THE REFLECTIVE LORENTZIAN LATTICES OF RANK 3

DANIEL ALLCOCK

ABSTRACT. We classify all the symmetric integer bilinear forms of signature $(2,1)$ whose isometry groups are generated up to finite index by reflections. There are 8595 of them up to scale, whose 374 distinct Weyl groups fall into 39 commensurability classes. This extends Nikulin's enumeration of the strongly square-free cases. Our technique is an analysis of the shape of the Weyl chamber, followed by computer work using Vinberg's algorithm and our "method of bijections". We also correct a minor error in Conway and Sloane's definition of their canonical 2-adic symbol.

1. INTRODUCTION

Lorentzian lattices, that is, integral symmetric bilinear forms of signature $(n,1)$, play a major role in K3 surface theory and the structure theory of hyperbolic Kac-Moody algebras. In both cases, the lattices which are reflective, meaning that their isometry groups are generated by reflections up to finite index, play a special role. In the KM case they provide candidates for root lattices of KMA's with hyperbolic Weyl groups that are large enough to be interesting. For K3's they are important because a K3 surface has finite automorphism group if and only if its Picard group is "2-reflective", meaning that its isometry group is generated up to finite index by reflections in classes with self-intersection -2 (see [20]). Even if the Picard group is not 2-reflective, but is still reflective, then one can often describe the automorphism group of the surface very explicitly [26][6][15]. The problem of classifying all reflective lattices of given rank is also of interest in its own right from the perspectives of Coxeter groups and arithmetic subgroups of $\mathrm{O}(n,1)$.

Date: February 22, 2011.

2000 *Mathematics Subject Classification.* 11H56 (20F55, 22E40).

Key words and phrases. Lorentzian lattice, Weyl group, Coxeter group, Vinberg's algorithm.

Partly supported by NSF grant DMS-0600112.

In this paper we give the classification of reflective Lorentzian lattices of rank 3, that is, of signature $(2, 1)$. There are 8595 of them up to scale, but this large number makes the result seem more complicated than it is. For example, 176 lattices share a single Weyl group (W_{68} in the table).

Our work is logically independent of Nikulin's monumental paper [16], but follows it conceptually. Most of our techniques run parallel to or are refinements of Nikulin's methods. His work had the same motivations as ours, and solves a problem both more general and more specific. He considered Lorentzian lattices that satisfy the more general condition of being "almost reflective", meaning that the Weyl chamber is small enough that its isometry group is $\mathbb{Z}^l$ by finite. This is a very natural class of lattices from both the K3 and KM perspectives. He also restricts attention to lattices that are strongly square-free (SSF), which in rank 3 means they have square-free determinant. The full classification is desirable from the KM perspective, because for example the root lattice of even a finite-dimensional simple Lie algebra need not be SSF. On the other hand, every lattice canonically determines a SSF lattice, so that his classification does yield all lattices whose reflection groups are maximal under inclusion. The SSF condition also enabled Nikulin to use some tools from the theory of quadratic forms that we avoid. There are 1097 lattices in Nikulin's 160 SSF lattices' duality classes (defined in section 2), while our 8595 lattices fall into 1419 duality classes. In fact, as Nikulin explains, his 1097 lattices are exactly the "elementary" ones, i.e., those with discriminant group a product of cyclic groups of prime order. The reason is that each duality class containing an elementary lattice consists entirely of such lattices and contains a unique SSF lattice.

We have striven to keep the table of lattices to a manageable size, while remaining complete in the sense that the full classification can be mechanically recovered from the information we give. So we have printed only 704 of them and given operations to obtain the others. Even after this reduction, the table is still large. The unabridged list of lattices is available separately as [1], including the Gram matrices if desired. Anyone wanting to work seriously with the lattices will need computer-readable data, which is very easy to extract from the TeX source file of this paper. Namely, by arcane trickery we have arranged for the TeX source to be simultaneously a Perl script that prints out all 8595 lattices (or just some of them) in computer-readable format. If the file is saved as `file.tex` then simply enter `perl file.tex` at the unix command line.

For each reflective lattice L, we worked out the conjugacy class of its Weyl group $W(L)$ in $\mathrm{O}(2,1)$, the normalizers of $W(L)$ in $\mathrm{Aut}\,L$ and $\mathrm{O}(2,1)$, the area of its Weyl chamber, its genus as a bilinear form over $\mathbb{Z}$, one construction of L for each corner of its chamber, and the relations among the lattices under p-duality and p-filling (defined in section 2). Considerable summary information appears in section 7, for example largest and smallest Weyl chambers, most and fewest lattices with given Weyl group, most edges (28), numbers of chambers with interesting properties like regularity or compactness, genera containing multiple reflective lattices, and the most-negative determinants and largest root norms.

Here is a brief survey of related work. Analogues of Nikulin's work have been done for ranks 4 and 5 by R. Scharlau [21] and C. Walhorn [29]; see also [23]. In rank ≥ 21, Esselmann [11] proved there are only 2 reflective lattices, both in rank 22—the even sublattice of $\mathbb{Z}^{21,1}$ (recognized as reflective by Borcherds [2]) and its dual lattice. Also, Nikulin [13] proved that there are only finitely many reflective lattices in each rank, so that a classification is possible. The lattices of most interest for K3 surfaces are the 2-reflective ones, and these have been completely classified by Nikulin [17][18] and Vinberg [27].

Beyond these results, the field is mostly a long list of examples found by various authors, with only glimmers of a general theory, like Borcherds' suggested almost-bijection between "good" reflection groups and "good" automorphic forms [4, §12]. To choose a few highlights, we mention Coxeter and Whitrow's analysis [10] of $\mathbb{Z}^{3,1}$, Vinberg's famous paper [25] treating $\mathbb{Z}^{n,1}$ for $n \leq 17$, its sequel with Kaplinskaja [28] extending this to $n \leq 19$, Conway's analysis [7] of the even unimodular Lorentzian lattice of rank 26, and Borcherds' method of deducing reflectivity of a lattice from the existence of a suitable automorphic form [5]. Before this, Vinberg's algorithm [25] was the only systematic way to prove a given lattice reflective.

Here is an overview of the paper. After providing background material in section 2, we consider the shape of a hyperbolic polygon P in section 3. The main result is the existence of 3, 4 or 5 consecutive edges such that the lines in the hyperbolic plane H^2 containing them are "not too far apart". This is our version of Nikulin's method of narrow parts of polyhedra [16, §4][13], and some of the same constants appear. Our technique is based the bisectors of angles and edges of P, while his is based on choosing a suitable point of P's interior and considering how the family of lines through it meet ∂P. Our method is simpler and improves certain bounds, but has no extension to higher-dimensional hyperbolic spaces, while his method works in arbitrary dimensions. For a more detailed comparison, please see the end of section 3.

In section 4 we convert these bounds on the shape of the Weyl chamber into an enumeration of inner product matrices of 3, 4 or 5 consecutive simple roots of $W(L)$. In section 5 we introduce "corner symbols". Essentially, given L and two consecutive simple roots r and s, we define a symbol like $2^*_4 4$ or $26^{4,3}_{\infty b}104$ displaying their norms, with the super- and subscripts giving enough information to recover the inclusion $(r,s) \to L$. So two lattices, each with a chamber and a pair of consecutive simple roots, are isometric by an isometry identifying these data if and only if the corner symbols and lattice determinants coincide. Some device of this sort is required, even for sorting the reflective lattices into isometry classes, because the genus of a lattice is not a strong enough invariant to distinguish it. In one case (see section 7), two lattices with the same genus even have different Weyl groups. We also use corner symbols in the proof of the main theorem (theorem 13 in section 6) and for tabulating our results.

Besides the proof of the main theorem, section 6 also introduces what we call the "method of bijections", which is a general algorithm for determining whether a finite-index subgroup of a Coxeter group is generated by reflections up to finite index. Section 7 explains how to read the table at the end of the paper. That table displays 704 lattices explicitly, together with operations to apply to obtain the full list of lattices. Finally, in section 8 we discuss the effects of various "moves" between lattices on the Conway-Sloane genus symbol [9, ch. 15]. We also resolve a small error in their definition of the canonical 2-adic symbol.

Computer calculations were important at every stage of this project, as explained in detail in section 6. We wrote our programs in C++, using the PARI software library [19], and the full classification runs in around an hour on the author's laptop computer. The source code is available on request. Although there is no a posteriori check, like the mass formula used when enumerating unimodular lattices [8], we can offer several remarks in favor of the reliability of our results. First, in the SSF case we recovered Nikulin's results exactly; second, many steps of the calculation provided proofs of their results (e.g., that a lattice is reflective or not); and third, our list of lattices is closed under the operations of p-duality and p-filling (see section 2) and "mainification" (section 7). This is a meaningful check because our enumeration methods have nothing to do with these relations among lattices.

I am grateful to the Max Planck Institute in Bonn, where some of this work was carried out.

2. Background

For experts, the key points in this section are (i) lattices need not be integral and (ii) the definition of "reflective hull". The notation $\langle x,\ldots,z\rangle$ indicates the $\mathbb{Z}$-span of some vectors.

Lattices: A lattice means a free abelian group L equipped with a $\mathbb{Q}$-valued symmetric bilinear form, called the inner product. If L has signature $(n,1)$ then we call it Lorentzian. If the inner product is $\mathbb{Z}$-valued then L is called integral. In this case we call the gcd of L's inner products the scale of L. We call L unscaled if it is integral and its scale is 1. The norm r^2 of a vector means its inner product with itself, and r is called primitive if L contains none of the elements r/n, $n>1$. L is called even if all its vectors have even norm, and odd otherwise. If L is nondegenerate then its dual, written L^*, means the set of vectors in $L\otimes\mathbb{Q}$ having integer inner products with all of L. So L is integral just if $L\subseteq L^*$. In this case L's discriminant group is defined as $\Delta(L):=L^*/L$, a finite abelian group. The determinant $\det L$ means the determinant of any inner product matrix, and when L is integral we have $|\det L|=|\Delta(L)|$.

Roots: A root means a primitive lattice vector r of positive norm, such that reflection in r, given by

$$x\mapsto x-2\frac{x\cdot r}{r^2}r$$

preserves L. Under the primitivity hypothesis, this condition on the reflection is equivalent to $L\cdot r\subseteq \frac{1}{2}r^2\mathbb{Z}$. Now suppose $r_1,\ldots,r_n$ are roots in a lattice L, whose span is nondegenerate. Then their reflective hull $\langle r_1,\ldots,r_n\rangle^{\mathrm{rh}}$ means

$$\langle r_1,\ldots,r_n\rangle^{\mathrm{rh}}:=\{v\in\langle r_1,\ldots,r_n\rangle\otimes\mathbb{Q}\,:\,v\cdot r_i\in\tfrac{1}{2}r_i^2\mathbb{Z}\text{ for all }i\}.$$

The key property of the reflective hull is that it contains the projection of L to $\langle r_1,\ldots,r_n\rangle\otimes\mathbb{Q}$. In the special case that $r_1,\ldots,r_n$ generate L up to finite index, we have

$$\langle r_1,\ldots,r_n\rangle\subseteq L\subseteq\langle r_1,\ldots,r_n\rangle^{\mathrm{rh}}.$$

Then there are only finitely many possibilities for L, giving us some control over a lattice when all we know is some of its roots. As far as I know, the reflective hull was first introduced in [22], in terms of the "reduced discriminant group". As examples and for use in the proof of theorem 10, we mention that if r and s are simple roots for an A_1^2, A_2, B_2 or G_2 root system, then $\langle r,s\rangle^{\mathrm{rh}}/\langle r,s\rangle$ is $(\mathbb{Z}/2)^2$, $\mathbb{Z}/3$, $\mathbb{Z}/2$ or trivial, respectively.

Weyl Group: We write $W(L)$ for the Weyl group of L, meaning the subgroup of $O(L)$ generated by reflections in roots. When L is Lorentzian, $O(L)$ acts on hyperbolic space H^n, which is defined as the image of the negative-norm vectors of $L \otimes \mathbb{R}$ in $P(L \otimes \mathbb{R})$. For r a root, we refer to $r^\perp \subseteq L \otimes \mathbb{R}$ or the corresponding subset of H^n as r's mirror. The mirrors form a locally finite hyperplane arrangement in H^n, and a Weyl chamber (or just chamber) means the closure in H^n of a component of the complement of the arrangement. Often one chooses a chamber C and calls it "the" Weyl chamber. The general theory of Coxeter groups (see e.g. [24]) implies that C is a fundamental domain for $W(L)$. Note that C may have infinitely many sides, or even be all of H^n (if L has no roots). A Lorentzian lattice L is called reflective if C has finite volume. The purpose of this paper is to classify all reflective lattices of signature $(2,1)$. For a discrete group Γ of isometries of H^n, not necessarily generated by reflections, its reflection subgroup means the group generated by its reflections, and the Weyl chamber of Γ means the Weyl chamber of this subgroup.

Simple roots: The preimage of C in $\mathbb{R}^{2,1} - \{0\}$ has two components; choose one and call it $\tilde{C}$. For every facet of $\tilde{C}$, there is a unique outward-pointing root r orthogonal to that facet (outward-pointing means that $r^\perp$ separates r from $\tilde{C}$). These are called the simple roots, and their pairwise inner products are ≤ 0. Changing the choice of component simply negates the simple roots, so the choice of component will never matter to us. These considerations also apply to subgroups of $W(L)$ that are generated by reflections.

Vinberg's algorithm: Given a Lorentzian lattice L, a negative-norm vector k (called the controlling vector) and a set of simple roots for the stabilizer of k in $W(L)$, there is a unique extension of these roots to a set of simple roots for $W(L)$. Vinberg's algorithm [25] finds this extension, by iteratively adjoining batches of new simple roots. If L is reflective then there are only finitely many simple roots, so after some number of iterations the algorithm will have found them all. One can recognize when this happens because the polygon defined by the so-far-found simple roots has finite area. Then the algorithm terminates. On the other hand, if C has infinite area then the algorithm will still find all the simple roots, but must run forever to get them. In this case one must recognize L as non-reflective, which one does by looking for automorphisms of L preserving C. See section 6 for the details of the method we used. There is a variation on Vinberg's algorithm using a null vector as controlling vector [7], but the original algorithm was sufficient for our purposes.

p-duality: For p a prime, the p-dual of a nondegenerate lattice L is the lattice in $L \otimes \mathbb{Q}$ characterized by

$$p\text{-dual}(L) \otimes \mathbb{Z}_q = \begin{cases} L^* \otimes \mathbb{Z}_q & \text{if } q = p \\ L \otimes \mathbb{Z}_q & \text{if } q \neq p \end{cases}$$

for all primes q, where $\mathbb{Z}_q$ is the ring of q-adic integers. Applying p-duality twice recovers L, so L and its p-dual have the same automorphism group. When L is integral one can be much more concrete: p-dual(L) is the sublattice of L^* corresponding to the p-power part of $\Delta(L)$.

In our applications L will be unscaled, so the p-power part of $\Delta(L)$ has the form $\oplus_{i=1}^n \mathbb{Z}/p^{a_i}$ where $n = \dim L$ and at least one of the a_i's is 0. In this case we define the "rescaled p-dual" of L as p-dual(L) with all inner products multiplied by p^a, where $a = \max\{a_1, \dots, a_n\}$. This lattice is also unscaled, and the p-power part of its discriminant group is $\oplus_{i=1}^n \mathbb{Z}/p^{a-a_i}$. Repeating the operation recovers L. We say that unscaled lattices are in the same duality class if they are related by a chain of rescaled p-dualities. Our main interest in rescaled p-duality is that all the lattices in a duality class have the same isometry group, so we may usually replace any one of them by a member of the class with smallest $|\det|$. We will usually suppress the word "rescaled" except for emphasis, since whether we intend p-duality or rescaled p-duality will be clear from context. The effect of p-duality on the Conway-Sloane genus symbol is explained in section 8.

p-filling: Suppose L is an integral lattice, p is a prime, and $\Delta(L)$ has some elements of order p^2. Then we define the p-filling of L as the sublattice of L^* corresponding to $p^{n-1}A \subseteq \Delta(L)$, where A is the p-power part of $\Delta(L)$ and p^n is the largest power of p among the orders of elements of $\Delta(L)$. One can check that p-fill(L) is integral, and obviously its determinant is smaller in absolute value. The operation is not reversible, but we do have the inclusion $\operatorname{Aut} L \subseteq \operatorname{Aut}(p\text{-fill}(L))$. As for p-duality, the effect of p-filling on the Conway-Sloane genus symbol is explained in section 8.

SSF lattices: Given any integral lattice L, one can apply p-filling operations until no more are possible, arriving at a lattice whose discriminant group is a sum of cyclic groups of prime order. Then applying rescaled p-duality for some primes p leaves a lattice whose discriminant group has rank (i.e., cardinality of its smallest generating set) at most $\frac{1}{2}\dim L$. Such a lattice is called strongly square-free (SSF). For rank 3 lattices this is equivalent to $|\Delta(L)|$ being square-free. Nikulin's paper [16] classified the SSF reflective Lorentzian lattices of rank 3. Watson

[30][31] introduced operations leading from any lattice to a SSF one, but ours are not related to his in any simple way.

3. The shape of a hyperbolic polygon

In this section we develop our version of Nikulin's method of narrow parts of polyhedra [13][16]. We will prove the following theorem, asserting that any finite-sided finite-area polygon P in H^2 has one of three geometric properties, and then give numerical consequences of them. These consequences have the general form: P has several consecutive edges, such that the distances between the lines containing them are less than some a priori bounds. See the remark at the end of the section for a detailed comparison of our method with Nikulin's.

Theorem (and Definition) 1. *Suppose P is a finite-sided finite-area polygon in H^2. Then one of its edges is* **short**, *meaning that the angle bisectors based at its endpoints cross. Furthermore, one of the following holds:*

(i) *P has a short edge orthogonal to at most one of its neighbors, with its neighbors not orthogonal to each other.*
(ii) *P has at least 5 edges and a* **short pair** *(S, T), meaning: S is a short edge orthogonal to its neighbors, T is one of these neighbors, and the perpendicular bisector of S meets the angle bisector based at the far vertex of T.*
(iii) *P has at least 6 edges and a* **close pair** *of short edges $\{S, S'\}$, meaning: S and S' are short edges orthogonal to their neighbors, their perpendicular bisectors cross, and some* **long** *(i.e., not short) edge is adjacent to both of them.*

For our purposes, the perpendicular bisector of a finite-length edge of a polygon P means the ray orthogonal to that edge, departing from the midpoint of that edge, into the interior of P. The angle bisector at a finite vertex is a ray defined in the usual way, and the angle bisector at an ideal vertex means that vertex together with the set of points equidistant from (the lines containing) the two edges meeting there. We think of it as a ray based at the ideal vertex.

Lemma 2. *Suppose P is a convex polygon in $H^2 \cup \partial H^2$ and $x_1, \ldots, x_{n\geq 2}$ are distinct points of ∂P, numbered consecutively in the positive direction around ∂P, with subscripts read modulo n. Suppose a ray b_i departs from each x_i, into the interior of P. Then for some neighboring pair b_i, b_{i+1} of these rays, either they meet or one of them meets the open arc (x_i, x_{i+1}) of ∂P.*

Remarks. When speaking of arcs of ∂P, we always mean those traversed in the positive direction. Also, the proof works for the closure of any open convex subset of H^2; one should interpret ∂P as the topological boundary of P in $\mathbb{R}P^2$.

Proof. By convexity of P and the fact that b_i enters the interior of P, its closure $\bar{b}_i$ meets ∂P at one other point y_i. (We refer to $\bar{b}_i$ rather than b_i because y_i might lie in ∂H^2.) Now, y_i lies in exactly one arc of ∂P of the form $(x_j, x_{j+1}]$, and we define s_i as the number of steps from x_i to x_j. Formally: s_i is the number in $\{0, \ldots, n-1\}$ such that $i + s_i \equiv j \bmod n$.

