/4}(xj(x))\xi_+\,,\, \xi_-) \\ &= (\xi_+\,,\, \sigma_{t-i/4}(xj(x))\xi_-).\end{aligned} \tag{3.9}$$

It follows from (3.7)-(3.9) that

$$\mathrm{II} = -\int \left((\xi_+, \sigma_{t+i/4}(xj(x))\xi_-) + (\xi_+, \sigma_{t-i/4}(xj(x))\xi_-)\right) f(t)\, dt$$

Using the Cauchy integral theorem, the assumption in the theorem and the property (c) in the Definition 3.1, we obtain that

$$\mathrm{II} = -\int (\xi_+, \sigma_t(xj(x))\xi_-)\Big(f(t-i/4) + f(t+i/4)\Big)\, dt.$$

Since $\sigma_t(xj(x))\xi_- \in \mathcal{P}$, $(\xi_+, \sigma_t(xj(x))\xi_-) \geq 0$ for $t \in \mathbb{R}$. By the property (b) in the Definition 3.1, we conclude that $\mathrm{II} \leq 0$. This proved the part (c) of the theorem.

Clearly $\mathcal{E}[\cdot] \geq 0$. By Theorem 3.3 (a)-(c) and Proposition 2.3, the semigroup $\{T_t\}_{t\geq 0}$ generated by $(\mathcal{E}, \mathcal{H})$ is sub-Markovian. By Theorem 2.4, $(\mathcal{E}, \mathcal{H})$ is a Dirichlet form. ■

4 Symmetric Markovian Semigroups on von Neumann Algebras

In this section we give the explicit expression of the generator L of the symmetric Markovian semigroup $P_t = e^{-tL}$ on $\mathcal{M}$ associated to a Markov semigroup $T_t = e^{-tH}$ on $\mathcal{H}$ constructed in Section 3, and investigate detailed properties of L.

Let us consider the symmetric embedding:

$$\begin{aligned} i_0 &: \mathcal{M} \to \mathcal{H} \\ i_0(A) &= \Delta^{1/4} A \Delta^{-1/4}\xi_0. \end{aligned}$$

Then L and H are related by $i_0 \circ L = H \circ i_0$ [Cip2]. Note that $j(A)\xi_0 = \Delta^{1/2}A^*\xi_0$ for any $A \in \mathcal{M}$. A direct computation yields that for $x = x^* \in \mathcal{M}_0, \quad A \in \mathcal{M}_0$,

$$
\begin{aligned}
&\big(\sigma_{t-i/4}(x) - j\,(\sigma_{t-i/4})\big)^*\big(\sigma_{t-i/4}(x) - j(\sigma_{t-i/4}(x))\big)\, i_0(A)\xi_0 \\
&\quad = \big(\sigma_{t-i/4}(x) - j(\sigma_{t-i/4}(x))\big)^*[\sigma_{t-i/4}(x), \Delta^{1/4}A\Delta^{-1/4}]\xi_0 \\
&\quad = [\,\sigma_{t+i/4}(x)\,,\,[\,\sigma_{t-i/4}(x)\,,\,\Delta^{1/4}A\Delta^{-1/4}\,]\,\Delta\,]\xi_0 \\
&\quad = [\,\sigma_{t+i/4}(x)\,,\,\Delta^{1/4}[\,\sigma_t(x)\,,\,A]\,\Delta^{3/4}\,]\xi_0 \\
&\quad = \Delta^{-1/4}[\,\sigma_t(x)\,,\,\Delta^{1/2}\,[\,\sigma_t(x)\,,\,A]\Delta^{1/2}]\xi_0,
\end{aligned}
\tag{4.1}
$$

where $[A, B] = AB - BA \quad$ for $A, B \in \mathcal{M}$. Thus it follows (3.4) and (4.1) that for given admissible function f and $x = x^* \in \mathcal{M}_0$,

$$D(L) = \mathcal{M}$$

$$
\begin{aligned}
L(A) \;&=\; \int \Delta^{-1/2}[\sigma_t(x), \Delta^{1/2}[\sigma_t(x), A]\Delta^{1/2}]\,\Delta^{-1/2}\, f(t)\, dt \\
&=\; \int \big(\sigma_{t+i/2}(x)[\sigma_t, A] - [\sigma_t(x), A]\sigma_{t-i/2}(x)\big)\, f(t)\, dt.
\end{aligned}
\tag{4.2}
$$

The first expression in (4.2) is a formal expression, which has a symmetric form.