Now choose i with s_i minimal. If $s_i = 0$ then b_i meets (x_i, x_{i+1}) or contains $x_{i+1} \in b_{i+1}$ and we're done, so suppose $s_i > 0$. Now, either b_{i+1} meets (x_i, x_{i+1}), so we're done, or it meets b_i, so we're done, or else y_{i+1} lies in $(x_{i+1}, y_i] \subseteq (x_{i+1}, x_{j+1}]$. In the last case $s_{i+1} < s_i$, so we have a contradiction of minimality. □

Proof of theorem 1. To find a short edge, apply lemma 2 with $x_1, \ldots, x_n$ the vertices of P and b_i the angle bisector at x_i. No b_i can meet $[x_{i-1}, x_i)$ or $(x_i, x_{i+1}]$, so some neighboring pair of bisectors must cross, yielding a short edge.

Suppose for the moment that some short edge is non-orthogonal to at least one of its neighbors. Then conclusion (i) applies, except if its neighbors, call them T and T', are orthogonal to each other. This can only happen if P is a triangle. In a triangle all edges are short and at most one angle equals $\pi/2$. Therefore conclusion (i) applies with T or T' taken as the short edge.

We suppose for the rest of the proof that every short edge of P is orthogonal to its neighbors, because this is the only case where anything remains to prove.

Next we show that P has ≥ 5 sides; suppose otherwise. We already treated triangles, so suppose P is a quadrilateral. Because H^2 has negative curvature, P has a vertex with angle $\neq \pi/2$; write E and F for the edges incident there. The angle bisector based there meets one of the other two edges (perhaps at infinity), say the one next to E. Drawing a picture shows that E is short. So P has a short edge involved in an angle $\neq \pi/2$, a contradiction.

Next we claim that if S and S' are adjacent short edges, say with S' at least as long as S, then (S, S') is a short pair. Write b for the perpendicular bisectors of S, γ for the angle bisector at $S \cap S'$, and α (resp. α') for the angle bisector at the other vertex of S (resp. S'). We hope the left half of figure 3 will help the reader; the intersection point whose existence we want to prove is marked. Now, α meets γ because

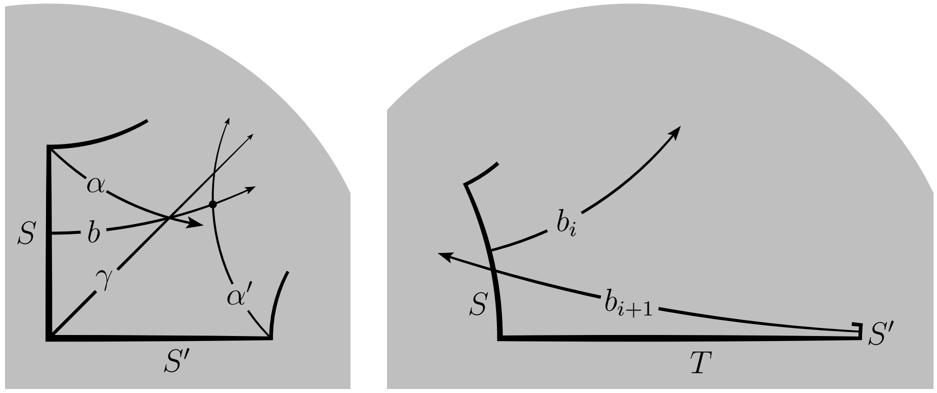

FIGURE 1. See the proof of theorem 1.

S is short, and b passes though $\alpha \cap \gamma$ because it is the perpendicular bisector of an edge whose two angles are equal. Similarly, γ meets α' because S' is short. Furthermore, the intersection point $\alpha' \cap \gamma$ is at least as far away from $S \cap S'$ as $\alpha \cap \gamma$ is, because S' is at least as long as S. If S' is strictly longer than S, then b enters the interior of the triangle bounded by γ, α' and S' by passing through γ. Since b cannot meet S' (by $S \perp S'$), it must meet α' as it exits the triangle. So (S, S') is a short pair. (In the limiting case where S and S' have the same length, α, b, γ and α' all meet at a single point.)

Now an argument similar to the one used for quadrilaterals shows that a pentagon has a short pair. Namely, suppose S is a short edge, and write T, T' for its neighbors and U, U' for their other neighbors. The perpendicular bisector of S cannot meet T or T' (since $S \perp T, T'$), so it meets U or U' (perhaps at infinity), say U. Then the angle bisector at $T \cap U$ either meets b, so (S, T) is a short pair, or it meets S. In the latter case, T is also short, and the previous paragraph assures us that either (S, T) or (T, S) is a short pair. This finishes the treatment of a pentagon, so we take P to have ≥ 6 edges in the rest of the proof.

At this point it suffices to prove that P has a short pair, or a close pair of short edges. We apply lemma 2 again. Let $x_1, \dots, x_n$ be the midpoints of the short edges and the vertices not on short edges, in whatever order they occur around ∂P. If x_i is the midpoint of an edge, let b_i be the perpendicular bisector of that edge. If x_i is a vertex not on a short edge, let b_i be the angle bisector there. Lemma 2 now gives us some i such that either b_i and b_{i+1} meet, or one of them meets (x_i, x_{i+1}). We now consider the various possibilities:

If both are angle bisectors then x_i and x_{i+1} must be the endpoints of one edge, or else there would be some other x_j between them. An angle bisector based at an endpoint of the edge cannot meet the interior of that edge, so b_i and b_{i+1} must cross, so the edge must be short. But this is impossible because then b_i and b_{i+1} would not be among $b_1, \ldots, b_n$. So this case cannot occur.

If one, say b_i, is the perpendicular bisector of a short edge S, and the other is an angle bisector, then we may suppose the following holds: the edge T after S is long, and b_{i+1} is the angle bisector based at the endpoint of T away from S. Otherwise, some other x_j would come between x_i and x_{i+1}. If b_i and b_{i+1} meet then (S, T) is a short pair. Also, b_i cannot meet (x_i, x_{i+1}) since $S \perp T$. The only other possibility is that b_{i+1} crosses S between x_i and $S \cap T$. But in this case the angle bisectors based at the endpoints of T would cross, in which case T is short, a contradiction.

Finally, suppose b_i and b_{i+1} are the perpendicular bisectors of short edges S, S'. If S, S' are adjacent then one of the pairs (S, S'), (S', S) is a short pair; so we suppose they are not adjacent. There must be exactly one edge between them, which must be long, for otherwise some other x_j would come between x_i and x_{i+1}. If b_i and b_{i+1} cross then S and S' forms a close pair of short edges and we are done. The only other possibility (up to exchanging $b_i \leftrightarrow b_{i+1}$, $S \leftrightarrow S'$) is pictured in the right half of figure 3. Namely, b_{i+1} meets S between $S \cap T$ and the midpoint of S. In this case, the angle bisector based at $S \cap T$ either meets b_{i+1}, so (S', T) is a short pair, or it meets S', which implies that T is short, a contradiction. □

In lemmas 3–5 we give numerical inequalities expressing the geometric configurations from theorem 1. We have tried to formulate the results as uniformly as possible. In each of the cases (short edge, short pair, close pair of short edges), we consider three edges R, T, R' of a convex polygon in H^2, consecutive except that there might be a short edge S (resp. S') between R (resp. R') and T. We write $\hat{r}, \hat{t}, \hat{r}'$ for the corresponding outward-pointing unit vectors in $\mathbb{R}^{2,1}$ (and $\hat{s}, \hat{s}'$ when S, S' are present). We define $\mu = -\hat{r} \cdot \hat{t}$ and $\mu' = -\hat{r}' \cdot \hat{t}$, and the hypotheses of the lemmas always give upper bounds on them (e.g., if R, T are adjacent then $\mu \leq 1$). The main assertion of each lemma is a constraint on $\lambda := -\hat{r} \cdot \hat{r}'$, namely

$$\begin{aligned} \lambda < K &:= 1 + \mu + \mu' + 2\sqrt{1+\mu}\sqrt{1+\mu'} \\ &= \big(\sqrt{1+\mu} + \sqrt{1+\mu'}\big)^2 - 1. \end{aligned} \tag{1}$$

This may be regarded as giving an upper bound on the hyperbolic distance between (the lines containing) R and R'. The bound (1) plays a crucial role in section 4.

Lemma 3. *Suppose R, T, R' are consecutive edges of a polygon in H^2. Then $K \leq 7$, and T is short if and only if $\lambda < K$.*

Proof. Observe that $\hat{r} - \hat{t}$ and $\hat{r}' - \hat{t}$ are vectors orthogonal to the angle bisectors based at $R \cap T$ and $R' \cap T$; in fact they point outward from the region bounded by T and these two bisectors. The bisectors intersect if and only if

$$(\hat{r} - \hat{t}\,) \cdot (\hat{r}' - \hat{t}\,) < -\sqrt{(\hat{r} - \hat{t}\,)^2}\sqrt{(\hat{r}' - \hat{t}\,)^2}.$$

(This is the condition that $\hat{r} - \hat{t}$ and $\hat{r}' - \hat{t}$ have positive-definite inner product matrix.) Using $\hat{r}^2 = \hat{r}'^2 = \hat{t}^2 = 1$ and the definitions of μ and μ', then rearranging, yields $\lambda < K$. And K is largest when μ and μ' are, namely when T is parallel to R, R', yielding $\mu = \mu' = 1$ and $K = 7$. □

If a short edge is orthogonal to its neighbors, then its neighbors' unit vectors' inner product lies in $(-3, -1)$. This is because $K = 3$ when $\mu = \mu' = 0$ in lemma 3. We will use this in the next two lemmas.

Lemma 4. *Suppose R, S, T, R' are distinct consecutive edges of a polygon in H^2, with S short and orthogonal to R and T. Then $\mu \in (1, 3)$ and $K < 5 + 4\sqrt{2} \approx 10.657...$, and a necessary condition for (S, T) to be a short pair is $\lambda < K$. Furthermore, the projection s_0 of $\hat{r}'$ to the $\mathbb{Q}$-span of $\hat{s}$ satisfies*

$$s_0^2 = s_0 \cdot \hat{r}' = 1 + \frac{\lambda^2 + 2\lambda\mu\mu' + \mu'^2}{\mu^2 - 1}. \tag{2}$$

Remark. In this lemma, $\lambda < K$ is necessary but not sufficient, while in lemma 3 it was both. The reason for the difference is that here, if T is short enough then the angle bisector at $T \cap R'$ will exit P before it can cross the perpendicular bisector of S. A similar phenomenon occurs in the next lemma also.

Proof. First we note that $\mu > 1$ since R and T are ultraparallel. Also, $\mu < 3$ by the remark before the lemma. The proof that $\lambda < K$ is exactly the same as in the previous proof, using the fact that $\hat{r} - \hat{t}$ is orthogonal to the perpendicular bisector of S. (Lemmas 3–5 acquire a certain unity when the perpendicular bisector of S is regarded as the "angle bisector" of the ultraparallel edges R and T.) The inequality $K < 5 + 4\sqrt{2}$ follows from $\mu < 3, \mu' \leq 1$.

Next, the $\mathbb{Q}$-span of $\hat{s}$ is $\langle \hat{r}, \hat{t}\,\rangle^\perp$, so s_0 is got from $\hat{r}'$ by adding suitable multiples of $\hat{r}$ and $\hat{t}$. Computation reveals

$$s_0 = \hat{r}' + \frac{\lambda + \mu\mu'}{1 - \mu^2}\hat{r} + \frac{\lambda\mu + \mu'}{1 - \mu^2}\hat{t},$$

and then one can work out $s_0 \cdot \hat{r}'$. Finally, $s_0^2 = s_0 \cdot \hat{r}'$ because s_0 was defined as an orthogonal projection of $\hat{r}'$. ☐

Lemma 5. *Suppose R, S, T, S', R' are distinct consecutive edges of a polygon in H^2, and assume S and S' are short and orthogonal to their neighbors. Then $\mu, \mu' \in (1, 3)$, $K < 15$, and a necessary condition for (S, S') to be a close pair of short edges is $\lambda < K$. Furthermore, the projection s_0 of $\hat{r}'$ to the $\mathbb{Q}$-span of $\hat{s}$ satisfies (2), and similarly with primed and unprimed letters exchanged. Finally,*

$$s_0 \cdot s_0' = \lambda + \mu\mu' - \frac{(\lambda + \mu\mu')^3}{(\mu^2 - 1)(\mu'^2 - 1)}. \tag{3}$$

Proof. Essentially the same as the previous one. The computation of $s_0 \cdot s_0'$ is long but mechanical. ☐

Remarks. The constants 7, $5+4\sqrt{2}$ and 15 appearing in lemmas 3–5 are essentially the same as the constants 14, $10 + 8\sqrt{2}$ and 30 in Nikulin's (4.1.53), (4.1.55) and (4.1.58) in [16, Theorem 4.1.8]. The factor of 2 comes from the fact that he uses norm 2 vectors where we use unit vectors. And our short edge, short pair, close pair cases in theorem 1 correspond to his type I, II and III narrow parts of polygons. In the short edge case, our condition that the short edge be non-orthogonal to one of its neighbors corresponds to Nikulin's condition [16, Theorem 4.1.4] that the Dynkin diagram of the 3 roots be connected.

Our results simplify his slightly in that we can suppose P has ≥ 5 (resp. 6) edges in the short pair (resp. close pair) cases. More important is our bound on $\hat{r} \cdot \hat{r}'$ in terms of $\hat{r} \cdot \hat{t}$ and $\hat{r}' \cdot \hat{t}$, which we believe is optimal. The analogous bounds in Nikulin's paper are the second part of (4.1.4), formula (4.1.10) or (4.1.55), and formula (4.1.13). Our bounds are better and simpler in the short pair and close pair cases. They limit the size of the enumeration in the next section enough that we can avoid considering class numbers of imaginary quadratic fields, which Nikulin uses in his section 3.2.

4. The shape of a 2-dimensional Weyl chamber

In this section we consider a Lorentzian lattice L of rank 3, and assume that its Weyl chamber C possesses one of the features (i)–(iii) from theorem 1. This always holds if C has finite area. The goal is to restrict L to one of finitely many possibilities (up to scale). This is achieved in section 6, but we do most of the work here, by showing that the simple roots corresponding to 3, 4 or 5 suitable consecutive edges have one of finitely many inner product matrices (up to scale). The point is not really this finiteness, which was already known by Nikulin's finiteness theorem [13]. What is important is that all possibilities are worked out explicitly. The main tool is lemma 6 below, which allows one to obtain information about two roots of L, given information about the corresponding unit vectors.

In fact we will work with "quasiroots" of L rather than roots; the point is that the methods of this section can be used to gain control of the 3-dimensional lattice associated to a finite-area 2-dimensional face of a higher-dimensional Weyl chamber. The author intends to use this in higher-dimensional analogues of this paper. For purposes of classifying rank 3 reflective lattices, we will not need this generalization, so the reader may read "root" in place of "quasiroot" and assume that ρ, σ, τ, σ' and ρ' are always 2. For use in the general case, here is the definition. If L is any lattice then we say that $a \in L$ is a quasiroot of L of weight $\alpha \in \{1, 2, \dots\}$ if a is primitive, $a^2 > 0$, and $L \cdot a \subseteq \frac{1}{\alpha} a^2 \mathbb{Z}$. Notice that scaling a lattice up or down doesn't affect the quasiroot weights.

When a roman letter represents a quasiroot we will use the corresponding greek letter for its weight. A quasiroot of weight 2 is the same thing as a root, and quasiroot of weight 1 is the same thing as a generator of a 1-dimensional positive-definite summand of L. Of course, every primitive positive-norm vector of L is a quasiroot of weight equal to its norm. But sometimes one has an a priori bound on the weights of quasiroots involved, independent of their norms.

Lemma 6. *Suppose a, b are quasiroots of weights α, β in a lattice L, whose associated unit vectors satisfy $\hat{a} \cdot \hat{b} = -K$ for some $K > 0$. Then the inner product matrix of a and b is a rational number times*

$$\begin{pmatrix} \alpha u & -uv \\ -uv & \beta v \end{pmatrix}, \tag{4}$$

for some positive integers u, v satisfying $uv = K^2 \alpha\beta$.

We will also use versions of this lemma with both $=$ signs replaced by $\leq$ or both by $<$. These are formal consequences of this version.

Proof. We know $a \cdot b \in \frac{1}{\alpha} a^2 \mathbb{Z}$ and $a \cdot b \in \frac{1}{\beta} b^2 \mathbb{Z}$, so

$$a \cdot b = -va^2/\alpha \qquad \text{and} \qquad a \cdot b = -ub^2/\beta \tag{5}$$

for some positive integers u, v. This gives

$$-a \cdot b = \sqrt{(-a \cdot b)^2} = \sqrt{(va^2/\alpha)(ub^2/\beta)} = \sqrt{a^2}\sqrt{b^2}\sqrt{uv/\alpha\beta}.$$

Rewriting $-\hat{a} \cdot \hat{b} = K$ as $-a \cdot b = K\sqrt{a^2}\sqrt{b^2}$ yields $uv = K^2 \alpha\beta$. For convenience we scale the inner products so that $a \cdot b = -uv$, and then (5) gives $a^2 = \alpha u$ and $b^2 = \beta v$. $\square$

The hypothesis $a \cdot b \neq 0$ in the lemma is essential and annoying: when two quasiroots are orthogonal, their norms need not satisfy any relation at all. For example, an essential part of the proof of lemma 7 uses $R \not\perp T, R'$ to relate the norms of r, t and r'. If we could do away with the non-orthogonality hypothesis in lemma 6 then we could carry out our enumeration of lattices using only the existence of a short edge. The possibility of orthogonality is what forces us to consider the more-complicated configurations of theorem 1.

The setup for lemmas 7–9 below is similar to that for lemmas 3–5. Their hypotheses are exactly the "outputs" of the short edge, short pair and close pair cases of theorem 1. P is a finite-sided finite-area polygon in H^2, with angles at most $\pi/2$, whose edges are the orthogonal complements of quasiroots of a rank 3 Lorentzian lattice L. R, T and R' are distinct edges of P, consecutive except that there might be a short edge S (resp. S') between R (resp. R') and T. We write r, t, r' (and s, s' when S, S' are present) for the outward-pointing quasiroots of L orthogonal to $R, \dots, R'$. We suppose that r, t and r' (and s and s' if present) are quasiroots of weights ρ, τ and ρ' (and σ and σ' if relevant). As usual, we use hats to indicate the corresponding unit vectors. As in section 3 we define $\mu = -\hat{r} \cdot \hat{t}$, $\mu' = -\hat{r}' \cdot \hat{t}$,

$$K = 1 + \mu + \mu' + 2\sqrt{1+\mu}\sqrt{1+\mu'},$$

and $\lambda = -\hat{r} \cdot \hat{r}'$. In lemmas 7–9 μ, μ' and λ will be given by the formulas $\mu = \sqrt{AB/\rho\tau}$, $\mu' = \sqrt{A'B'/\rho'\tau}$ and $\lambda = \sqrt{CC'/\rho\rho'}$, where A, B, A', B', C, C' are non-negative integers described in the lemmas. In particular, K is a function of A, B, A', B'. We have tried to make the treatment as uniform as possible, but the essential difference between orthogonal and non-orthogonal quasiroots forces separate treatment of the cases $T \perp R'$, $T \not\perp R'$ in each of lemmas 7 and 8.

Lemma 7. *Suppose R, T, R' are consecutive edges of P, T is short, and $R \not\perp T, R'$. Then the inner product matrix of r, t, r' is one of finitely many possibilities, up to scale.*

More precisely, let (A, B) vary over all pairs of positive integers satisfying $AB \leq \rho\tau$ and let (A', B') vary over $(0, 0)$ and all pairs of positive integers satisfying $A'B' \leq \rho'\tau$. For fixed such A, B, A', B', let (C, C') vary over all pairs of positive integers satisfying

$$CC' < K^2\rho\rho'$$

and

$$AB'C' = A'BC; \text{ call the common value } \beta. \tag{6}$$

Then for some such A, B, A', B', C, C', the inner product matrix of r, t, r' is a rational multiple of

$$\begin{pmatrix} \rho A B' & -ABB' & -\beta \\ -ABB' & \tau BB' & -A'B'B \\ -\beta & -A'B'B & \rho' A'B \end{pmatrix} \quad \text{if } A', B' \neq 0 \tag{7}$$

$$\begin{pmatrix} \rho AC & -ABC & -ACC' \\ -ABC & \tau BC & 0 \\ -ACC' & 0 & \rho' AC' \end{pmatrix} \quad \text{if } A', B' = 0. \tag{8}$$

Proof. We have $\mu \in (0, 1]$ by hypothesis, so lemma 6 says there exist positive integers A, B with $AB \leq \rho\tau$, such that r, t have inner product matrix a rational multiple of

$$\begin{pmatrix} \rho A & -AB \\ -AB & \tau B \end{pmatrix}. \tag{9}$$

Now suppose $T \not\perp R'$. Then the argument of the previous paragraph applies with primed and unprimed symbols exchanged. And by shortness of T, $\lambda < K$ (lemma 3). Then another application of lemma 6 (using the hypothesis $R \not\perp R'$) shows there exist positive integers C, C' with $CC' < K^2 \rho\rho'$, such that r, r' have inner product matrix a rational multiple of

$$\begin{pmatrix} \rho C & -CC' \\ -CC' & \rho' C' \end{pmatrix}. \tag{10}$$

Now we see

$$\frac{\rho' C'}{\rho C} = \frac{r'^2}{r^2} = \frac{r'^2}{t^2}\frac{t^2}{r^2} = \frac{\rho' A'}{\tau B'}\frac{\tau B}{\rho A},$$

so $AB'C' = A'BC$. This establishes (6). Now we put all three 2×2 matrices together by choosing the scale at which $t^2 = \tau BB'$, leading to (7).

It remains to consider the case $T \perp R'$. Then we take $A' = B' = 0$ and note that $\sqrt{A'B'/\rho'\tau}$ is indeed equal to $\mu' = -\hat{r}' \cdot \hat{t} = 0$, so this is consistent with the definition of μ'. The same argument as before shows that r, r' have inner product matrix a rational multiple of (10). This time (6) is trivially true because both sides vanish. Now we put together (9) and (10) by choosing the scale at which $r^2 = \rho AC$. Finally, using $r' \cdot t = 0$, the resulting inner product matrix is (8). $\square$

Lemma 8. *Suppose P has at least 5 edges, and R, S, T, R' are consecutive edges with (S, T) a short pair. Then the inner product matrix of r, s, t, r' is one of finitely many possibilities, up to scale.*

More precisely, let (A, B) vary over all pairs of positive integers with $\rho\tau < AB < 9\rho\tau$ and let (A', B') vary over $(0, 0)$ and all pairs of positive *integers with $A'B' \leq \rho'\tau$. For fixed such A, B, A', B', let (C, C') vary over all pairs of positive integers satisfying*

$$\rho\rho' < CC' < K^2\rho\rho' \tag{11}$$

$$AB'C' = A'BC\text{; call the common value } \beta\text{, and} \tag{12}$$

$$N := \rho'\sigma + \sigma\frac{CC'\tau + 2\beta + A'B'\rho}{AB - \rho\tau} \text{ is an integer.} \tag{13}$$

For fixed such A, B, A', B', C, C', let k vary over all positive integers dividing N. Then for some such A, B, A', B', C, C', k, the inner product matrix of r, s, t, r' is a rational multiple of

$$\begin{pmatrix} \rho AB' & 0 & -ABB' & -\beta \\ 0 & \sigma A'BN/k^2 & 0 & -A'BN/k \\ -ABB' & 0 & \tau BB' & -A'B'B \\ -\beta & -A'BN/k & -A'B'B & \rho'A'B \end{pmatrix} \text{ if } A', B' \neq 0 \tag{14}$$

$$\begin{pmatrix} \rho AC & 0 & -ABC & -ACC' \\ 0 & \sigma AC'N/k^2 & 0 & -AC'N/k \\ -ABC & 0 & \tau BC & 0 \\ -ACC' & -AC'N/k & 0 & \rho'AC' \end{pmatrix} \text{ if } A', B' = 0. \tag{15}$$

Proof. Lemma 3 says that $1 < \mu < 3$. So apply the previous proof with this inequality in place of $0 < \mu \leq 1$. The result is that r, t, r' have inner products (up to scale) as in (14) or (15), where A, B, A', B', C, C' have the stated properties up to and including (12). For the treatment of the inner product matrix of r, r' in that argument, note that $R \not\perp R'$ by the fact that P has at least 5 edges. Indeed R and R' are not even adjacent. Since P has all angles $\leq \pi/2$, this says that the lines containing R and R' are ultraparallel. This gives $\lambda > 1$, which proves the $CC' > \rho\rho'$ inequality in (11).

Now we consider s. As in lemma 4, let s_0 be the projection of $\hat{r}'$ to $\mathbb{Q} \cdot s$, so the projection of r' is $\sqrt{r'^2}s_0$, which lies in $\frac{1}{\sigma}s\mathbb{Z}$ since s is a quasiroot of weight σ. Therefore there is a positive integer k such that $\sqrt{r'^2}s_0 = -ks/\sigma$, i.e.,

$$s = -\sigma\sqrt{r'^2}s_0/k. \tag{16}$$

On the other hand, r' is a quasiroot of weight ρ', so $s \cdot r' \in \frac{1}{\rho'}r'^2\mathbb{Z}$. To work with this condition we define the integer

$$\begin{aligned} M := \rho'\frac{s \cdot r'}{r'^2} &= \rho'\frac{s \cdot \sqrt{r'^2}s_0}{r'^2} = \rho'\frac{-\sigma\sqrt{r'^2}\sqrt{r'^2}}{kr'^2}s_0^2 = -\frac{\rho'\sigma}{k}s_0^2 \\ &= -\frac{\rho'\sigma}{k}\Big(1 + \frac{\lambda^2 + 2\lambda\mu\mu' + 2\mu'^2}{\mu^2 - 1}\Big), \end{aligned} \tag{17}$$

where the last step uses (4). A calculation shows $M = -N/k$ where N is defined in (13). Since M is an integer we must have $N \in \mathbb{Z}$ and $k|N$.

We have shown that A, B, A', B', C, C', k have all the properties claimed, and all that remains is to work out the remaining entries in the inner product matrix. We have $s \cdot r = s \cdot t = 0$ by the short pair hypothesis. Referring to (17) gives

$$s \cdot r' = M{r'}^2/\rho' = -N{r'}^2/k\rho'$$

and referring to (16) and (17) gives

$$s^2 = \frac{\sigma^2}{k^2}{r'}^2 s_0^2 = \frac{\sigma^2}{k^2}{r'}^2\Big(-\frac{Mk}{\rho'\sigma}\Big) = \frac{\sigma}{k}{r'}^2\Big(-\frac{-N/k}{\rho'}\Big) = N\sigma{r'}^2/\rho' k^2.$$

This allows one to fill in the missing entries of (14) and (15), using the values for ${r'}^2$ given there. □

Lemma 9. *Suppose P has at least 6 edges and R, S, T, S', R' are consecutive edges, with S and S' forming a close pair of short edges. Then the inner product matrix for r, s, t, s', r' is one of finitely many possibilities, up to scale.*

More precisely, let (A, B) vary over all pairs of positive integers satisfying $\rho\tau < AB < 9\rho\tau$, and let (A', B') vary over all pairs of positive integers satisfying $\rho'\tau < A'B' < 9\rho'\tau$. For fixed such A, B, A', B', let (C, C') vary over all pairs of positive integers satisfying

$$\rho\rho' < CC' < K^2\rho\rho' \tag{18}$$

$$AB'C' = A'BC;\ \textit{call the common value } \beta, \textit{ and} \tag{19}$$

$$N \textit{ and } N' \textit{ are integers,} \tag{20}$$

where N is defined as in (13) and N' similarly, with primed and unprimed letters exchanged. For fixed such A, B, A', B', C, C', let (k, k') vary over all pairs of positive integers with $k|N$ and $k'|N'$, such that

$$\gamma k^2/A'BN \textit{ and } \gamma {k'}^2/AB'N' \textit{ are integers,} \tag{21}$$

where

$$\gamma = \frac{\sigma\sigma'\beta}{kk'\tau}\left(\tau + \frac{\beta}{CC'} - \frac{(\tau CC' + \beta)^3}{(AB - \rho\tau)(A'B' - \rho'\tau)C^2{C'}^2}\right). \tag{22}$$

Then for some such $A, B, A', B', C, C', k, k'$, the inner product matrix of r, s, t, s', r' is a rational multiple of

(23)
$$\begin{pmatrix} \rho AB' & 0 & -ABB' & -AB'N'/k' & -\beta \\ 0 & \sigma A'BN/k^2 & 0 & \gamma & -A'BN/k \\ -ABB' & 0 & \tau BB' & 0 & -A'B'B \\ -AB'N'/k' & \gamma & 0 & \sigma' AB'N'/k'^2 & 0 \\ -\beta & -A'BN/k & -A'B'B & 0 & \rho' A'B \end{pmatrix}.$$

Proof. Mostly this is a repeat of the $\mu' \neq 0$ cases of the previous proofs. Follow the proof of lemma 7, using $\mu, \mu' \in (1, 3)$ from lemma 3 and $\lambda \in (1, K)$ from lemma 5. The result is that after rescaling, r, t, r' have inner products as in (7), where A, B, A', B', C, C' satisfy the conditions up to and including (19). Then lemma 8's analysis of N and k applies, and also applies with primed and unprimed symbols exchanged. This shows that N, N', k, k' have all the properties claimed except (21), and justifies all entries in (23) except the $s \cdot s'$ entry. To work this out we use $s = -\sigma\sqrt{r'^2}s_0/k$ by (16), $s' = -\sigma'\sqrt{r^2}s'_0/k'$ similarly, and formula (3) for $s_0 \cdot s'_0$. The result is

$$\begin{aligned} s \cdot s' &= \frac{\sigma\sigma'}{kk'}\sqrt{r^2\, r'^2}\, s_0 \cdot s'_0 \\ &= \frac{\sigma\sigma'}{kk'}\sqrt{r^2\, r'^2}\Big(\lambda + \mu\mu' - \frac{(\lambda + \mu\mu')^3}{(\mu^2 - 1)(\mu'^2 - 1)}\Big). \end{aligned}$$

A calculation shows that this equals γ, defined in (22). This gives the last entry in the matrix. Finally, (21) is the condition that $\gamma = s \cdot s'$ lies in $\frac{1}{\sigma}s^2\mathbb{Z}$ and in $\frac{1}{\sigma'}s'^2\mathbb{Z}$, which holds because s and s' are quasiroots. □

5. Corner symbols

In this section we introduce a compact notation called a corner symbol for describing a corner of the Weyl chamber of a lattice L of signature $(2, 1)$. It is a formalization of the ideas in [16, pp. 59–62], and is needed for the proof of our main theorem in section 6 and presenting our results in the table. But the reader should skip it until it is needed.

Throughout this section suppose L is a 3-dimensional Lorentzian lattice and e, f are consecutive edges of some Weyl chamber. We will define the "corner symbol" $S(L, e, f)$, whose key property is that together with $D := \det L$ it characterizes (L, e, f) up to isometry. First, there is a unique chamber $C \subseteq H^2$ of which e, f are edges. Choose a component $\tilde{C}$ of its preimage in $L \otimes \mathbb{R} - \{0\}$. Take x, y to be simple roots corresponding to e, f; implicit in this definition is that they point away from $\tilde{C}$. Define z as the primitive lattice vector in $\langle x, y\rangle^{\perp}$ that lies in (the closure of) $\tilde{C}$. We will define the corner symbol in terms of x, y, z, so strictly speaking we should write $S(L, e, f, \tilde{C})$ or $S(L, x, y, z)$. But in the end the choice of $\tilde{C}$ is irrelevant because changing it negates x, y, z, which doesn't affect $S(L, x, y, z)$. So $S(L, e, f)$ will be well-defined. We will write X and Y for x^2 and y^2. First we will define corner symbols for corners in H^2, and then for ideal vertices.

The definitions involve some standard lattice-theoretic terms, which we define here for clarity. If L is a lattice and M a sublattice, then M's saturation M_{sat} is defined as $L \cap (M \otimes \mathbb{Q})$, and M is called saturated if $M = M_{\mathrm{sat}}$. Now suppose L is integral and M is saturated. Then L lies between $M \oplus M^\perp$ and $(M \oplus M^\perp)^*$, so it corresponds to a subgroup of $\Delta(M) \oplus \Delta(M^\perp)$. This subgroup is called the glue group of the inclusion $M \oplus M^\perp \to L$. Because of saturation, it meets the two summands $\Delta(M)$, $\Delta(M^\perp)$ trivially.

Theorem (and Definition) 10. *Suppose L, x, y and z are as stated, with z defining a finite vertex of L's Weyl chamber. Then for exactly one row of table 1 do they satisfy the conditions listed in the middle three columns. We define their* **corner symbol** *as the entry in the first column. The corner symbol and $D = \det L$ determine the quadruple (L, x, y, z) up to isometry, with z^2 given in the last column in terms of D, X, Y and $M := \min\{X, Y\}$.*

Here are some mnemonics for the notation. The subscript always gives the angle at that corner. The absence of a superscript indicates the absence of saturation and glue, so that $\langle x, y\rangle$ is a summand of L. An asterisk indicates the most complicated saturation and/or glue possible. The remaining superscripts are s (saturation but no glue), l (glue involving the left root), r (glue involving the right root), b (glue involving both roots) and $\pm$ (self-explanatory).

Proof. At a finite vertex of a Weyl chamber, the angle is π/n for $n \in \{2, 3, 4, 6\}$, with x and y being simple roots for an A_1^2, A_2, B_2 or G_2 root system. The projection of L to $\langle x, y\rangle \otimes \mathbb{Q}$ lies in the reflective hull of x and y (see section 2), and x and y are primitive in L. This constrains the possible enlargements $\langle x, y\rangle \to \langle x, y\rangle_{\mathrm{sat}}$ and $\langle x, y\rangle_{\mathrm{sat}} \oplus \langle z\rangle \to L$ so tightly that all possibilities are listed. (In the A_2 case there must be trivial saturation and nontrivial glue because otherwise every element of $W(G_2) = \mathrm{Aut}\langle x, y\rangle$ would extend to L, so x and y would not be simple roots. Similarly, most of the A_1^2 cases can only occur if $X \neq Y$.) Also, z^2 is easy to work out in each case. The table shows how to reconstruct (L, x, y, z) from knowledge of D and the corner symbol. $\square$

Corner symbol	**Angle**	$\langle x,y\rangle \to \langle x,y\rangle_{\mathrm{sat}}$ **by adjoining**	$\langle x,y\rangle_{\mathrm{sat}} \oplus \langle z\rangle \to L$ **by adjoining**	z^2
X_2Y	$\pi/2$	nothing	nothing	D/XY
X_2^sY	$\pi/2$	$(x+y)/2$	nothing	$4D/XY$
X_2^bY	$\pi/2$	nothing	$(x+y+z)/2$	$4D/XY$
X_2^lY	$\pi/2$	nothing	$(x+z)/2$	$4D/XY$
X_2^rY	$\pi/2$	nothing	$(y+z)/2$	$4D/XY$
X_2^*Y	$\pi/2$	$(x+y)/2$	$(x+z)/2$, $(y+z)/2$	$16D/XY$
$X_3^{\pm}X$	$\pi/3$	nothing	$(2x+y\pm z)/3$	$12D/X^2$
X_4Y	$\pi/4$	nothing	nothing	D/M^2
X_4^*Y	$\pi/4$	nothing	$(z+\text{longer of } x,y)/2$	$4D/M^2$
X_6Y	$\pi/6$	nothing	nothing	$4D/3M^2$

Table 1. Corner symbols for finite vertices of the Weyl chamber of a 3-dimensional Lorentzian lattice L; see theorem 10. Here $D = \det L$ and $M = \min\{X,Y\}$. When the angle is $\pi/4$ (resp. $\pi/6$), one of X and Y is 2 (resp. 3) times the other.

Corner symbols at ideal vertices are more complicated. Most of the information is contained in the structure of $x^\perp$, which we encode as follows.

Lemma 11. *Suppose L is a 2-dimensional integral Lorentzian lattice and $z \in L$ is primitive isotropic. Then there is a unique primitive null vector $u \in L$ with $z \cdot u < 0$, and we define λ as $z \cdot u$ and s as $[L : \langle z,u\rangle]$. There exists a unique $a \in L$ of the form $a = \frac{1}{s}(\sigma z + u)$ with $\sigma \in \{0,\dots,s-1\}$. The triple (λ,s,σ) characterizes (L,z) up to isometry. The triples that arise by this construction are exactly the triples of integers with $\lambda < 0$, $s > 0$, $s^2|2\lambda$, $0 \le \sigma < s$ and $(\sigma,s) = 1$.*

Proof. The uniqueness of u is clear, and $\lambda < 0$ by construction. Since $[L : \langle z,u\rangle] = s$, L lies between $\frac{1}{s}\langle z,u\rangle$ and $\langle z,u\rangle$, so it corresponds to a subgroup of order s in $\frac{1}{s}\langle z,u\rangle/\langle z,u\rangle \cong (\mathbb{Z}/s)^2$. By the primitivity of z and u, this subgroup meets the subgroups $\langle \frac{1}{s}z\rangle/\langle z\rangle$ and $\langle \frac{1}{s}u\rangle/\langle u\rangle$ trivially, so it must be the graph of an isomorphism between them. The existence and uniqueness of a follows, as does the condition $(\sigma,s) = 1$. The inner product matrix of L with respect to the basis z, a is

$$\begin{pmatrix} 0 & \lambda/s \\ \lambda/s & 2\lambda\sigma/s^2 \end{pmatrix}$$

Corner symbol	**invariants of** $(x^\perp, z)$	$\langle x\rangle \oplus x^\perp \to L$ **by adjoining**
$X^{s,\sigma}_{\infty}Y$	$(-s\sqrt{-D/X}, s, \sigma)$	nothing
$X^{s,\sigma}_{\infty z}Y$	$(-2s\sqrt{-D/X}, s, \sigma)$	$(x+z)/2$
$X^{s,\sigma}_{\infty a}Y$	$(-2s\sqrt{-D/X}, s, \sigma)$	$(x+a)/2$
$X^{s,\sigma}_{\infty b}Y$	$(-2s\sqrt{-D/X}, s, \sigma)$	$(x+z+a)/2$

Table 2. Corner symbols for ideal vertices of the Weyl chamber of a 3-dimensional Lorentzian lattice of determinant D; see theorem 12. Note that the values of s and σ appear, while the letters z, a, b appear literally when relevant, for example $26^{4,3}_{\infty b}104$.

so integrality forces $s^2|2\lambda\sigma$, hence $s^2|2\lambda$. If s satisfies this, then the rest of the matrix is automatically integral, which justifies our claim about which triples can arise. That (λ, s, σ) determines (L, z) follows from the intrinsic nature of our constructions. □

Theorem (and Definition) 12. *Suppose (L, x, y, z) are as in theorem 10, except that z defines an ideal vertex of L's Weyl chamber. Suppose also that z and a are the basis of $x^\perp$ associated to the pair $(x^\perp, z)$ by lemma 11. Then for exactly one row of table 2 do the conditions in the last two columns hold. We define the* **corner symbol** *of (L, x, y, z) to be the entry in the first column. The quadruple can be recovered up to isometry from its symbol and knowledge of $D = \det L$.*

Proof. Because x is a root, the glue group of $\langle x\rangle \oplus x^\perp \to L$ is trivial or $\mathbb{Z}/2$. The determinant of $x^\perp$ is D/X or $4D/X$ in these two cases. If (λ, s, σ) are the invariants of $(x^\perp, z)$, then $\det x^\perp = \lambda^2/s^2$ gives us an equation for λ in terms of D, X and s. If $\lambda = -s\sqrt{-D/X}$ then there is no glue, so the conditions of the top row are satisfied. If $\lambda = -2s\sqrt{-D/X}$ then there is $\mathbb{Z}/2$ glue, and the last three rows of the table list all the possible gluings of $\langle x\rangle$ to $x^\perp$. This shows that the conditions of exactly one row of the table are satisfied, so the corner symbol is defined.