Theorem 4.1. *For given admissible function f and $x = x^* \in \mathcal{M}_0$, let $L : \mathcal{M} \to \mathcal{M}$ be defined as in (4.2). Assume that there exists a constant $M > 0$ such that the bound*

$$\sup_{s\in[-1/2,1/2]} ||\sigma_{t+is}(x)\,|| \;\le M$$

holds uniformly in $t \in \mathbb{R}$. Then the following properties hold:

(a) $L(\mathbf{1}) = 0$

(b) $*$-invariance: $L(A^*) = L(A)^*, \quad \forall A \in \mathcal{M}$

(c) Dissipativity: $L(A^*A) \le L(A^*)A + A^*L(A), \quad \forall A \in \mathcal{M}$

(d) KMS-symmetry:

$$(L(A)\xi_0, \sigma_{-i/2}(B)\xi_0) = (\sigma_{-i/2}(A)\xi_0, L(B)\xi_0), \quad \forall A, B \in \mathcal{M}.$$

Before producing the proof of the above theorem, let us discuss consequences of the theorem. Let $P_t = \exp(-tL)$ be the semigroup on $\mathcal{M}$ generated by L. Then $\{P_t\}_{t\ge0}$ has the following properties:

$$
\begin{aligned}
&\text{(a)}\quad \text{Unit preserving: } P_t(\mathbf{1}) = \mathbf{1}, \quad t \ge 0 \\
&\text{(b)}\quad \mathrm{Reality}: \mathrm{P_t(A^*) = P_t(A)^*}, \quad \forall \mathrm{t} \ge 0, \mathrm{A} \in \mathcal{M} \\
&\text{(c)}\quad \text{Positivity preserving: } 0 \le A \Rightarrow 0 \le P_t(A), \quad \forall t \ge 0, A \in \mathcal{M} \\
&\text{(d)}\quad \mathrm{KMS-symmetry}: \\
&\qquad\; (P_t(A)\xi_0\,, \sigma_{-i/2}(B)\xi_0) = (\sigma_{-i/2}(A)\xi_0\,,\, P_t(B)\xi_0), \quad \forall t \ge 0, A, B \in \mathcal{M}.
\end{aligned}
\tag{4.3}
$$

The properties (a), (b) and (d) follow from the properties (a), (b) and (d) of Theorem 4.1. The property (c) follows from Theorem 4.1 (c). See [MZ2,BR] for the details. Thus we have a KMS-symmetric Markovian semigroup $\{P_t\}_{t\geq 0}$ on $\mathcal{M}$.

Proof of Theorem 4.1. The properties (a), (b) and (d) follows from the definition of $L(A)$ in (4.2). We will show that (c) holds. In order to give the main idea of the proof clearly and also to avoid unnecessary notational complications, let us perform a formal calculation. A direct computation shows that

$$
\begin{aligned}
&\Delta^{-1/2}[\,\sigma_t(x), \Delta^{1/2}\,[\,\sigma_t(x)\,,\, A^*A\,]\,\Delta^{1/2}]\,\Delta^{-1/2} \\
&\quad = \Delta^{-1/2}[\sigma_t(x)\,,\, \Delta^{1/2}\,[\,\sigma_t(x), A^*\,]\Delta^{1/2}\Delta^{-1/2}A\Delta^{1/2}]\Delta^{-1/2} \\
&\qquad + \Delta^{-1/2}[\,\sigma_t(x)\,, \Delta^{1/2}A^*\Delta^{-1/2}\Delta^{1/2}[\,\sigma_t(x), A\,]\Delta^{1/2}]\Delta^{-1/2} \\
&\quad = \Delta^{-1/2}[\,\sigma_t(x)\,, \Delta^{1/2}[\,\sigma_t(x)\,,\, A^*\,]\,\Delta^{1/2}]\Delta^{-1/2}A \\
&\qquad + [\,\sigma_t(x),\, A^*\,]\Delta^{1/2}[\sigma_t(x)\,, \Delta^{-1/2}A\Delta^{1/2}]\Delta^{-1/2} \\
&\qquad + \Delta^{-1/2}[\sigma_t(x)\,, \Delta^{1/2}A^*\Delta^{-1/2}]\Delta^{1/2}[\,\sigma_t(x),\, A] \\
&\qquad + A^*\Delta^{-1/2}[\,\sigma_t(x)\,, \Delta^{1/2}[\,\sigma_t(x), A]\,\Delta^{1/2}]\Delta^{-1/2}.
\end{aligned}
$$