Now we show that it and D determine (L, x, y, z). By referring to the table we see that they determine $(x^\perp, z)$ up to isometry. Since the norm of x is part of the symbol, they determine $\big(\langle x\rangle \oplus x^\perp, x, z\big)$ up to isometry. Since the symbol also indicates the glue, they determine (L, x, z) up to isometry. These determine the Weyl chamber uniquely, since of all the Weyl chambers of L incident at z, there is only one for which x is a simple root. This in turn determines y, because only one simple root of that chamber besides x is orthogonal to z. $\square$

6. The classification theorem

Here is the main theorem of the paper; the list of lattices appears at the end of the paper.

Theorem 13. *Up to scale, there are 8595 lattices of signature* $(2,1)$ *whose isometry groups are generated up to finite index by reflections.*

Proof. Suppose L is such a lattice, with Weyl group W. Since its Weyl chamber C has finitely many edges, theorem 1 applies, so that C has one of the three features described there: a short edge that is non-orthogonal to at least one of its neighbors (which are non-orthogonal to each other), at least 5 edges and a short pair, or at least 6 edges and a close pair. Depending on which of those conditions holds, one of lemmas 7–9 applies, with the quasiroot weights $\rho,\dots,\rho'$ all 2 because roots are quasiroots of weight 2. The conclusion is that L has 3, 4 or 5 consecutive simple roots $r,\dots,r'$ whose inner product matrix is one of finitely many possibilities, up to scale. These possibilities are listed explicitly there, and we enumerated them using a computer: the numbers of matrices of the forms (7), (8), (14), (15) and (23) are 416, 3030, 9150, 168075 and 137235 respectively.

Some of these matrices are not really possibilities: of those of form (7), 1 has only rank 2. Of those of form (8), 8 have rank 2 and 5 are positive-definite. Matrices of the form (14) are 4×4, so the span of $r,\dots,r'$ is obtained by quotienting $\mathbb{Z}^4$ by the kernel of the inner product. After quotienting, $r,\dots,r'$ may no longer be primitive vectors. When this happens we reject the matrix because roots are always primitive. In this way we reject 4153 matrices, and similarly we reject 84357 and 101189 of the forms (15) and (23). This leaves 128193 possible inner product matrices for $r,\dots,r'$, up to scale.

For M any one of of these matrices, say size $n\times n$, we regard M as an inner product on $\mathbb{Z}^n$ and define L_M as the quotient by the kernel of the form, equipped with the induced inner product. Then $r,\dots,r'$ are the images in this quotient of the standard basis vectors of $\mathbb{Z}^n$ and are roots of L_M. Let E vary over all lattices that lie between the span of these roots and their reflective hull (defined in section 2). There are only finitely many such enlargements of L_M, because E corresponds to a subgroup of the finite abelian group $\langle r,\dots,r'\rangle^{\mathrm{rh}}/\langle r,\dots,r'\rangle$. We enumerated the possible pairs (M,E) with a computer: a total of 512207. Because $r,\dots,r'$ are roots of L, L lies between their span and their reflective hull, so $(L,r,\dots,r')$ is isometric to one of our candidates

$(E, r, \ldots, r')$, up to scale. Because roots are always primitive, we may discard any $(E, r, \ldots, r')$ for which $r, \ldots, r'$ are not primitive, leaving a total of 214928 candidate tuples $(E, r, \ldots, r')$ to consider. We rescaled each candidate to make it unscaled.

Vinberg's algorithm is the essential tool for determining whether a given lattice is reflective. But we are not ready to apply it, because we need to choose a controlling vector k and find a set of simple roots for its stabilizer W_k in W. A natural choice is to take k to represent one of the 2, 3 or 4 vertices of C determined by consecutive pairs among $r, \ldots, r'$, and one would expect W_k to be generated by the reflections in those two roots. But for some of the candidates $(E, r, \ldots, r')$, W_k is actually larger. For example, the two roots might be simple roots for an A_2 root system, but their span might contain additional roots of L, enlarging the A_2 to G_2. We discard the candidate if this occurs at one of the 2, 3 or 4 vertices. The reason is that $r, \ldots, r'$ are not simple roots of E, because some other mirror of E passes through the interior of the angle at that vertex. Since we started with the assumption that $r, \ldots, r'$ are simple roots of L, the candidate $(E, r, \ldots, r')$ cannot be isometric to $(L, r, \ldots, r')$. In this manner we discard 10408 candidates. This does not much reduce the number of cases that must be analyzed, but does allow us to apply Vinberg's algorithm in the remaining cases.

In principle we could have applied Vinberg's algorithm to the 204520 remaining candidates $(E, r, \ldots, r')$. In practice we considered the 2497 SSF cases first. Because SSF lattices may be distinguished by their genera, one can immediately sort them into isometry classes and keep only one candidate from each class. There turn out to be 857 distinct SSF lattices among our candidates. To these we applied Vinberg's algorithm, obtaining Nikulin's 160 lattices [16] and simple roots for them. We checked that our results agree with his.

We mentioned in section 2 that the original form of Vinberg's algorithm (i.e., with negative-norm controlling vector) was enough for us. This is because in every one of the SSF cases, at least one of the 2, 3 or 4 possible starting corners lies in H^2, rather than being an ideal vertex. One can show a priori that this must happen: the Weyl chamber of a 3-dimensional SSF lattice cannot have consecutive ideal vertices. But this isn't really needed since one can just look at the corners in the 857 cases.

When applying the algorithm to one of our candidates $(E, r, \ldots, r')$, we checked after each new batch of simple roots whether the polygon defined by the so-far-found simple roots closed up. If this happened then E is reflective and we had a complete set of simple roots. Otherwise, we looked at the corner symbols of E at the so-far-found corners.

If some corner symbol appears twice then E admits an automorphism carrying one corner to the other and preserving the chamber C being studied. (Note that each corner can give two corner symbols since its incident edges can be taking in either order.) If we find an infinite-order automorphism this way, or finite-order automorphisms with distinct fixed points, then C has infinite area and E is non-reflective. Conversely, if E is non-reflective then eventually enough corners of C will be found to show this. With these stopping conditions, the algorithm is guaranteed to terminate, either proving non-reflectivity or finding a set of simple roots.

We remark that a nontrivial automorphism of E preserving C must be an involution, by the argument for [21, Prop. 3.4]. When we found such an involution before finishing the search for simple roots, we used it to speed up the search.

To analyze reflectivity in the non-SSF cases, we sorted them in increasing order of $|\det E|$ and proceeded inductively. Suppose when studying a particular case E that we have found all the reflective lattices of smaller $|\det|$ and obtained simple roots for them. Since E is not SSF, its $|\det|$ may be reduced by p-filling or (rescaled) p-duality, as explained in section 2. Write F for the resulting lattice. If F is not reflective then E cannot be either, since $W(E) \subseteq W(F)$. And if F is reflective then we can determine whether E is by a much faster method than Vinberg's algorithm (explained below).

But it is a nontrivial problem to determine whether F is on the list of known reflective lattices. We solved this as follows. If say x and y are consecutive simple roots of E's Weyl chamber, then they (or scalar multiples of them) are roots of F. These two roots might not be extendible to a set of simple roots for F, because of phenomena like the $A_2 \subseteq G_2$ inclusion encountered above. But we can look in their rational span to obtain a pair of simple roots for F. Then we have a corner of F's Weyl chamber and we compute its corner symbol, say S. We compare this corner symbol with all the corner symbols for all the reflective lattices of F's determinant. If S does not appear then F is not reflective and we are done. If S does appear then we obtain an isomorphism of F with a known reflective lattice, extending the two known simple roots to a full set of simple roots.

We have reduced to the case that applying p-filling or p-duality to E yields a reflective lattice F with a known set of simple roots. In the p-duality case, the isometry groups of E and F are equal, so E is reflective and we can obtain simple roots for it by rescaling F's. So we assume the p-filling case. Then E is a known sublattice of F, lying between F and pF, and we must determine whether E is reflective and obtain simple roots for it when it is. We did this using what we call the "method of bijections", which we explain in a more general context below.

In this manner we determined all reflective lattices and found simple roots for them. There might be some redundancy in the list, but this can be easily removed using the lists of corner symbols: two reflective lattices are isometric if and only if they have the same determinant and share a corner symbol. □

We now explain the "method of bijections", referred to in the proof. The general setup is that W is a Coxeter group with chamber C, X is a finite set on which it acts, $x \in X$ and we want to know whether the stabilizer W_x is generated up to finite index by reflections. In the proof of the theorem we used the case that F is a Lorentzian lattice, W its Weyl group, X the set of subgroups of F/pF for some prime p, x is the subgroup corresponding to the sublattice E of F, and W_x is the subgroup of $W(F)$ preserving E. It contains $W(E)$ because $\operatorname{Aut} E \subseteq \operatorname{Aut}(p\text{-fill}(E))$.

The method is a minor variation on traditional coset-enumeration algorithms. The idea is to build up a bijection between a set of chambers and the orbit of x in X. Then the union of the chambers is a fundamental domain for W_x. I imagine a little man making notes as he walks from chamber to chamber, being careful never to cross a mirror of W_x. For any chamber D we write w_D for the (unique) element of W that sends D to C. Two chambers D, D' are W_x-equivalent if and only if $w_D(x) = w_{D'}(x)$.

During the algorithm we maintain a list $\mathcal{D}$ of chambers, and also the list $\mathcal{F}$ of all pairs (D, f) where $D \in \mathcal{D}$ and f is an facet of D. We mark each $(D, f) \in \mathcal{F}$ by one of the following:

"unexplored":	no assertions are made about (D, f);
"mirror":	the reflection R_f across f lies in W_x;
"interior":	$R_f \notin W_x$ and $R_f(D) \in \mathcal{D}$;
"matched":	$R_f \notin W_x$ and $R_f(D) \notin \mathcal{D}$ but $w_D \circ R_f(x) = w_{D'}(x)$ for some $D' \in \mathcal{D}$.

In the initial state, $\mathcal{D} = \{C\}$ and all pairs (C, f) are marked "unexplored".

At each iteration of the algorithm we choose an "unexplored" pair $(D, f) \in \mathcal{F}$ if one exists. If none does then the algorithm terminates. If $R_f \in W_x$ then we mark the pair "mirror". Otherwise, if $R_f(D) \in \mathcal{D}$ then we mark the pair "interior". Otherwise, if $w_D \circ R_f(x) = w_{D'}(x)$ for some $D' \in \mathcal{D}$ then we mark the pair "matched". Otherwise we mark (D, f) as "interior" and add $D' := R_f(D)$ to $\mathcal{D}$ and all pairs

(D', facet of D') to $\mathcal{F}$, and mark them "unexplored". Then we continue to the next iteration.

In the last case of this chain of otherwise's, note that $w_{D'} = w_D \circ R_f$, and by the conditions under which this case applies, $w_{D'}$ lies in a coset of W_x not represented by any previous member of $\mathcal{D}$. Since there are only finitely many cosets of W_x in W, the algorithm must terminate. Furthermore, at termination $R_f(D)$ is W_x-equivalent to some member of $\mathcal{D}$, for every $(D, f) \in \mathcal{F}$, or else $R_f(D)$ would get adjoined to $\mathcal{D}$ and the algorithm would continue. So $\mathcal{D}$ is a complete set of coset representatives for W_x in W and the union $U = \cup_{D\in\mathcal{D}} D$ is a fundamental domain for W_x. Also, whenever the algorithm adjoins a chamber $D' = R_f(D)$ to $\mathcal{D}$, it is already known that $R_f \notin W_x$. It follows that both D, D' lie in one Weyl chamber of W_x, and by induction that all of U does.

At termination, each (D, f) is marked "interior", "matched" or "mirror". Each facet marked "interior" really is in U's interior, except perhaps for its boundary. If (D, f) is marked "matched", then there is a unique $D' \in \mathcal{D}$ for which $w_D \circ R_f(x) = w_{D'}(x)$, and we define $\gamma_{D,f} = w_{D'}^{-1} \circ w_D \circ R_f$. This carries $R_f(D)$ to D', and lies in W_x because it preserves $x \in X$. Our main claim about the algorithm is the following:

Theorem 14. *W_x is generated by the reflections across those facets f for which (D, f) is marked "mirror" and the $\gamma_{D,f}$ for which (D, f) is marked "matched". The latter generate the stabilizer in W_x of W_x's Weyl chamber. In particular, W_x is generated by reflections up to finite index if and only if the group Γ generated by all the $\gamma_{D,f}$ is finite.*

Proof. The algorithm only marks (D, f) as "mirror" if $R_f \in W_x$, and all the $\gamma_{D,f}$ lie in W_x by construction. So the supposed generators do in fact lie in W_x. Next we observe that $\gamma_{D,f}$ preserves the Weyl chamber of W_x that contains U; this is just the fact that $\gamma_{D,f}$ sends $R_f(D)$ to D', and $R_f(D)$ lies in the same Weyl chamber of W_x as D because $R_f \notin W_x$. It follows that Γ preserves this chamber of W_x. Now consider the polytope V which is the union of the Γ-translates of U; it lies in this single chamber of W_x. The reflection across any one of its facets is a Γ-conjugate of one of the reflections in our generating set, so lies in W_x. It follows that V is the whole of the Weyl chamber for W_x that contains it. All our claims follow from this. $\square$

In our situation, checking whether Γ is finite amounts to checking whether the $\gamma_{D,f}$ have a common fixed point in hyperbolic space, which is just linear algebra. When Γ is finite, we take the Γ-images of the facets marked "mirror". They bound W_x's chamber, yielding simple roots for W_x, which in our case is $W(E)$.

7. How to read the table

At the end of the paper we exhibit the 8595 unscaled reflective Lorentzian lattices of rank 3. Only 704 lattices are displayed explicitly, and the rest are obtained by some simple operations. They are grouped according to the conjugacy classes of their Weyl groups in $O(2,1;\mathbb{R})$. The first 122 correspond to the "main" lattices in Nikulin [16] and are listed in the same order. Then come the Weyl groups of the lattices of Nikulin's table 2 whose Weyl groups have not been listed yet. The remainder are listed in no particular order. Though we do not give details here, we classified the 374 Weyl groups up to commensurability; they represent 39 classes.

General Information: For each Weyl group, its Euler characteristic is $-\chi/24$ and the area of its Weyl chamber is $\pi\chi/12$, where χ is given in the table. The largest area is 12π (W_{329}, W_{365} and W_{371}) and the smallest is $\pi/12$ (W_3). The number of lattices having that Weyl group is listed; the largest number is 176 (W_{68}) and the smallest is 1 (W_{286}). For each W_n we have numbered the printed lattices having that Weyl group as $L_{n.1}, L_{n.2}, \ldots$

Chamber Angles: At the right side of the page we give the number of edges of the Weyl chamber and the list of angles $\pi/(2$, 3, 4, 6 or $\infty)$ in cyclic order. The largest number of edges is 28 (W_{365}). Of the 374 different Weyl chambers, 2 are regular (W_{234} and W_{286}), 3 have all ideal vertices (W_{247}, W_{285} and W_{286}), 46 are triangles, 163 have all right angles, and 269 are compact.

Chamber Isometries: If the chamber has nontrivial isometry group then the list of angles is followed by "$\rtimes C_n$" or "$\rtimes D_n$". The subscript n gives the number of isometries and we write D (resp. C) when there are (resp. are not) reflections in this finite group. Note the distinction between C_2 and D_2. The largest C_n appearing is C_3 (W_{174}) and the maximal dihedral groups appearing are D_{12} (W_{234}, W_{320} and W_{350}) and D_8 (W_{182}, W_{184}, W_{238}, W_{285}, W_{286}, W_{307}, W_{324}, W_{349}, W_{359} and W_{365}). It happens very often that the chamber admits reflections, but in this case the reflections don't preserve any of the lattices having that Weyl group. In the dihedral case, we have also indicated how the mirrors of the reflections meet the boundary of the chamber. A vertical line through a digit (i.e., $\mathrlap{\,|}2$, $\mathrlap{\,|}3$, $\mathrlap{\;|}\infty$) means that a mirror passes through that vertex, and a vertical line between two digits means that a mirror bisects the edge joining the two corresponding vertices.

Genus: For each lattice L listed explicitly we give its genus, in the notation of Conway and Sloane [9, ch. 15]. That is, for each prime p dividing twice the determinant, we give the p-adic symbol for $L \otimes \mathbb{Z}_p$. Even though it could be suppressed, we have printed the symbol for the p^0 constituent explicitly to allow easy application of p-duality and p-filling (see section 8). For $p = 2$ we give their canonical 2-adic symbol in the "compartment" notation, but see section 8 because there is a minor error in their paper which requires adjusting the meaning of "canonical". We have also suppressed their initial I or II because it can be read from the 2^0-constituent of the 2-adic symbol, and we have suppressed the signature because it is always $(2, 1)$. For example, they would write $\mathrm{II}_{2,1}(2^1_1)$ in place of our $1^2_{\mathrm{II}} 2^1_1$.

The 8595 reflective lattices represent 8488 genera, the largest number of reflective lattices in a genus is 2, and 107 genera contain this many. When a listed lattice is not the only reflective lattice in its genus, this is noted in the table and we give additional information; see "Distinct reflective lattices sharing a genus" below.

Discriminant group and determinant: The structure of the discriminant group $\Delta(L)$, and therefore the determinant of L, can be read from the genus symbol. To do this one simply reads each symbol $q^{\pm k}$ as the group $(\mathbb{Z}/q)^k$, where q is a prime power, and the string of symbols as the direct sum of groups. One ignores the sign, and when q is even one also ignores any subscripts. For example, if L has genus $1^1_7 4^{-2}_{\mathrm{II}}{\cdot}1^{-}3^1 9^1{\cdot}1^1 5^{-2}$ then discriminant group is $(\mathbb{Z}/4)^2 \oplus (\mathbb{Z}/3) \oplus (\mathbb{Z}/9) \oplus (\mathbb{Z}/5)^2 \cong (\mathbb{Z}/180) \oplus (\mathbb{Z}/60)$. The most-negative determinant among all reflective lattices is -29811600 (2.3.5.7.13-dual($L_{122.1}$) and 2.3.5.7.13-dual($L_{121.1}$)). But a better measure of complexity considers only lattices which are minimal in their duality class, yielding -108000 ($L_{365.1}$ and 3-dual($L_{365.1}$) $\cong$ 5-dual($L_{365.1}$)). Among all primes occurring among the factors of our lattices' determinants, the largest is 97 (all lattices with Weyl group W_{99}).

Implicitly printed lattices: For some L's in the table, $\langle \ldots \rangle$ appears after the genus; it indicates how to construct some other lattices, not explicitly printed, from L. A digit indicates the operation of p-filling (see section 2) and m indicates the operation of "mainification". This has meaning when L is odd and the 2^0 Jordan constituent of the 2-adic lattice $L \otimes \mathbb{Z}_2$ has dimension 1 or 2. It means passage to L's even sublattice, followed by multiplying inner products by $\frac{1}{4}$ or $\frac{1}{2}$ to obtain an unscaled lattice. Every automorphism of L preserves L's mainification. For rank 3 lattices, not admitting mainification is equivalent to being "main" in Nikulin's terminology [16]. We dislike our name for the operation because it is ugly and the mainification of a lattice need not be main.

A string of these symbols indicates that these operations should be applied in order, but usually the order is not important because the operations commute, except for mainification and 2-filling. Since all the primes involved are less than 10, we have printed them without spaces. So for example $\langle 3, 32, 3m, m\rangle$ indicates that L's line on the table implicitly represents 4 additional lattices. One is obtained by 3-filling, one by 3-filling followed by 2-filling, one by 3-filling followed by mainification, and one by mainification. A total of 715 lattices are implicitly printed in this way. We only use this convention when each lattice obtained by the stated operations has the same Weyl group as L, so that simple roots for the derived lattices are scalar multiples of L's. See "Applying lattice operations" below for how to construct the inner product matrix and simple roots for lattices obtained in this way.

It turns out that all of Nikulin's lattices are implicitly printed in this fashion. We have indicated this by for example $\langle 23 \to N_4, 3, 2\rangle$ for $L_{4.1}$. This means that the lattice got by applying 2-filling and then 3-filling to $L_{4.1}$ is numbered $N = 4$ in Nikulin's [16, table 1]. We replace N by N' to indicate lattices in his table 2. We prepared our tables so that our simple roots come in the same order as his. Also, a few implicitly printed lattices share their genera with other reflective lattices; we indicate this with an asterisk "$*$" and provide some additional information in "Distinct reflective lattices sharing a genus" below.

Corner Symbols and description of the lattices: At the right we give the corner symbols at all the corners of the chamber in the same cyclic order used to list the angles of the Weyl chamber. When $\operatorname{Aut} L$ contains a nontrivial isometry preserving the chamber, this isometry is always an involution, and we have written "$(\times 2)$" after the corner symbols. The full sequence of corner symbols is then got by concatenating two copies of the ones listed.

The corner symbols display the root norms; the largest norm among all reflective lattices is 14400 (2.3-dual$(L_{365.1}) \cong$ 2.5-dual$(L_{365.1})$) and the largest among those minimal in their duality classes is 7200 ($L_{365.1}$ and 3-dual$(L_{365.1}) \cong$ 5-dual$(L_{365.1})$).

By giving the corner symbols we have given enough information to reconstruct L in several different ways. For example, the genus $1_7^1 8_1^1 256_1^1$ mentioned below contains $L_{342.1} = 256_2^r 4_2^b 256_2^l 8_{\infty}^{32,17} 32_{\infty z}^{32,25} 8_2$, and to construct it we find a consecutive pair of roots, say $8_2 256$ (the last root followed by the first one). So L has roots of norms 8 and 256; the subscript says they are orthogonal, and the absence of a superscript says that L is generated by them and their orthogonal complement. Since the determinant is $-8 \cdot 256$, a generator for the complement has norm -1. So L has inner product matrix $\operatorname{diag}[-1, 8, 256]$. Refer to section 5 for full instructions on constructing L from one of its corner symbols.

Distinct reflective lattices sharing a genus: When one of our listed lattices L shares its genus with another reflective lattice, we describe the other lattice, either as a lattice in L's duality class or in terms of another $L_{n.i}$. We also give all compositions of p-dualities that carry L to a lattice isometric to L. This information is required to reconstitute the full table from what we have printed. See "Completeness of the table" below. A few implicitly-listed lattices also share their genera with other reflective lattices. The corresponding information for these lattices is the following:

2-fill($L_{10.1}$) shares genus with its 5-dual
$1_{\mathrm{II}}^{-2} 2_1^1, 1^- 5^- 25^-$

main($L_{248.2}$) shares genus with $L_{248.1}$
$[1^1 2^1]_0 128_1^1$

2-fill($L_{194.1}$) shares genus with its 5-dual
$1_{\mathrm{II}}^{-2} 2_1^1, 1^2 9^-, 1^- 5^- 25^-$

2-fill($L_{195.1}$) shares genus with its 5-dual
$1_{\mathrm{II}}^{-2} 2_1^1, 1^{-2} 9^1, 1^- 5^- 25^-$

2-fill($L_{73.1}$) shares genus with its 5-dual
$1_{\mathrm{II}}^{2} 2_7^1, 1^- 5^- 25^-, 1^{-2} 11^-$

2-fill($L_{366.1}$) shares genus with its 13-dual
$1_{\mathrm{II}}^{-2} 2_1^1, 1^- 13^- 169^-$

2-fill($L_{345.1}$) shares genus with its 3-dual; isometric to its own 7-dual
$1_{\mathrm{II}}^{-2} 2_1^1, 1^- 3^1 9^-, 1^1 7^1 49^1$

Reflective lattices having the same genus have the same Weyl group, except for the genus $1_7^1 8_1^1 256_1^1$ (and its 2-dual). This genus contains $L_{341.1}$ and $L_{342.1}$, with Weyl groups W_{341} and W_{342}.

Inner product matrix: This is the first displayed matrix. We did not seek bases in which the matrix entries are small, or bases related to the corners of the chamber. Rather, they are in Smith normal form relative to their duals, which is useful when applying p-filling and p-duality. Namely, let a, b, c be the elementary divisors of the inclusion $L \to L^*$, where $c|b|a$. These can be read from the structure of $\Delta(L)$, which can be read from the genus as explained under "Discriminant group and determinant". With respect to the basis having the displayed inner product matrix, L^* has basis $(1/a, 0, 0)$, $(0, 1/b, 0)$, $(0, 0, 1/c)$. Our lattices are unscaled, so c is always 1.

Diagram automorphism: If L admits a diagram automorphism, that is, an isometry preserving the Weyl chamber (other than negation), then this is an involution and we display it right after the inner product matrix.

Simple roots: Next we list the simple roots of L. If L has no diagram automorphism then all the simple roots are listed, in the order used for the corner symbols. If L has a diagram automorphism then only the first half of the simple roots are displayed and the rest are got by applying the diagram automorphism.

Completeness of the table: The table of lattices is complete in the following sense. There are 1419 duality classes of reflective lattices, and each has exactly one member appearing in the table, 704 explicitly and 715 implicitly. The lattices printed explicitly always have minimal (i.e., least negative) determinant in their duality classes. The full list of reflective lattices is obtained from our explicitly printed lattices by applying the stated operations and then applying all possible p-dualities.

This requires a little more work to make explicit, because some chains of p-dualities may lead to lattices isomorphic to L. A fancy way to say this is that L's duality class is a homogeneous space for $(\mathbb{Z}/2)^\Omega$ where Ω is the set of primes dividing $\det L$. To find the other lattices in the duality class, without duplicates, one must find a subgroup of $(\mathbb{Z}/2)^\Omega$ acting simply-transitively, i.e., a complement to the subgroup $T \subseteq (\mathbb{Z}/2)^\Omega$ acting trivially. So one needs to know T in order to reconstruct the full list of lattices without duplication. If L is the unique reflective lattice in its genus (which is almost always the case) then one can determine whether a given sequence of p-dualities yields a lattice isometric to L by computing the genus of the result and comparing that to L's. (See section 8 for the easy computational details.) Otherwise, there is no easy way to determine this, and it is for this reason that we have listed the nontrivial elements of T explicitly when L shares its genus with another reflective lattice. We say "elements" but in this situation it happens that T is never larger than $\mathbb{Z}/2$.

Applying lattice operations: Because of our choice of basis for the lattices (see "Inner product matrix"), applying p-filling is easy. Let p^k be the p-part of the first elementary divisor a, and adjoin $(1/p, 0, 0)$ to L. This has the effect of dividing the first row and column of the inner product matrix by p, and multiplying the first coordinate of all roots by p. (If $p^k|b$ then we would treat the second coordinate similarly, but this never happens because the explicitly printed lattices are all minimal in their duality classes.) The result is the inner product matrix for

p-fill(L), and the simple roots of L expressed in a basis for p-fill(L). It may be that some of L's roots are divisible by p in the rescaled p-dual of L, so if necessary we divide by p to make them primitive. They will be roots of p-fill(L) since $\operatorname{Aut} L \subseteq \operatorname{Aut} p\text{-fill}(L)$. When the filling operation is indicated in our tables, these roots are a set of simple roots for p-fill(L). This is because we only indicate filling operations when they don't change the Weyl group. Similar but slightly more complicated operations treat mainification and p-duality.

8. The Conway-Sloane genus symbol

Genera of integer bilinear forms present some complicated phenomena, and Conway and Sloane introduced a notation for working with them that is as simple as possible [9, ch. 15]. In this section we state how to apply our operations of filling, duality and mainification in terms of their notation. We also correct a minor error in their formulation of the canonical 2-adic symbol.

p-filling: This affects only the p-adic symbol. One simply takes the Jordan constituent of largest scale p^{a+2} and replaces $a+2$ by a. (Recall that p-filling is only defined if the largest scale is $\geq p^2$.) The scale of the resulting summand of p-fill$(L) \otimes \mathbb{Z}_p$ may be the same as that of another constituent of L. In this case one takes the direct sum of these sublattices of p-fill$(L) \otimes \mathbb{Z}_2$ to obtain the p^a-constituent. The direct sum is computed as follows, where we write q for p^a:

$$q^{\varepsilon_1 n_1} \oplus q^{\varepsilon_2 n_2} = q^{(\varepsilon_1\varepsilon_2)(n_1+n_2)}$$

if p is odd, while

$$\begin{aligned} q_{t_1}^{\varepsilon_1 n_1} \oplus q_{t_2}^{\varepsilon_2 n_2} &= q_{t_1+t_2}^{(\varepsilon_1\varepsilon_2)(n_1+n_2)} \\ q_{t_1}^{\varepsilon_1 n_1} \oplus q_{\mathrm{II}}^{\varepsilon_2 n_2} &= q_{t_1}^{(\varepsilon_1\varepsilon_2)(n_1+n_2)} \\ q_{\mathrm{II}}^{\varepsilon_1 n_1} \oplus q_{\mathrm{II}}^{\varepsilon_2 n_2} &= q_{\mathrm{II}}^{(\varepsilon_1\varepsilon_2)(n_1+n_2)} \end{aligned}$$

if $p = 2$. Here the ε_i are signs, $n_i \geq 0$ are the ranks and $t_i \in \mathbb{Z}/8$ are the oddities. In short, signs multiply and ranks and subscripts add, subject to the special rules involving II. Here are some examples of 2-filling:

$$\begin{aligned} 1_1^1 8_7^1 256_1^1 \cdot 1^2 9^1 &\to 1_1^1 8_7^1 64_1^1 \cdot 1^2 9^1 \\ 1_2^{-2} 8_3^- &\to 1_2^{-2} 2_3^- = [1^{-2} 2^-]_5 = [1^2 2^1]_1 \\ 1_{-1}^1 [4^1 8^1]_2 &\to 1_{-1}^1 [4^1 2^1]_2 = [1^1 2^1 4^1]_1 \\ 1_1^1 4_7^1 16_1^1 &\to 1_1^1 4_7^1 \oplus 4_1^1 = 1_1^1 4_0^2 \\ 1_{\mathrm{II}}^2 4_1^1 &\to 1_{\mathrm{II}}^2 \oplus 1_1^1 = 1_1^3. \end{aligned}$$

Rescaling: Multiplying all inner products by a rational number s has the following effect on the l-adic symbols for all primes l that divide neither the numerator nor denominator of s. The sign of a constituent (power of l)$^{\pm n}$ is flipped just if n is odd and the Legendre symbol $\left(\frac{s}{l}\right)$ is -1. If $l = 2$ we must supplement this rule by describing how to alter the subscript. A subscript II is left alone, while a subscript $t \in \mathbb{Z}/8$ is multiplied by s. Scaling all inner products by l affects the l-adic symbol by multiplying each scale by l, leaving the rest of the symbol unchanged.

Rescaled p-duality: Say $q = p^a$ is the largest scale appearing in the p-adic symbol. One obtains the rescaled p-dual by an alteration that doesn't affect $L \otimes \mathbb{Z}_l$ for $l \neq p$, followed by rescaling by p^a. So for $l \neq p$ the effect of rescaled p-duality is the same as that of rescaling. The p-adic symbol is altered by replacing the scale p^{a_i} of each constituent by p^{a-a_i}, leaving superscripts and subscripts alone. Since one usually writes the constituents in increasing order of scale, this also reverses the order of the constituents. A single example of rescaled 2-duality should illustrate the process:

$$[1^1 2^1]_6 8^1_1 \cdot 3^2 9^- \cdot 1^1 7^- 49^1 \to 1^1_1 [4^1 8^1]_6 \cdot 3^2 9^1 \cdot 1^1 7^- 49^1$$

Mainification: This is an alteration of L that doesn't affect $L \otimes \mathbb{Z}_l$ for odd l, followed by rescaling by $\frac{1}{4}$ or $\frac{1}{2}$. So for odd l mainification has the same effect as rescaling. Recall that the mainification is only defined if L is odd with 2^0-constituent of rank 1 or 2. Referring to (33) and (34) of [9, ch. 15], this means that the constituent is one of $1^1_{\pm 1}$, $1^{-1}_{\pm 3}$, $1^2_{\pm 2}$, $1^{-2}_{\pm 2}$, 1^2_0 or 1^{-2}_4. To find the 2-adic symbol of main(L), begin by altering the 2^0-constituent as follows:

$$1^{\pm 1}_t \to 4^{\pm 1}_t \qquad 1^{\pm 2}_{\pm 2} \to 2^{\pm 2}_{\pm 2} \qquad 1^2_0 \to 2^2_{\mathrm{II}} \qquad 1^{-2}_4 \to 2^{-2}_{\mathrm{II}}.$$

The scale of the result may be the same as that of another constituent present, in which case one takes their direct sum as above. Finally, one divides every scale by the smallest scale present (2 or 4), leaving all superscripts and subscripts alone. This scaling factor is also the one to use to work out the effect on the odd l-adic symbols. Here are some examples:

$$1^2_2 8^1_7 \to 2^2_2 8^1_7 \to 1^2_2 4^1_7$$

$$1^2_6 8^-_5 \cdot 1^- 3^- 9^1 \to 2^2_6 8^-_5 \cdot 1^- 3^- 9^1 \to 1^2_6 4^-_5 \cdot 1^1 3^1 9^-$$

$$1^1_1 8^1_7 256^1_1 \cdot 1^2 9^1 \to 4^1_1 8^1_7 256^1_1 \cdot 1^2 9^1 = [4^1 8^1]_0 256^1_1 \cdot 1^2 9^1 \to [1^1 2^1]_0 64^1_1 \cdot 1^2 9^1$$

$$[1^1 2^- 4^1]_3 \cdot 1^{-2} 3^- \to [2^- 4^2]_3 \cdot 1^{-2} 3^- \to [1^- 2^2]_3 \cdot 1^{-2} 3^1.$$

The canonical 2*-adic symbol:* There is an error in the Conway-Sloane definition of the canonical 2-adic symbol. Distinct 2-adic symbols can represent isometric 2-adic lattices, and they give a calculus for reducing every 2-adic symbol to a canonical form. We have already used it in some of the examples above. First one uses "sign walking" to change some of the signs (and simultaneously some of the oddities), and then one uses "oddity fusion". "Compartments" and "trains" refer to subchains of the 2-adic symbol that govern these operations. (The compartments are delimited by brackets $[\cdots]$.) Conway and Sloane state that one can use sign walking to convert any 2-adic symbol into one which has plus signs everywhere except perhaps the first positive-dimensional constituent in each train. This is not true: sign walking would supposedly take the 2-dimensional lattice $1^1_1 2^-_3$ with total oddity $1+3=4$ to one of the form $1^-_{\dots} 2^1_{\dots}$ with total oddity 0. But the oddity of the first (resp. second) constituent must be ± 3 (resp. ± 1) by (33) of [9, ch. 15], so the total oddity cannot be 0.

So the canonical form we use is slightly more complicated: minus signs are allowed either on the first positive-dimensional constituent of a train, or on the second constituent of a compartment of the form $q^1_{\dots} r^-_{\dots}$ with total oddity 4 or $q^-_{\dots} r^-_{\dots}$ with total oddity 0. Here q and r are consecutive powers of 2. Oddity fusion remains unchanged. Note that the second form $[q^- r^-]_0$ can only occur if the q-constituent is the first positive-dimensional constituent of its train. Using [9, ch. 15, thm. 10] we checked that two 2-adic lattices are isomorphic if and only if their canonical symbols (in our sense) are identical.

References

[1] Allcock, Daniel, *Unabridged table of reflective Lorentzian lattices of rank* 3, available at *http://www.math.utexas.edu/∼allcock.*

[2] Borcherds, Richard E., Automorphism groups of Lorentzian lattices. *J. Algebra* **111** (1987) 133–153.

[3] Borcherds, Richard E., The monster Lie algebra. *Adv. Math.* **83** (1990) 30–47.

[4] Borcherds, Richard E., Automorphic forms with singularities on Grassmannians. *Invent. Math.* **132** (1998) 491–562.

[5] Borcherds, Richard E., Reflection groups of Lorentzian lattices. *Duke Math. J.* **104** (2000) 319–366.

[6] Borcherds, Richard E., Coxeter groups, Lorentzian lattices, and $K3$ surfaces. *Internat. Math. Res. Notices* 1998, no. 19, 1011–1031.

[7] Conway, J. H., The automorphism group of the 26-dimensional even unimodular Lorentzian lattice. *J. Algebra* **80** (1983) 159–163. Reprinted as ch. 27 of [9].

[8] Conway, J. H. and Sloane, N. J. A., On the enumeration of lattices of determinant one. *J. Number Theory* **15** (1982) 83–94. Reprinted as ch. 16 of [9]

[9] Conway, J. H. and Sloane, N. J. A., *Sphere Packings, Lattices and Groups,* 3rd ed., Grundlehren der Mathematischen Wissenschaften 290. Springer-Verlag, New York, 1999.

[10] Coxeter, H. S. M. and Whitrow, G. J., World-structure and non-Euclidean honeycombs. *Proc. Roy. Soc. London. Ser. A.* **201** (1950) 417–437.

[11] Esselmann, Frank, Über die maximale Dimension von Lorentz-Gittern mit coendlicher Spiegelungsgruppe. *J. Number Theory* **61** (1996) 103–144.

[12] Gritsenko, V. A. and Nikulin, V. V., On the classification of Lorentzian Kac-Moody algebras. *Uspekhi Mat. Nauk* **57** (2002) 79–138. English translation in *Russian Math. Surveys* **57** (2002) 921–979.

[13] Nikulin, V. V., On the arithmetic groups generated by reflections in Lobačevski spaces. *Izv. Akad. Nauk SSSR Ser. Mat.* **44** (1980), no. 3, 637–669, 719–720.

[14] Nikulin, V. V., On the classification of arithmetic groups generated by reflections in Lobachevsky spaces, *Izv. Akad. Nauk SSSR Ser. Mat.* **45** (1981) 113–142.

[15] Nikulin, V. V., K3 surfaces with interesting groups of automorphisms, *J. Math. Sci.* **95** (1999) 2028–2048.

[16] Nikulin, V. V., On the classification of hyperbolic root systems of rank three. *Tr. Mat. Inst. Steklova* **230** (2000), 256 pp.; English translation in *Proc. Steklov Inst. Math.* **230** (2000) 1–241.

[17] Nikulin, V. V., Quotient-groups of groups of automorphisms of hyperbolic forms of subgroups generated by 2-reflections. *Dokl. Akad. Nauk SSSR* **248** (1979) no. 6, 1307–1309. English translation: *Soviet Math. Dokl.* **20** (1979) no. 5, 1156–1158 (1980).

[18] Nikulin, V. V., K3 surfaces with a finite group of automorphisms and a Picard group of rank three, *Algebraic geometry and its applications. Trudy Mat. Inst. Steklov.* **165** (1984) 119–142.

[19] PARI/GP, version 2.4.3, Bordeaux, 2008, *http://pari.math.u-bordeaux.fr*

[20] Pjateckiĭ-Šapiro, I. I.; Šafarevič, I. R., Torelli's theorem for algebraic surfaces of type $K3$, *Izv. Akad. Nauk SSSR Ser. Mat.* **35** (1971) 530–572.

[21] Scharlau, R., On the classification of arithmetic groups on hyperbolic 3-space, preprint Bielefeld 1989.

[22] Scharlau, Rudolf and Blaschke, Britta, Reflective Integral Lattices, *J. Algebra* **181** (1996) 934–961.

[23] Scharlau, Rudolf and Walhorn, Claudia, Integral lattices and hyperbolic reflection groups. *Journées Arithmétiques, 1991 (Geneva). Astérisque* **209** (1992), 15–16, 279–291.

[24] Vinberg, È. B., Discrete linear groups that are generated by reflections. *Izv. Akad. Nauk SSSR Ser. Mat.* **35** (1971) 1072–1112.