Thus it follows from (4.2) and the above relation that

$$
\begin{aligned}
L(A^*A) \;&=\; L(A^*)A + A^*L(A) \\
&\quad + \int [\,\sigma_t(x),\, A^*\,]\,[\,\sigma_{t-i/2}(x)\,,\, A]f(t)\,dt \\
&\quad + \int [\,\sigma_{t+i/2}(x), A^*\,]\,[\,\sigma_t(x)\,,\, A\,]\,f(t)\,dt
\end{aligned} \tag{4.4}
$$

Clearly the above relation can be obtained from the rigorous expression of $L(A)$ in (4.2). Using the assumption in Theorem 4.1 and the Cauchy integral theorem, we obtain from (4.4) that

$$
\begin{aligned}
&L(A^*A) - L(A^*)A - A^*L(A) \\
&= \int \Big([\,\sigma_{t+i/4}(x)\,, A^*]\,[\,\sigma_{t-i/4}(x),\, A\,]\Big)\Big(f(t+i/4) + f(t-i/4)\Big)\,dt \\
&= -\int [\,\sigma_{t-i/4}(x)\,, A]^*\,[\,\sigma_{t-i/4}(x),\, A\,]\Big(f(t+i/4) + f(t-i/4)\Big)\,dt \\
&\leq 0
\end{aligned}
$$

Here we have used the property (b) in Definition 3.1. This proves Theorem 4.1 completely. ■

5 Translation Invariant Markovian Semigroups for Quantum Spin Systems

Using the results in section 3, we construct gradient type Dirichlet forms for quantum spin systems, which generate translation invariant Markovian semigroups.

Let us describe quantum spin systems briefly. For the details, we refer to [BR, 6.2]. Let $\mathbb{Z}^d$ be a d-dimensional lattice space and let $\mathcal{F}$ denote the family of all finite subsets of $\mathbb{Z}^d$. By $\mathcal{F}_0$ we denote an increasing sequence of finite volumes invading all the lattice $\mathbb{Z}^d$.

Let $\mathcal{A}$ be a C^*-algebra with norm $||\cdot||$ defined as the inductive limit of a finite dimensional matrix algebra $\mathbb{M}$. for any $X \in \mathcal{F}$, let $\mathcal{A}_X$ denote the subalgebra localized in X, i.e. the subalgebra in $\mathcal{A}$ isomorphic to $\mathbb{M}^X$. An element $A \in \mathcal{A}$ will be called *local* if there is some $Y \in \mathcal{F}$ such that $A \in \mathcal{A}_Y$. By $\mathcal{A}_0$ we denote the subset of all local elements, i.e. $\mathcal{A}_0 = \cup_{X\in\mathcal{F}}\mathcal{A}_X$.

Let $\Phi =: \{\Phi_X\}_{X\in\mathcal{F}}$ be an interaction, i.e. a family of self-adjoint elements in $\mathcal{A}$. Suppose that

$$||\Phi||_\lambda =: \sup_{i\in\mathbb{Z}^d} \sum_{X\in\mathcal{F};i\in X} e^{\lambda|X|}||\Phi_X|| < \infty \tag{5.1}$$

for some λ, where $|X| = \text{card }(X)$. Define a derivation δ by

$$\begin{aligned} D(\delta) &= \mathcal{A}_0 \\ \delta(A) &= i\sum_{X\cap\Lambda\neq\emptyset}[\Phi_X, A], \quad A \in \mathcal{A}_\Lambda. \end{aligned} \tag{5.2}$$

Then $\mathcal{A}_0$ is a norm-dense *-subalgebra of analytic elements of the closure $\overline{\delta}$ of δ. Thus, $\overline{\delta}$ generates a strongly continuous one-parameter group of *-automorphism τ of $\mathcal{A}$. Let ω be a τ-KMS states corresponding to the interaction Φ.