[25] Vinberg, È. B., The groups of units of certain quadratic forms. *Mat. Sb. (N.S.)* **87**(129) (1972), 18–36.

[26] Vinberg, È. B., The two most algebraic K3 surfaces. *Math. Ann.* **265** (1983) 1–21.

[27] Vinberg, È. B., Classification of 2-reflective hyperbolic lattices of rank 4, *Tr. Mosk. Mat. Obs.* **68** (2007) 44–76; English translation in *Trans. Moscow Math. Soc.* 2007, 39–66.

[28] Vinberg, È. B. and Kaplinskaja, I. M., The groups $O_{18,1}(Z)$ and $O_{19,1}(Z)$. *Dokl. Akad. Nauk SSSR* **238** (1978) 1273–1275

[29] Walhorn, Claudia, Arithmetische Spiegelungsgruppen auf dem 4-dimension--alen hyperbolische Raum, dissertation Bielefeld 1993.
[30] Watson, G. L., Transformations of a quadratic form which do not increase the class-number. *Proc. London Math. Soc. (3)* **12** (1962) 577–587.
[31] Watson, G. L., Transformations of a quadratic form which do not increase the class-number II. *Acta Arith.* **27** (1975), 171–189.

Table of Rank 3 Reflective Lorentzian Lattices

W_1 11 lattices, $\chi = 3$ — 3-gon: $\infty 24$

$L_{1.1} : 1^{2}_{\mathrm{II}}4^{1}_{1}$ $\langle 2 \to N_1 \rangle$ — $4^{1,0}_{\infty b}4^{r}_{2}2^{*}_{4}$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & -2 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$L_{1.2} : 1^{-2}_{2}8^{-}_{3}$ $\langle 2 \to N_1' \rangle$ — $2^{4,3}_{\infty a}8^{s}_{2}4^{*}_{4}$

$$\begin{bmatrix} -168 & 16 & 32 \\ 16 & -1 & -3 \\ 32 & -3 & -6 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & -2 \\ -1 & -4 & 6 \end{bmatrix}$$

$L_{1.3} : 1^{2}_{2}8^{1}_{7}$ $\langle m \rangle$ — $2^{4,3}_{\infty b}8^{l}_{2}1_{4}$

$$\begin{bmatrix} -328 & 32 & 56 \\ 32 & -3 & -6 \\ 56 & -6 & -7 \end{bmatrix} \qquad \begin{bmatrix} 2 & -3 & -1 \\ 15 & -24 & -7 \\ 3 & -4 & -2 \end{bmatrix}$$

W_2 4 lattices, $\chi = 2$ — 3-gon: $\infty 23$

$L_{2.1} : 1^{-2}_{\mathrm{II}}8^{-}_{5}$ $\langle 2 \to N_2 \rangle$ — $2^{4,1}_{\infty b}8^{b}_{2}2^{+}_{3}$

$$\begin{bmatrix} -24 & 8 & 16 \\ 8 & -2 & -5 \\ 16 & -5 & -10 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & 1 \\ 2 & -4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$$

W_3 6 lattices, $\chi = 1$ — 3-gon: 426

$L_{3.1} : 1^{-2}_{\mathrm{II}}4^{1}_{7},1^{2}3^{-}$ $\langle 2 \to N_3 \rangle$ — $2^{*}_{4}4^{b}_{2}6_{6}$

$$\begin{bmatrix} -516 & 36 & 72 \\ 36 & -2 & -5 \\ 72 & -5 & -10 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -3 \\ -1 & 8 & -6 \end{bmatrix}$$

W_4 44 lattices, $\chi = 6$ — 4-gon: $\infty 222$

$L_{4.1} : 1^{2}_{\mathrm{II}}4^{-}_{3},1^{1}3^{1}9^{-}$ $\langle 23 \to N_4, 3, 2 \rangle$ — $12^{3,2}_{\infty b}12^{r}_{2}18^{b}_{2}4^{*}_{2}$

$$\begin{bmatrix} 2124 & 0 & -108 \\ 0 & -6 & 3 \\ -108 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} -1 & 3 & 1 & -1 \\ -10 & 28 & 9 & -10 \\ -18 & 60 & 18 & -20 \end{bmatrix}$$

$L_{4.2} : 1^{2}_{6}8^{-}_{5},1^{-}3^{-}9^{1}$ $\langle 3m, 3, 2 \rangle$ — $6^{12,5}_{\infty a}24^{s}_{2}36^{*}_{2}8^{b}_{2}$

$$\begin{bmatrix} -3096 & -1440 & 432 \\ -1440 & -669 & 198 \\ 432 & 198 & -49 \end{bmatrix} \qquad \begin{bmatrix} -15 & 17 & 31 & -5 \\ 35 & -40 & -72 & 12 \\ 9 & -12 & -18 & 4 \end{bmatrix}$$

$L_{4.3} : 1^{-2}_{6}8^{1}_{1},1^{-}3^{-}9^{1}$ $\langle 32 \to N_2', 3, m \rangle$ — $6^{12,5}_{\infty b}24^{l}_{2}9_{2}8^{r}_{2}$

$$\begin{bmatrix} -9144 & -1800 & -1080 \\ -1800 & -354 & -213 \\ -1080 & -213 & -127 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & -4 & -5 \\ -5 & -8 & 15 & 16 \\ 0 & -12 & 9 & 16 \end{bmatrix}$$

W_5 22 lattices, $\chi = 9$ — 4-gon: $\infty 242$

$L_{5.1} : 1^{2}_{\mathrm{II}}4^{-}_{5},1^{2}5^{1}$ $\langle 2 \to N_5 \rangle$ — $20^{1,0}_{\infty b}20^{r}_{2}2^{*}_{4}4^{*}_{2}$

$$\begin{bmatrix} -15020 & 360 & 680 \\ 360 & -8 & -17 \\ 680 & -17 & -30 \end{bmatrix} \qquad \begin{bmatrix} -3 & 13 & 2 & -3 \\ -50 & 200 & 32 & -46 \\ -40 & 180 & 27 & -42 \end{bmatrix}$$

$L_{5.2}: 1^{2}_{2}8^{-}_{3},1^{2}5^{-}\ \langle 2 \to N'_5\rangle$ $\qquad 10^{4,3}_{\infty a}40^{l}_{2}1_{4}2^{b}_{2}$

$$\begin{bmatrix} -8360 & 240 & 360 \\ 240 & -6 & -11 \\ 360 & -11 & -15 \end{bmatrix} \qquad \begin{bmatrix} -2 & 9 & 1 & -1 \\ -25 & 100 & 12 & -11 \\ -30 & 140 & 15 & -16 \end{bmatrix}$$

$L_{5.3}: 1^{-2}_{2}8^{1}_{7},1^{2}5^{-}\ \langle m\rangle$ $\qquad 10^{4,3}_{\infty b}40^{s}_{2}4^{*}_{4}2^{s}_{2}$

$$\begin{bmatrix} -34760 & 200 & 1560 \\ 200 & -1 & -9 \\ 1560 & -9 & -70 \end{bmatrix} \qquad \begin{bmatrix} -2 & -1 & 1 & 0 \\ 0 & -20 & 2 & 6 \\ -45 & -20 & 22 & -1 \end{bmatrix}$$

W_6 6 lattices, $\chi = 2$ 4-gon: 2223

$L_{6.1}: 1^{-2}_{\mathrm{II}}4^{1}_{1},1^{-2}5^{-}\ \langle 2 \to N_6\rangle$ $\qquad 2^{b}_{2}10^{l}_{2}4^{r}_{2}2^{+}_{3}$

$$\begin{bmatrix} -1660 & 80 & 100 \\ 80 & -2 & -5 \\ 100 & -5 & -6 \end{bmatrix} \qquad \begin{bmatrix} -1 & -2 & 1 & 1 \\ -1 & -5 & 0 & 2 \\ -16 & -30 & 16 & 15 \end{bmatrix}$$

W_7 16 lattices, $\chi = 4$ 3-gon: $\infty 26$

$L_{7.1}: 1^{-2}_{\mathrm{II}}8^{1}_{7},1^{1}3^{-}9^{-}\ \langle 23 \to N_7, 3, 2\rangle$ $\qquad 6^{12,7}_{\infty a}24^{b}_{2}18_{6}$

$$\begin{bmatrix} -1224 & 360 & -144 \\ 360 & -102 & 39 \\ -144 & 39 & -14 \end{bmatrix} \qquad \begin{bmatrix} -1 & 3 & -1 \\ -7 & 20 & -3 \\ -9 & 24 & 0 \end{bmatrix}$$

W_8 6 lattices, $\chi = 6$ 4-gon: $4242 \rtimes C_2$

$L_{8.1}: 1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}7^{-}\ \langle 2 \to N_8\rangle$ $\qquad 2^{*}_{4}4^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -308 & 56 & 28 \\ 56 & -10 & -5 \\ 28 & -5 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -28 & 5 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \\ 1 & -6 \\ -3 & 0 \end{bmatrix}$$

W_9 22 lattices, $\chi = 12$ 5-gon: $\infty 2222$

$L_{9.1}: 1^{2}_{\mathrm{II}}4^{1}_{7},1^{-2}7^{1}\ \langle 2 \to N_9\rangle$ $\qquad 28^{1,0}_{\infty b}28^{r}_{2}2^{s}_{2}14^{b}_{2}4^{*}_{2}$

$$\begin{bmatrix} -21252 & 532 & 1204 \\ 532 & -12 & -33 \\ 1204 & -33 & -62 \end{bmatrix} \qquad \begin{bmatrix} -31 & 3 & 5 & 1 & -9 \\ -602 & 56 & 98 & 28 & -172 \\ -280 & 28 & 45 & 7 & -82 \end{bmatrix}$$

$L_{9.2}: 1^{2}_{6}8^{1}_{1},1^{-2}7^{1}\ \langle 2 \to N'_7\rangle$ $\qquad 14^{4,1}_{\infty a}56^{s}_{2}4^{l}_{2}7_{2}8^{r}_{2}$

$$\begin{bmatrix} -39928 & 224 & 1176 \\ 224 & -1 & -7 \\ 1176 & -7 & -34 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & -1 & -1 & 1 \\ 126 & 28 & -42 & -35 & 48 \\ 77 & 28 & -26 & -28 & 24 \end{bmatrix}$$

$L_{9.3}: 1^{-2}_{6}8^{-}_{5},1^{-2}7^{1}\ \langle m\rangle$ $\qquad 14^{4,1}_{\infty b}56^{l}_{2}1^{r}_{2}28^{*}_{2}8^{b}_{2}$

$$\begin{bmatrix} -655704 & 94472 & 11312 \\ 94472 & -13611 & -1630 \\ 11312 & -1630 & -195 \end{bmatrix} \qquad \begin{bmatrix} 48 & -5 & -7 & 3 & 29 \\ 287 & -28 & -42 & 14 & 172 \\ 385 & -56 & -55 & 56 & 244 \end{bmatrix}$$

W_{10} 16 lattices, $\chi = 6$ 4-gon: $\infty 222$

$L_{10.1}: 1^{-2}_{\mathrm{II}}8^{1}_{1},1^{-}5^{-}25^{-}\ \langle 25 \to N_{10}, 5, 2*\rangle$ $\qquad 10^{20,9}_{\infty a}40^{b}_{2}50^{l}_{2}8^{r}_{2}$

shares genus with its 5-dual

$$\begin{bmatrix} -95800 & 1200 & -10800 \\ 1200 & -10 & 165 \\ -10800 & 165 & -1042 \end{bmatrix} \qquad \begin{bmatrix} -14 & 15 & 28 & -3 \\ -443 & 476 & 885 & -96 \\ 75 & -80 & -150 & 16 \end{bmatrix}$$

W_{11} 6 lattices, $\chi = 5$ 4-gon: 4223

$L_{11.1} : 1_{\mathrm{II}}^{-2} 4_7^1, 1^2 11^- \ \langle 2 \to N_{11} \rangle$ $2_4^* 4_2^b 2 2_2^s 2_3^+$

$$\begin{bmatrix} -33924 & 748 & 1012 \\ 748 & -14 & -23 \\ 1012 & -23 & -30 \end{bmatrix} \qquad \begin{bmatrix} 2 & 3 & -4 & -2 \\ 16 & 22 & -33 & -15 \\ 55 & 84 & -110 & -56 \end{bmatrix}$$

W_{12} 22 lattices, $\chi = 36$ 8-gon: $\infty 222 \infty 222 \rtimes C_2$

$L_{12.1} : 1_{\mathrm{II}}^2 4_3^-, 1^{-2} 11^1 \ \langle 2 \to N_{12} \rangle$ $44_{\infty b}^{1,0} 44_2^r 2_2^b 4_2^* \ (\times 2)$

$$\begin{bmatrix} -2348148 & 9768 & 70268 \\ 9768 & -40 & -293 \\ 70268 & -293 & -2102 \end{bmatrix} \begin{bmatrix} 42041 & -154 & -1281 \\ 1285284 & -4709 & -39162 \\ 1225224 & -4488 & -37333 \end{bmatrix} \qquad \begin{bmatrix} -63 & -157 & -16 & -33 \\ -1826 & -4796 & -500 & -1040 \\ -1848 & -4576 & -465 & -958 \end{bmatrix}$$

$L_{12.2} : 1_6^2 8_5^-, 1^{-2} 11^- \ \langle 2 \to N_8' \rangle$ $22_{\infty b}^{4,1} 88_2^s 4_2^* 8_2^b \ (\times 2)$

$$\begin{bmatrix} -238040 & 528 & 4840 \\ 528 & -1 & -11 \\ 4840 & -11 & -98 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 10032 & -31 & -192 \\ -1672 & 5 & 31 \end{bmatrix} \qquad \begin{bmatrix} 5 & 1 & -1 & -3 \\ 330 & 44 & -66 & -160 \\ 209 & 44 & -42 & -132 \end{bmatrix}$$

$L_{12.3} : 1_6^{-2} 8_1^1, 1^{-2} 11^- \ \langle m \rangle$ $22_{\infty a}^{4,1} 88_2^l 1_2 8_2^r \ (\times 2)$

$$\begin{bmatrix} -1848 & -1848 & 968 \\ -1848 & -1826 & 945 \\ 968 & 945 & -483 \end{bmatrix} \begin{bmatrix} 527 & 472 & -218 \\ -1320 & -1181 & 545 \\ -1584 & -1416 & 653 \end{bmatrix} \qquad \begin{bmatrix} -105 & -21 & 11 & 77 \\ 209 & 44 & -22 & -160 \\ 198 & 44 & -21 & -160 \end{bmatrix}$$

W_{13} 22 lattices, $\chi = 42$ 8-gon: $\infty 242 \infty 242 \rtimes C_2$

$L_{13.1} : 1_{\mathrm{II}}^2 4_5^-, 1^2 13^1 \ \langle 2 \to N_{13} \rangle$ $52_{\infty b}^{1,0} 52_2^r 2_4^* 4_2^* \ (\times 2)$

$$\begin{bmatrix} -14425996 & 261508 & 523016 \\ 261508 & -4740 & -9481 \\ 523016 & -9481 & -18962 \end{bmatrix} \begin{bmatrix} 896635 & -16383 & -32508 \\ 125112 & -2287 & -4536 \\ 24667916 & -450723 & -894349 \end{bmatrix} \qquad \begin{bmatrix} -1159 & -2351 & -258 & -31 \\ -130 & -312 & -38 & -10 \\ -31902 & -64688 & -7097 & -850 \end{bmatrix}$$

$L_{13.2} : 1_2^{-2} 8_7^1, 1^2 13^- \ \langle 2 \to N_{10}' \rangle$ $26_{\infty b}^{4,3} 104_2^s 4_4^* 2_2^s \ (\times 2)$

$$\begin{bmatrix} 5304 & 104 & 2080 \\ 104 & -9 & -65 \\ 2080 & -65 & -198 \end{bmatrix} \begin{bmatrix} -11233 & 468 & 2196 \\ -527280 & 21969 & 103090 \\ 54912 & -2288 & -10737 \end{bmatrix} \qquad \begin{bmatrix} -412 & -1893 & -225 & -26 \\ -19344 & -88868 & -10562 & -1220 \\ 2015 & 9256 & 1100 & 127 \end{bmatrix}$$

$L_{13.3} : 1_2^2 8_3^-, 1^2 13^- \ \langle m \rangle$ $26_{\infty a}^{4,3} 104_2^l 1_4 2_2^b \ (\times 2)$

$$\begin{bmatrix} -2600 & -2600 & 1352 \\ -2600 & -2574 & 1325 \\ 1352 & 1325 & -675 \end{bmatrix} \begin{bmatrix} -17109 & -15463 & 7191 \\ 33488 & 30267 & -14076 \\ 31304 & 28294 & -13159 \end{bmatrix} \qquad \begin{bmatrix} -150 & -25 & 13 & 2 \\ 299 & 52 & -26 & -5 \\ 286 & 52 & -25 & -6 \end{bmatrix}$$

W_{14} 6 lattices, $\chi = 12$ 6-gon: $222222 \rtimes C_2$

$L_{14.1} : 1_{\mathrm{II}}^{-2} 4_1^1, 1^{-2} 13^- \ \langle 2 \to N_{14} \rangle$ $4_2^r 2_2^b 26_2^l \ (\times 2)$

$$\begin{bmatrix} -572 & -260 & 52 \\ -260 & -118 & 25 \\ 52 & 25 & 6 \end{bmatrix} \begin{bmatrix} 103 & 49 & 4 \\ -208 & -99 & -8 \\ -104 & -49 & -5 \end{bmatrix} \qquad \begin{bmatrix} 17 & 7 & -6 \\ -36 & -15 & 13 \\ 0 & 2 & 0 \end{bmatrix}$$

W_{15} 8 lattices, $\chi = 8$ 4-gon: $\infty 232$

$L_{15.1} : 1_{\mathrm{II}}^{-2} 8_3^-, 1^{-2} 7^1 \ \langle 2 \to N_{15} \rangle$ $14_{\infty a}^{4,3} 56_2^b 2_3^- 2_2^b$

$$\begin{bmatrix} -25256 & 392 & 672 \\ 392 & -6 & -11 \\ 672 & -11 & -14 \end{bmatrix} \qquad \begin{bmatrix} -2 & 11 & 1 & -1 \\ -105 & 560 & 52 & -51 \\ -14 & 84 & 7 & -8 \end{bmatrix}$$

W_{16} 24 lattices, $\chi = 4$ 4-gon: 2262

$L_{16.1} : 1^{-2}_{\mathrm{II}}4^{-}_{3},1^{-}3^{-}9^{1},1^{-2}5^{1}$ $\langle 23 \to N_{16}, 3, 2\rangle$ $36^{*}_{2}20^{b}_{2}6_{6}2^{b}_{2}$

$$\begin{bmatrix} -324180 & -9360 & 3600 \\ -9360 & -66 & 69 \\ 3600 & 69 & -34 \end{bmatrix} \qquad \begin{bmatrix} -5 & 9 & 3 & -4 \\ -138 & 250 & 83 & -111 \\ -810 & 1460 & 486 & -649 \end{bmatrix}$$

W_{17} 44 lattices, $\chi = 6$ 5-gon: 22222

$L_{17.1} : 1^{2}_{\mathrm{II}}4^{1}_{7},1^{2}3^{1},1^{-2}5^{1}$ $\langle 2 \to N_{17}\rangle$ $4^{*}_{2}12^{*}_{2}20^{b}_{2}2^{s}_{2}30^{b}_{2}$

$$\begin{bmatrix} -22020 & 360 & 240 \\ 360 & -4 & -5 \\ 240 & -5 & -2 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 3 & 1 & 1 \\ -28 & -30 & 80 & 28 & 30 \\ -50 & -48 & 150 & 49 & 45 \end{bmatrix}$$