Let $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ be the GNS representation of $(\mathcal{A}, \omega)$. For the standard form, we choose $\mathcal{H} = \mathcal{H}_\omega$, $\mathcal{M} = \pi(\mathcal{A})''$ and $\xi_0 = \Omega_\omega$. By the uniqueness of the modular automorphism [BR, Theorem 5.3.10], one may identify $\alpha_t = \tau_t,\ \ t \in \mathbb{R}$. In this section, we denote by $\mathcal{M}_0$ the algebra of local elements, i.e. $\mathcal{M}_0 = \pi(\mathcal{A}_0)$. We will write by A the element $\pi_\omega(A)$. Every element $A \in \mathcal{M}_0$ is an analytic element for α_t. For a given $\lambda > 0$, put $\beta_0 = (\frac{\lambda}{2||\Phi||_\lambda})$. Then for any $s \in (-\beta_0, \beta_0)$ the series

$$\alpha_{is}(A) = \sum_{n=0}^\infty \frac{(-is)^n}{n!}\delta^n(A), \quad A \in \mathcal{M}_0 \tag{5.3}$$

converge, where δ is the derivation given in (5.2). See the proof of [BR, Theorem 6.2.4]. Let us choose Φ sufficiently small so that $\beta_0 > \frac{1}{2}$.

We now turn to the construction of Dirichlet forms for quantum spin systems. Let $\{\tau_j\}_{j\in\mathbb{Z}^d}$ be the translational automorphism on $\mathcal{M}$ corresponding to the translation of the lattice by vectors $j \in \mathbb{Z}^d$. Let $x^a \in \pi_\omega(\mathbb{M})$, $a = 1, 2, \cdots, D$, be a basis consisting of self-adjoint elements of norm one and let $x_j^a = \tau_j(x^a), j \in \mathbb{Z}^d$. For a given admissible function f and x_j^a, let $\mathcal{E}_{a,j}$ be the Dirichlet form defined as in (3.5)

$$\mathcal{E}_{a,j}[\xi] = \int ||\Big(\sigma_{t-i/4}(x_j^a) - j(\sigma_{t-i/4}(x_j^a))\Big)\,\xi||^2\, f(t)\, dt, \tag{5.4}$$

Since $||\sigma_{t+is}(x)|| = ||\sigma_{is}(x)|| < \infty$ for any $s \in [-1/2, 1/2]$ and $x \in \mathcal{M}_0$, the assumption in Theorem 3.3 is satisfied. Define a quadratic form as follow:

$$\begin{aligned} D(\mathcal{E}) &= \{\xi \in \mathcal{H} : \sum_{j\in\mathbb{Z}^d}\sum_{a=1}^D \mathcal{E}_{a,j}[\xi] < \infty\} \\ \mathcal{E}[\xi] &= \sum_{j\in\mathbb{Z}^d}\sum_{a=1}^D \mathcal{E}_{a,j}[\xi], \quad \xi \in D(\mathcal{E}). \end{aligned} \tag{5.5}$$

Recall that an interaction $\Phi = \{\Phi_X\}_{X\in\mathcal{F}}$ is of finite range if there is $R > 0$ such that $\mathrm{dia}(X) > R$ implies $\Phi_X = 0$, and translation invariant if $\tau_j(\Phi_X) = \Phi_{X+j}$ for any $j \in \mathbb{Z}^d$. The following is the main result in this section:

Theorem 5.1. *Let f be an admissible function such that p in Definition 3.1(c) is greater than $d+1$, i.e. there exist $M > 0$ and $\epsilon > 0$ such that the bounds*

$$|f(t+is)| \le M(1+|t|)^{-(d+1+\epsilon)}$$

holds uniformly in $s \in [-1/4, 1/4]$. Let the interaction Φ is of finite range and translation invariant. Then the form $(\mathcal{E}, D(\mathcal{E}))$ is a Dirichlet form which generates a translation invariant Markovian semigroup.

Proof. By Theorem 3.3, each component $\mathcal{E}_{a,j}, a = 1, \cdots, D, j \in \mathbb{Z}^d$, is a Dirichlet form. Thus, by [Cip2, Theorem 5.2] it is sufficient to show that $D(\mathcal{E})$ is dense in $\mathcal{H}$.

We will show that $\Delta^{1/4}\mathcal{M}\xi_0$ is a dense subset contained in $D(\mathcal{E})$. Using α-KMS condition, it can be checked that

$$||\Delta^{1/4}A\xi_0|| \le ||A||, \quad A \in \mathcal{M}. \tag{5.6}$$

See also [MZ1, Theorem 2.4 (iii)]. Since $\Delta^{1/4}\mathcal{M}\xi_0$ is dense in $\mathcal{H}$, $\Delta^{1/4}\mathcal{M}_0\xi_0$ is also dense in $\mathcal{H}$ by the above inequality.