$L_{17.2} : 1^{-2}_{6}8^{-}_{5},1^{2}3^{-},1^{-2}5^{-}$ $\langle 2 \to N'_{13}\rangle$ $8^{b}_{2}6^{l}_{2}40_{2}1^{r}_{2}60^{*}_{2}$

$$\begin{bmatrix} 9960 & 3240 & -120 \\ 3240 & 1054 & -39 \\ -120 & -39 & 1 \end{bmatrix} \qquad \begin{bmatrix} 9 & -1 & -13 & 0 & 29 \\ -28 & 3 & 40 & 0 & -90 \\ -8 & 0 & 0 & -1 & -30 \end{bmatrix}$$

$L_{17.3} : 1^{2}_{6}8^{1}_{1},1^{2}3^{-},1^{-2}5^{-}$ $\langle m\rangle$ $8^{r}_{2}6^{b}_{2}40^{*}_{2}4^{l}_{2}15_{2}$

$$\begin{bmatrix} -126840 & -3360 & 600 \\ -3360 & -89 & 16 \\ 600 & 16 & -1 \end{bmatrix} \qquad \begin{bmatrix} -3 & -2 & 7 & 3 & 2 \\ 112 & 75 & -260 & -112 & -75 \\ -8 & -3 & 20 & 6 & 0 \end{bmatrix}$$

W_{18} 88 lattices, $\chi = 18$ 6-gon: ∞22222

$L_{18.1} : 1^{2}_{\mathrm{II}}4^{1}_{7},1^{1}3^{-}9^{-},1^{2}5^{-}$ $\langle 23 \to N_{18}, 3, 2\rangle$ $60^{3,2}_{\infty b}60^{r}_{2}18^{b}_{2}10^{s}_{2}6^{b}_{2}4^{*}_{2}$

$$\begin{bmatrix} -18654660 & 443520 & 38700 \\ 443520 & -10542 & -921 \\ 38700 & -921 & -80 \end{bmatrix} \qquad \begin{bmatrix} -53 & 3 & 13 & 2 & -10 & -21 \\ -1750 & 100 & 429 & 65 & -331 & -694 \\ -5490 & 300 & 1350 & 220 & -1026 & -2168 \end{bmatrix}$$

$L_{18.2} : 1^{2}_{6}8^{1}_{1},1^{-}3^{1}9^{1},1^{2}5^{1}$ $\langle 3m, 3, 2\rangle$ $30^{12,5}_{\infty b}120^{s}_{2}36^{*}_{2}20^{l}_{2}3_{2}8^{r}_{2}$

$$\begin{bmatrix} -116280 & -4680 & -5040 \\ -4680 & -186 & -201 \\ -5040 & -201 & -217 \end{bmatrix} \qquad \begin{bmatrix} -2 & -1 & 1 & 1 & 0 & -1 \\ -145 & -40 & 72 & 50 & -10 & -88 \\ 180 & 60 & -90 & -70 & 9 & 104 \end{bmatrix}$$

$L_{18.3} : 1^{-2}_{6}8^{-}_{5},1^{-}3^{1}9^{1},1^{2}5^{1}$ $\langle 32 \to N'_{12}, 3, m\rangle$ $30^{12,5}_{\infty a}120^{l}_{2}9_{2}5^{r}_{2}12^{*}_{2}8^{b}_{2}$

$$\begin{bmatrix} -1572120 & -310680 & 109800 \\ -310680 & -61395 & 21696 \\ 109800 & 21696 & -7663 \end{bmatrix} \qquad \begin{bmatrix} -219 & 43 & 44 & -14 & -113 & -195 \\ 1315 & -260 & -264 & 85 & 680 & 1172 \\ 585 & -120 & -117 & 40 & 306 & 524 \end{bmatrix}$$

W_{19} 12 lattices, $\chi = 3$ 4-gon: 4222

$L_{19.1} : 1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}3^{1},1^{2}5^{-}$ $\langle 2 \to N_{19}\rangle$ $2^{*}_{4}4^{b}_{2}10^{l}_{2}12^{r}_{2}$

$$\begin{bmatrix} -27060 & 300 & 540 \\ 300 & -2 & -7 \\ 540 & -7 & -10 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -2 & 1 \\ 27 & -26 & -55 & 24 \\ 35 & -36 & -70 & 36 \end{bmatrix}$$

W_{20} 22 lattices, $\chi = 27$ 7-gon: ∞222242

$L_{20.1} : 1^{2}_{\mathrm{II}}4^{1}_{1},1^{2}17^{1}$ $\langle 2 \to N_{20}\rangle$ $68^{1,0}_{\infty b}68^{r}_{2}2^{b}_{2}34^{l}_{2}4^{r}_{2}2^{*}_{4}4^{*}_{2}$

$$\begin{bmatrix} -53788 & 1156 & 2652 \\ 1156 & -24 & -59 \\ 2652 & -59 & -126 \end{bmatrix} \qquad \begin{bmatrix} -7 & 75 & 17 & 132 & 17 & 4 & -5 \\ -170 & 1768 & 402 & 3128 & 404 & 96 & -118 \\ -68 & 748 & 169 & 1309 & 168 & 39 & -50 \end{bmatrix}$$

$L_{20.2} : 1^{2}_{2}8^{1}_{7},1^{2}17^{1}$ $\langle 2 \to N'_{15}\rangle$ $34^{4,3}_{\infty b}136^{l}_{2}1_{2}17^{r}_{2}8^{l}_{2}1_{4}2^{s}_{2}$

$$\begin{bmatrix} -2045576 & 8160 & 11424 \\ 8160 & -30 & -47 \\ 11424 & -47 & -63 \end{bmatrix} \qquad \begin{bmatrix} -4 & 59 & 7 & 56 & 15 & 2 & -2 \\ -289 & 4216 & 501 & 4012 & 1076 & 144 & -143 \\ -510 & 7548 & 895 & 7157 & 1916 & 255 & -256 \end{bmatrix}$$

$L_{20.3}:\ 1^{-2}_{2}8^{-}_{3},1^{2}17^{1}\ \langle m\rangle$ $\qquad 34^{4,3}_{\infty a}136^{s}_{2}4^{*}_{2}68^{s}_{2}8^{s}_{2}4^{*}_{4}2^{b}_{2}$

$$\begin{bmatrix}-62696 & -15640 & 952\\ -15640 & -3901 & 237\\ 952 & 237 & -14\end{bmatrix} \qquad \begin{bmatrix}-40 & -175 & -21 & -87 & 1 & 7 & -1\\ 170 & 748 & 90 & 374 & -4 & -30 & 4\\ 153 & 748 & 94 & 408 & 0 & -32 & -1\end{bmatrix}$$

W_{21} 6 lattices, $\chi = 18$ — 6-gon: $422422 \rtimes C_2$

$L_{21.1}:\ 1^{-2}_{\mathrm{II}}4^{1}_{7},1^{2}19^{-}\ \langle 2\to N_{21}\rangle$ $\qquad 2^{*}_{4}4^{b}_{2}38^{s}_{2}\ (\times 2)$

$$\begin{bmatrix}-26372 & -18468 & 1672\\ -18468 & -12930 & 1171\\ 1672 & 1171 & -106\end{bmatrix}\begin{bmatrix}3191 & 2233 & -203\\ -1368 & -958 & 87\\ 35112 & 24563 & -2234\end{bmatrix} \qquad \begin{bmatrix}-4 & 7 & 139\\ 2 & -2 & -57\\ -41 & 88 & 1558\end{bmatrix}$$

W_{22} 12 lattices, $\chi = 4$ — 4-gon: 2622

$L_{22.1}:\ 1^{-2}_{\mathrm{II}}4^{1}_{1},1^{2}3^{-},1^{-2}7^{-}\ \langle 2\to N_{22}\rangle$ $\qquad 4^{r}_{2}6_{6}2^{b}_{2}42^{l}_{2}$

$$\begin{bmatrix}-25788 & 588 & 252\\ 588 & -10 & -7\\ 252 & -7 & -2\end{bmatrix} \qquad \begin{bmatrix}-1 & -1 & 1 & 2\\ -20 & -21 & 20 & 42\\ -56 & -54 & 55 & 105\end{bmatrix}$$

W_{23} 88 lattices, $\chi = 48$ — 10-gon: $\infty 2222\infty 2222 \rtimes C_2$

$L_{23.1}:\ 1^{2}_{\mathrm{II}}4^{-}_{5},1^{1}3^{1}9^{-},1^{-2}7^{-}\ \langle 23\to N_{23},3,2\rangle$ $\qquad 84^{3,2}_{\infty b}84^{r}_{2}18^{b}_{2}12^{*}_{2}4^{*}_{2}\ (\times 2)$

$$\begin{bmatrix}-16363116 & 948024 & 53676\\ 948024 & -54906 & -3111\\ 53676 & -3111 & -176\end{bmatrix}\begin{bmatrix}505343 & -29696 & -1632\\ 4563888 & -268193 & -14739\\ 73432800 & -4315200 & -237151\end{bmatrix}\begin{bmatrix}-333 & -899 & -403 & -259 & -49\\ -3010 & -8120 & -3639 & -2338 & -442\\ -48342 & -130620 & -58572 & -37656 & -7130\end{bmatrix}$$

$L_{23.2}:\ 1^{-2}_{2}8^{1}_{7},1^{-}3^{-}9^{1},1^{-2}7^{-}\ \langle 3m,3,2\rangle$ $\qquad 42^{12,11}_{\infty b}168^{s}_{2}36^{*}_{2}24^{b}_{2}2^{s}_{2}\ (\times 2)$

$$\begin{bmatrix}-8168328 & -56448 & -59472\\ -56448 & -390 & -411\\ -59472 & -411 & -433\end{bmatrix}\begin{bmatrix}6299 & 43 & 46\\ 88200 & 601 & 644\\ -945000 & -6450 & -6901\end{bmatrix} \qquad \begin{bmatrix}-4 & -27 & -13 & -9 & -1\\ -133 & -448 & -156 & -68 & 1\\ 672 & 4116 & 1926 & 1296 & 136\end{bmatrix}$$

$L_{23.3}:\ 1^{2}_{2}8^{-}_{3},1^{-}3^{-}9^{1},1^{-2}7^{-}\ \langle 32\to N'_{16},3,m\rangle$ $\qquad 42^{12,11}_{\infty a}168^{l}_{2}9_{2}24^{r}_{2}2^{b}_{2}\ (\times 2)$

$$\begin{bmatrix}-22810536 & 3272976 & -990864\\ 3272976 & -469623 & 142176\\ -990864 & 142176 & -43039\end{bmatrix}\begin{bmatrix}-6504625 & 932578 & -283932\\ -39054960 & 5599369 & -1704780\\ 20738592 & -2973324 & 905255\end{bmatrix}$$

$$\begin{bmatrix}-2617 & -13491 & -2972 & -3747 & -338\\ -15715 & -81004 & -17844 & -22496 & -2029\\ 8337 & 43008 & 9477 & 11952 & 1079\end{bmatrix}$$

W_{24} 44 lattices, $\chi = 9$ — 5-gon: 22224

$L_{24.1}:\ 1^{2}_{\mathrm{II}}4^{-}_{5},1^{2}3^{-},1^{2}7^{1}\ \langle 2\to N_{24}\rangle$ $\qquad 4^{*}_{2}28^{b}_{2}6^{b}_{2}14^{s}_{2}2^{*}_{4}$

$$\begin{bmatrix}-278796 & 2436 & 2856\\ 2436 & -20 & -27\\ 2856 & -27 & -26\end{bmatrix} \qquad \begin{bmatrix}-3 & -3 & 4 & 13 & 3\\ -198 & -196 & 264 & 854 & 196\\ -124 & -126 & 165 & 539 & 125\end{bmatrix}$$

$L_{24.2}:\ 1^{2}_{2}8^{-}_{3},1^{2}3^{1},1^{2}7^{1}\ \langle 2\to N'_{19}\rangle$ $\qquad 2^{b}_{2}56^{*}_{2}12^{*}_{2}28^{l}_{2}1_{4}$

$$\begin{bmatrix}-1129128 & -560616 & 11088\\ -560616 & -278347 & 5504\\ 11088 & 5504 & -107\end{bmatrix} \qquad \begin{bmatrix}74 & 151 & -197 & -645 & -75\\ -151 & -308 & 402 & 1316 & 153\\ -99 & -196 & 264 & 854 & 98\end{bmatrix}$$

$L_{24.3}:\ 1^{-2}_{2}8^{1}_{7},1^{2}3^{1},1^{2}7^{1}\ \langle m\rangle$ $\qquad 2^{l}_{2}56_{2}3_{2}7^{r}_{2}4^{*}_{4}$

$$\begin{bmatrix}-514248 & 3024 & 1848\\ 3024 & -17 & -12\\ 1848 & -12 & -5\end{bmatrix} \qquad \begin{bmatrix}1 & 15 & 1 & -1 & -1\\ 119 & 1792 & 120 & -119 & -120\\ 83 & 1232 & 81 & -84 & -82\end{bmatrix}$$

W_{25} 24 lattices, $\chi = 6$ 5-gon: 22222

$L_{25.1}: 1^{-2}_{\mathrm{II}}4^{1}_{1},1^{-}3^{1}9^{1},1^{2}7^{1}\ \langle 23 \to N_{25}, 3, 2\rangle$ $12^{*}_{2}252^{b}_{2}2^{l}_{2}36^{r}_{2}14^{b}_{2}$

$$\begin{bmatrix} 1925028 & -70560 & 2520 \\ -70560 & 2586 & -93 \\ 2520 & -93 & 2 \end{bmatrix} \qquad \begin{bmatrix} -5 & -47 & 0 & 17 & 6 \\ -134 & -1260 & 0 & 456 & 161 \\ 72 & 630 & -1 & -216 & -70 \end{bmatrix}$$

W_{26} 8 lattices, $\chi = 24$ 6-gon: $\infty 22 \infty 22 \rtimes C_2$

$L_{26.1}: 1^{-2}_{\mathrm{II}}8^{1}_{7},1^{-2}11^{-}\ \langle 2 \to N_{26}\rangle$ $22^{4,3}_{\infty b}88^{b}_{2}2^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -19976 & -9240 & -880 \\ -9240 & -4274 & -407 \\ -880 & -407 & -38 \end{bmatrix} \begin{bmatrix} 4223 & 1956 & 192 \\ -9152 & -4239 & -416 \\ 352 & 163 & 15 \end{bmatrix} \qquad \begin{bmatrix} -25 & 61 & 5 \\ 55 & -132 & -11 \\ -11 & 0 & 2 \end{bmatrix}$$

W_{27} 6 lattices, $\chi = 22$ 6-gon: $423423 \rtimes C_2$

$L_{27.1}: 1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}23^{-}\ \langle 2 \to N_{27}\rangle$ $2^{*}_{4}4^{b}_{2}2^{-}_{3}\ (\times 2)$

$$\begin{bmatrix} -182068 & 2024 & 4048 \\ 2024 & -22 & -45 \\ 4048 & -45 & -90 \end{bmatrix} \begin{bmatrix} 17939 & -185 & -400 \\ -21528 & 221 & 480 \\ 814476 & -8399 & -18161 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & -5 \\ -1 & -44 & 92 \end{bmatrix}$$

W_{28} 8 lattices, $\chi = 14$ 5-gon: $\infty 2322$

$L_{28.1}: 1^{-2}_{\mathrm{II}}8^{1}_{1},1^{2}13^{-}\ \langle 2 \to N_{28}\rangle$ $26^{4,1}_{\infty a}104^{b}_{2}2^{+}_{3}2^{l}_{2}8^{r}_{2}$

$$\begin{bmatrix} -255855288 & 3601312 & -238784 \\ 3601312 & -50690 & 3357 \\ -238784 & 3357 & -194 \end{bmatrix} \qquad \begin{bmatrix} -1062 & 343 & 161 & -168 & -795 \\ -76154 & 24596 & 11545 & -12047 & -57008 \\ -10621 & 3432 & 1610 & -1681 & -7952 \end{bmatrix}$$

W_{29} 6 lattices, $\chi = 28$ 8-gon: $22232223 \rtimes C_2$

$L_{29.1}: 1^{-2}_{\mathrm{II}}4^{1}_{1},1^{-2}29^{-}\ \langle 2 \to N_{29}\rangle$ $2^{b}_{2}58^{l}_{2}4^{r}_{2}2^{-}_{3}\ (\times 2)$

$$\begin{bmatrix} -42108 & 1508 & 812 \\ 1508 & -54 & -29 \\ 812 & -29 & -14 \end{bmatrix} \begin{bmatrix} 3189 & -115 & -65 \\ 90596 & -3267 & -1846 \\ -3828 & 138 & 77 \end{bmatrix} \qquad \begin{bmatrix} 2 & 133 & 31 & 16 \\ 56 & 3770 & 880 & 455 \\ -1 & -145 & -36 & -20 \end{bmatrix}$$

W_{30} 48 lattices, $\chi = 24$ 6-gon: $\infty 22 \infty 22 \rtimes C_2$

$L_{30.1}: 1^{-2}_{\mathrm{II}}8^{-}_{3},1^{1}3^{1}9^{-},1^{-}5^{1}25^{-}\ \langle 235 \to N_{30}, 35, 25, 23, 5, 3, 2\rangle$

$30^{60,31}_{\infty b}120^{b}_{2}18^{b}_{2}30^{60,19}_{\infty a}120^{b}_{2}450^{b}_{2}$

$$\begin{bmatrix} -1348200 & 21600 & -55800 \\ 21600 & -330 & 915 \\ -55800 & 915 & -2282 \end{bmatrix} \qquad \begin{bmatrix} -15 & -131 & -23 & -14 & 15 & 28 \\ -317 & -2756 & -483 & -293 & 316 & 585 \\ 240 & 2100 & 369 & 225 & -240 & -450 \end{bmatrix}$$

W_{31} 16 lattices, $\chi = 4$ 4-gon: 6222

$L_{31.1}: 1^{-2}_{\mathrm{II}}8^{-}_{3},1^{2}3^{-},1^{-2}5^{-}\ \langle 2 \to N_{31}\rangle$ $2_{6}6^{b}_{2}10^{l}_{2}24^{r}_{2}$

$$\begin{bmatrix} -221160 & 960 & 1800 \\ 960 & -2 & -9 \\ 1800 & -9 & -14 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -2 & 1 \\ 52 & -51 & -105 & 48 \\ 95 & -96 & -190 & 96 \end{bmatrix}$$

W_{32} 24 lattices, $\chi = 20$ 6-gon: $226226 \rtimes C_2$

$L_{32.1}: 1^{-2}_{\mathrm{II}}4^{-}_{5},1^{1}3^{-}9^{-},1^{2}11^{-}\ \langle 23 \to N_{32}, 3, 2\rangle$ $18^{s}_{2}22^{b}_{2}6_{6}\ (\times 2)$

$$\begin{bmatrix} -1983564 & -559152 & 9108 \\ -559152 & -157602 & 2559 \\ 9108 & 2559 & -38 \end{bmatrix} \begin{bmatrix} -99001 & -27675 & 350 \\ 364320 & 101843 & -1288 \\ 803880 & 224721 & -2843 \end{bmatrix} \qquad \begin{bmatrix} -322 & -278 & -3 \\ 1185 & 1023 & 11 \\ 2619 & 2255 & 21 \end{bmatrix}$$

W_{33} 44 lattices, $\chi = 15$ 6-gon: 222224

$L_{33.1}: 1^{2}_{\mathrm{II}}4^{1}_{1},1^{2}3^{1},1^{2}11^{-}\ \langle 2 \to N_{33}\rangle$ $4^{*}_{2}12^{b}_{2}22^{l}_{2}4^{r}_{2}66^{b}_{2}2^{*}_{4}$

$$\begin{bmatrix} -292380 & 1584 & 1848 \\ 1584 & -8 & -11 \\ 1848 & -11 & -10 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 4 & 3 & 20 & 1 \\ -110 & -108 & 440 & 328 & 2178 & 108 \\ -64 & -66 & 253 & 192 & 1287 & 65 \end{bmatrix}$$