Next we show that $\Delta^{1/4}\mathcal{M}_0\xi_0$ is contained in $D(\mathcal{E})$. Notice that for any $A \in \mathcal{M}_0$ and $x = x^* \in \mathcal{M}_0$

$$\begin{aligned} ||\Big(\sigma_{t-i/4}(x) - j(\sigma_{t-i/4}(x))\Big)\Delta^{1/4}A\xi_0|| &= ||\Delta^{1/4}[\sigma_t(x),\, A]\,\xi_0\,|| \\ &\le ||\,[\sigma_t(x)\,,\, A]\,||. \end{aligned} \tag{5.7}$$

Here we have used the inequality (5.6). Thus it follows from (5.4), (5.5) and (5.7) that for $A \in \mathcal{M}_0$

$$\mathcal{E}[\Delta^{1/4}A\xi_0] \le \sum_{j\in\mathbb{Z}^d}\sum_{a=1}^{D}\int ||\,[\sigma_t(x_j^a),\, A]\,||^2 f(t)\,dt. \tag{5.8}$$

Replacing the hyper asymptotic abelianess in [MZ1] by the decay property of f in Theorem 5.1, and using the method employed in the proof of [MZ1, Theorem 5.5] it can be shown that the right hand side of (5.8) is finite. For the completeness, we produce the proof.

The following finite speed of propagation property for α_t is true for any $A \in \pi_\omega(\mathcal{A}_\Lambda)$, $\Lambda \in \mathcal{F}$,

$$||\,[\sigma_t(x_j^a),\, A]\,|| \le D(A)\exp\big(-\kappa(d(j,\Lambda) - \nu s)\big) \tag{5.9}$$

with some positive constant $D(A), \kappa$ and ν possibly dependent on $A \in \mathcal{M}_0[\mathrm{LR}]$. As in [MZ1], for a given $a \in \{1, 2, \cdots, D\}$ we represent the sum on the right hand side of (5.8) as follows:

$$\sum_{N\in\mathbb{N}}\ \sum_{N-1\le d(j,\Lambda)<N}\int ||\,[\sigma_t(x_j^a),\, A]\,||^2 f(t)\,dt = \mathrm{I}_1 + \mathrm{I}_2 \tag{5.10}$$

where

$$\begin{aligned}
\mathrm{I}_1 &= \sum_{N\in\mathbb{N}} \sum_{N-1\le d(j,\Lambda)<N} \int_{|t|\le \frac{N}{2\nu}} ||\,[\sigma_t(x_j^a),\, A]\,||^2 f(t)\,dt \\
\mathrm{I}_2 &= \sum_{N\in\mathbb{N}} \sum_{N-1\le d(j,\Lambda)<N} \int_{|t|> \frac{N}{2\nu}} ||\,[\sigma_t(x_j^a),\, A]\,||^2 f(t)\,dt. \qquad (5.11)
\end{aligned}$$

Now, using the finite speed of propagation (5.9), one has that

$$\mathrm{I}_1 \le \sum_{\mathrm{N}\in\mathbb{N}} \sum_{\mathrm{N}-1\le \mathrm{d(j,\Lambda)<N}} \frac{\mathrm{N}}{\nu}\mathrm{D(A)e^{-\kappa N}} < \infty \qquad (5.12)$$

On the other hand, we use the decay property of f to conclude that

$$\begin{aligned}
\mathrm{I}_2 &\le \sum_{N\in\mathbb{N}} \sum_{N-1\le d(j,\Lambda)<N} M(A) \int_{|t|> \frac{N}{2\nu}} (1+|t|)^{-(d+1+\epsilon)}\,dt \\
&\le \sum_{N\in\mathbb{N}} \sum_{N-1\le d(j,\Lambda)<N} M(A)(N+1)^{-(d+\epsilon)} \\
&< \infty, \qquad (5.13)
\end{aligned}$$

with some positive constant $M(A)$ dependent on A. Combining (5.10)-(5.13), we proved that the right hand side of (5.8) is finite for each $A \in \pi(\mathcal{A}_0)$. This ends the proof of Theorem 5.1. ■

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