$L_{33.2}: 1^{2}_{2}8^{1}_{7},1^{2}3^{-},1^{2}11^{1}\ \langle 2 \to N'_{22}\rangle$ $2^{b}_{2}24^{*}_{2}44^{s}_{2}8^{l}_{2}33_{2}1_{4}$

$$\begin{bmatrix} -1159752 & -577368 & 7128 \\ -577368 & -287435 & 3548 \\ 7128 & 3548 & -43 \end{bmatrix} \quad \begin{bmatrix} 37 & 77 & -293 & -223 & -749 & -38 \\ -75 & -156 & 594 & 452 & 1518 & 77 \\ -55 & -108 & 440 & 328 & 1089 & 54 \end{bmatrix}$$

$L_{33.3}: 1^{-2}_{2}8^{-}_{3},1^{2}3^{-},1^{2}11^{1}\ \langle m\rangle$ $2^{l}_{2}24_{2}11^{r}_{2}8^{s}_{2}132^{*}_{2}4^{*}_{4}$

$$\begin{bmatrix} -24359016 & 35904 & 78936 \\ 35904 & -49 & -119 \\ 78936 & -119 & -254 \end{bmatrix} \quad \begin{bmatrix} 4 & 35 & 29 & 9 & -7 & -5 \\ 640 & 5592 & 4631 & 1436 & -1122 & -798 \\ 943 & 8256 & 6842 & 2124 & -1650 & -1180 \end{bmatrix}$$

W_{34} 88 lattices, $\chi = 72$ 14-gon: $\infty 222222 \infty 222222 \rtimes C_2$

$L_{34.1}: 1^{2}_{\mathrm{II}}4^{1}_{1},1^{1}3^{-}9^{-},1^{-2}11^{1}\ \langle 23 \to N_{34},3,2\rangle$

$132^{3,2}_{\infty b}132^{r}_{2}18^{l}_{2}4^{r}_{2}6^{b}_{2}396^{*}_{2}4^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -6574916700 & 52718688 & 1769328 \\ 52718688 & -422706 & -14187 \\ 1769328 & -14187 & -476 \end{bmatrix} \begin{bmatrix} 4245449 & -34075 & -1125 \\ 496887468 & -3988139 & -131670 \\ 971019324 & -7793634 & -257311 \end{bmatrix}$$

$$\begin{bmatrix} -311 & 3 & 31 & 5 & -41 & -3505 & -209 \\ -36388 & 352 & 3627 & 584 & -4801 & -410322 & -24466 \\ -71478 & 660 & 7128 & 1180 & -9306 & -798732 & -47662 \end{bmatrix}$$

$L_{34.2}: 1^{-2}_{2}8^{-}_{3},1^{-}3^{1}9^{1},1^{-2}11^{-}\ \langle 3m,3,2\rangle$ $66^{12,11}_{\infty a}264^{s}_{2}36^{s}_{2}8^{l}_{2}3_{2}792^{r}_{2}2^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -2375208 & -49104 & -50688 \\ -49104 & -1014 & -1047 \\ -50688 & -1047 & -1081 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 92400 & 1849 & 1925 \\ -88704 & -1776 & -1849 \end{bmatrix} \quad \begin{bmatrix} -5 & -1 & 1 & 1 & 1 & 83 & 2 \\ -781 & -88 & 156 & 104 & 53 & 792 & -21 \\ 990 & 132 & -198 & -148 & -99 & -4752 & -76 \end{bmatrix}$$

$L_{34.3}: 1^{2}_{2}8^{1}_{7},1^{-}3^{1}9^{1},1^{-2}11^{-}\ \langle 32 \to N'_{21},3,m\rangle$

$66^{12,11}_{\infty b}264^{l}_{2}9^{r}_{2}8^{s}_{2}12^{*}_{2}792^{b}_{2}2^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -35211528 & 6384312 & -1161864 \\ 6384312 & -1157559 & 210660 \\ -1161864 & 210660 & -38335 \end{bmatrix} \begin{bmatrix} -2299969 & 417252 & -76428 \\ -13817232 & 2506672 & -459147 \\ -6220368 & 1128477 & -206704 \end{bmatrix}$$

$$\begin{bmatrix} 1076 & -95 & -98 & 71 & 673 & 46759 & 1336 \\ 6457 & -572 & -588 & 428 & 4046 & 281028 & 8029 \\ 2871 & -264 & -261 & 200 & 1836 & 127116 & 3629 \end{bmatrix}$$

W_{35} 18 lattices, $\chi = 12$ 6-gon: $222222 \rtimes C_2$

$L_{35.1}: 1^{-2}_{\mathrm{II}}4^{-}_{5},1^{-}3^{1}9^{-},1^{-2}11^{1}\ \langle 23 \to N_{35},3,2\rangle$ $12^{*}_{2}396^{b}_{2}2^{b}_{2}12^{*}_{2}44^{b}_{2}18^{b}_{2}$

$$\begin{bmatrix} -16236 & 1584 & 0 \\ 1584 & -150 & -3 \\ 0 & -3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 13 & 0 & -5 & -19 & -2 \\ 10 & 132 & 0 & -52 & -198 & -21 \\ 18 & 198 & -1 & -78 & -286 & -27 \end{bmatrix}$$

W_{36} 8 lattices, $\chi = 36$ 8-gon: $\infty 222 \infty 222 \rtimes C_2$

$L_{36.1}: 1^{-2}_{\mathrm{II}}8^{-}_{5},1^{2}17^{1}\ \langle 2 \to N_{36}\rangle$ $34^{4,1}_{\infty b}136^{b}_{2}2^{b}_{2}8^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -171224 & -40800 & -4352 \\ -40800 & -9722 & -1037 \\ -4352 & -1037 & -110 \end{bmatrix} \begin{bmatrix} 39167 & 9336 & 984 \\ -164832 & -39290 & -4141 \\ 4896 & 1167 & 122 \end{bmatrix} \quad \begin{bmatrix} -32 & 81 & 4 & -41 \\ 136 & -340 & -17 & 172 \\ -17 & 0 & 2 & 0 \end{bmatrix}$$

W_{37} 12 lattices, $\chi = 8$ — 5-gon: 22322

$L_{37.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{7},1^{-2}5^{1},1^{-2}7^{-}$ $\langle 2 \to N_{37}\rangle$ — $4^{*}_{2}20^{b}_{2}2^{+}_{3}2^{s}_{2}70^{b}_{2}$

$$\begin{bmatrix} -246820 & 3080 & -3360 \\ 3080 & -38 & 39 \\ -3360 & 39 & -26 \end{bmatrix} \qquad \begin{bmatrix} -3 & 7 & 2 & -3 & -37 \\ -286 & 670 & 191 & -287 & -3535 \\ -42 & 100 & 28 & -43 & -525 \end{bmatrix}$$

W_{38} 44 lattices, $\chi = 36$ — 10-gon: $2222222222 \rtimes C_2$

$L_{38.1}$: $1^{2}_{\mathrm{II}}4^{-}_{3},1^{-2}5^{1},1^{2}7^{1}$ $\langle 2 \to N_{38}\rangle$ — $20^{*}_{2}28^{*}_{2}4^{b}_{2}2^{s}_{2}14^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 2380 & 140 & -140 \\ 140 & -4 & -3 \\ -140 & -3 & 6 \end{bmatrix} \begin{bmatrix} -71 & -11 & 7 \\ -700 & -111 & 70 \\ -1820 & -286 & 181 \end{bmatrix} \qquad \begin{bmatrix} 23 & 29 & 7 & 0 & -2 \\ 240 & 294 & 68 & -2 & -28 \\ 610 & 756 & 178 & -3 & -63 \end{bmatrix}$$

$L_{38.2}$: $1^{-2}_{6}8^{1}_{1},1^{-2}5^{-},1^{2}7^{1}$ $\langle 2 \to N'_{25}\rangle$ — $40^{b}_{2}14^{l}_{2}8_{2}1^{r}_{2}28^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -133560 & 560 & 840 \\ 560 & -2 & -5 \\ 840 & -5 & 1 \end{bmatrix} \begin{bmatrix} -2521 & 14 & 1 \\ -446040 & 2477 & 177 \\ -110880 & 616 & 43 \end{bmatrix} \qquad \begin{bmatrix} -39 & -29 & -17 & -1 & -3 \\ -6900 & -5131 & -3008 & -177 & -532 \\ -1720 & -1274 & -744 & -43 & -126 \end{bmatrix}$$

$L_{38.3}$: $1^{2}_{6}8^{-}_{5},1^{-2}5^{-},1^{2}7^{1}$ $\langle m\rangle$ — $40^{r}_{2}14^{b}_{2}8^{*}_{2}4^{l}_{2}7_{2}$ $(\times 2)$

$$\begin{bmatrix} 264040 & -2240 & -840 \\ -2240 & 19 & 7 \\ -840 & 7 & -2 \end{bmatrix} \begin{bmatrix} -36961 & 309 & -42 \\ -4385920 & 36667 & -4984 \\ 258720 & -2163 & 293 \end{bmatrix} \qquad \begin{bmatrix} 119 & 86 & 49 & 5 & 3 \\ 14120 & 10206 & 5816 & 594 & 357 \\ -840 & -595 & -332 & -30 & -14 \end{bmatrix}$$

W_{39} 12 lattices, $\chi = 9$ — 5-gon: 42222

$L_{39.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{7},1^{2}5^{-},1^{2}7^{1}$ $\langle 2 \to N_{39}\rangle$ — $2^{*}_{4}4^{b}_{2}14^{s}_{2}10^{l}_{2}28^{r}_{2}$

$$\begin{bmatrix} -691460 & 2240 & 3640 \\ 2240 & -6 & -13 \\ 3640 & -13 & -18 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -2 & 2 & 15 \\ 115 & -114 & -231 & 225 & 1708 \\ 119 & -120 & -238 & 240 & 1792 \end{bmatrix}$$

W_{40} 8 lattices, $\chi = 40$ — 8-gon: $\infty 232 \infty 232 \rtimes C_2$

$L_{40.1}$: $1^{-2}_{\mathrm{II}}8^{1}_{7},1^{-2}19^{-}$ $\langle 2 \to N_{40}\rangle$ — $38^{4,3}_{\infty b}152^{b}_{2}2^{+}_{3}2^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} -1205512 & 6080 & 8968 \\ 6080 & -30 & -47 \\ 8968 & -47 & -62 \end{bmatrix} \begin{bmatrix} 67031 & -360 & -441 \\ 8557752 & -45961 & -56301 \\ 3202640 & -17200 & -21071 \end{bmatrix} \qquad \begin{bmatrix} 346 & 619 & 2 & -2 \\ 44175 & 79040 & 256 & -255 \\ 16530 & 29564 & 95 & -96 \end{bmatrix}$$

W_{41} 12 lattices, $\chi = 7$ — 4-gon: 2264

$L_{41.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}3^{-},1^{2}13^{1}$ $\langle 2 \to N_{41}\rangle$ — $4^{*}_{2}52^{b}_{2}6_{6}2^{*}_{4}$

$$\begin{bmatrix} -235092 & 2496 & 4836 \\ 2496 & -26 & -53 \\ 4836 & -53 & -94 \end{bmatrix} \qquad \begin{bmatrix} -3 & -3 & 4 & 2 \\ -178 & -182 & 237 & 120 \\ -54 & -52 & 72 & 35 \end{bmatrix}$$

W_{42} 104 lattices, $\chi = 36$ — 10-gon: $2222222222 \rtimes C_2$

$L_{42.1}$: $1^{-2}_{6}8^{-}_{5},1^{2}3^{1},1^{-2}13^{1}$ $\langle 2m \to N_{42}, 2 \to N'_{26}\rangle$ — $1_{2}13^{r}_{2}12^{*}_{2}8^{*}_{2}156^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -2834520 & 5616 & 5928 \\ 5616 & -10 & -13 \\ 5928 & -13 & -11 \end{bmatrix} \begin{bmatrix} 4679 & -10 & -9 \\ 1212120 & -2591 & -2331 \\ 1085760 & -2320 & -2089 \end{bmatrix} \qquad \begin{bmatrix} 1 & 9 & 7 & 9 & 71 \\ 258 & 2327 & 1812 & 2332 & 18408 \\ 233 & 2093 & 1626 & 2088 & 16458 \end{bmatrix}$$

$L_{42.2}$: $1^{2}_{0}8^{1}_{7},1^{2}3^{1},1^{-2}13^{1}$ $\langle m\rangle$ — $1^{r}_{2}52^{*}_{2}12^{s}_{2}8^{s}_{2}156^{*}_{2}4^{l}_{2}13_{2}3^{r}_{2}8^{l}_{2}39_{2}$

$$\begin{bmatrix} -3004872 & 12168 & 6240 \\ 12168 & -49 & -26 \\ 6240 & -26 & -11 \end{bmatrix} \qquad \begin{bmatrix} 2 & 31 & 11 & 13 & 97 & 5 & 4 & -1 & -1 & 10 \\ 414 & 6422 & 2280 & 2696 & 20124 & 1038 & 832 & -207 & -208 & 2067 \\ 155 & 2392 & 846 & 996 & 7410 & 380 & 299 & -78 & -76 & 780 \end{bmatrix}$$

$L_{42.3} : 1_6^2 8_1^1, 1^2 3^1, 1^{-2} 13^1 \ \langle m \rangle$ $\qquad 4_2^* 52_2^l 3_2 8_2 39_2^r \ (\times 2)$

$$\begin{bmatrix} -50843832 & -1271088 & 49920 \\ -1271088 & -31777 & 1248 \\ 49920 & 1248 & -49 \end{bmatrix} \begin{bmatrix} 47423 & 1186 & -46 \\ -1873248 & -46848 & 1817 \\ 592800 & 14825 & -576 \end{bmatrix} \qquad \begin{bmatrix} 7 & 49 & 8 & 17 & 58 \\ -274 & -1924 & -315 & -672 & -2301 \\ 152 & 910 & 126 & 200 & 468 \end{bmatrix}$$

$L_{42.4} : [1^- 2^1]_2 16_5^-, 1^2 3^1, 1^{-2} 13^1 \ \langle 2 \rangle$ $\qquad 16_2^s 52_2^* 48_2^s 8_2^* 624_2^l 1_2 208_2 3_2^r 8_2^* 156_2^s$

$$\begin{bmatrix} 341328 & 169728 & -624 \\ 169728 & 84398 & -310 \\ -624 & -310 & 1 \end{bmatrix} \qquad \begin{bmatrix} 201 & 333 & 65 & -1 & -77 & 0 & 205 & 34 & 135 & 1345 \\ -408 & -676 & -132 & 2 & 156 & 0 & -416 & -69 & -274 & -2730 \\ -1016 & -1690 & -336 & 0 & 312 & -1 & -1040 & -171 & -680 & -6786 \end{bmatrix}$$

$L_{42.5} : [1^1 2^1]_0 16_7^1, 1^2 3^1, 1^{-2} 13^1 \ \langle m \rangle$ $\qquad 16_2^* 52_2^s 48_2^l 2_2 624_2 1_2^r 208_2^* 12_2^l 2_2 39_2^r$

$$\begin{bmatrix} -7951632 & 7488 & 18096 \\ 7488 & -2 & -18 \\ 18096 & -18 & -41 \end{bmatrix} \qquad \begin{bmatrix} -1 & -3 & 1 & 3 & 167 & 4 & 71 & 7 & 2 & 7 \\ -76 & -234 & 72 & 229 & 12792 & 307 & 5460 & 540 & 155 & 546 \\ -408 & -1222 & 408 & 1222 & 68016 & 1629 & 28912 & 2850 & 814 & 2847 \end{bmatrix}$$

$L_{42.6} : [1^- 2^1]_4 16_3^-, 1^2 3^1, 1^{-2} 13^1$ $\qquad 16_2^l 13_2 48_2 2_2^r 624_2^s 4_2^* 208_2^l 3_2 2_2^r 156_2^*$

$$\begin{bmatrix} -501072 & 1872 & 2496 \\ 1872 & -2 & -12 \\ 2496 & -12 & -11 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 1 & 2 & 107 & 5 & 43 & 2 & 1 & 5 \\ -76 & -78 & 72 & 151 & 8112 & 380 & 3276 & 153 & 77 & 390 \\ -144 & -143 & 144 & 286 & 15288 & 714 & 6136 & 285 & 142 & 702 \end{bmatrix}$$

$L_{42.7} : [1^1 2^1]_6 16_1^1, 1^2 3^1, 1^{-2} 13^1 \ \langle m \rangle$ $\qquad 16_2 13_2^r 48_2^* 8_2^s 624_2^* 4_2^s 208_2^s 12_2^* 8_2^l 39_2$

$$\begin{bmatrix} 243984 & -1872 & 1872 \\ -1872 & 14 & -12 \\ 1872 & -12 & -1 \end{bmatrix} \qquad \begin{bmatrix} 7 & 4 & -1 & -1 & 1 & 1 & 23 & 5 & 7 & 29 \\ 1072 & 611 & -156 & -154 & 156 & 154 & 3536 & 768 & 1074 & 4446 \\ 160 & 91 & -24 & -24 & 0 & 22 & 520 & 114 & 160 & 663 \end{bmatrix}$$

W_{43} 24 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{43.1} : 1_{\mathrm{II}}^{-2} 4_3^-, 1^- 3^1 9^1, 1^{-2} 13^- \ \langle 23 \to N_{43}, 3, 2 \rangle$ $\qquad 12_2^r 26_2^b 36_2^b 2_2^l \ (\times 2)$

$$\begin{bmatrix} -805428 & 2808 & 4212 \\ 2808 & -6 & -15 \\ 4212 & -15 & -22 \end{bmatrix} \begin{bmatrix} -1561 & 7 & 8 \\ -21840 & 97 & 112 \\ -285480 & 1281 & 1463 \end{bmatrix} \qquad \begin{bmatrix} -9 & -14 & -5 & 0 \\ -128 & -195 & -66 & 1 \\ -1644 & -2561 & -918 & -1 \end{bmatrix}$$

W_{44} 32 lattices, $\chi = 16$ 5-gon: $\infty 2622$

$L_{44.1} : 1_{\mathrm{II}}^{-2} 8_1^1, 1^1 3^- 9^-, 1^{-2} 7^- \ \langle 23 \to N_{44}, 3, 2 \rangle$ $\qquad 42_{\infty a}^{12,1} 168_2^b 18_6 6_2^l 72_2^r$

$$\begin{bmatrix} -1218168 & -337176 & -519624 \\ -337176 & -91686 & -159513 \\ -519624 & -159513 & -71678 \end{bmatrix} \qquad \begin{bmatrix} 42396 & -28393 & -12611 & 13671 & 131419 \\ -131908 & 88340 & 39237 & -42535 & -408888 \\ -13797 & 9240 & 4104 & -4449 & -42768 \end{bmatrix}$$

W_{45} 24 lattices, $\chi = 12$ 6-gon: $222222 \rtimes C_2$

$L_{45.1} : 1_{\mathrm{II}}^{-2} 8_1^1, 1^- 3^1 9^-, 1^2 7^1 \ \langle 23 \to N_{45}, 3, 2 \rangle$ $\qquad 72_2^r 14_2^b 18_2^l 8_2^r 126_2^b 2_2^l$

$$\begin{bmatrix} -20664 & 3024 & 0 \\ 3024 & -438 & -3 \\ 0 & -3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 7 & 1 & -4 & -13 & -31 & 0 \\ 48 & 7 & -27 & -88 & -210 & 0 \\ 72 & 14 & -36 & -128 & -315 & -1 \end{bmatrix}$$

W_{46} 24 lattices, $\chi = 16$ 6-gon: 222262

$L_{46.1} : 1_{\mathrm{II}}^{-2} 4_7^1, 1^- 3^- 9^1, 1^{-2} 17^1 \ \langle 23 \to N_{46}, 3, 2 \rangle$ $\qquad 36_2^* 68_2^b 6_2^s 306_2^b 2_6 6_2^b$

$$\begin{bmatrix} -85866660 & -89964 & 332928 \\ -89964 & -66 & 333 \\ 332928 & 333 & -1282 \end{bmatrix} \qquad \begin{bmatrix} 17 & -63 & -21 & -215 & -8 & 7 \\ 2892 & -10710 & -3571 & -36567 & -1361 & 1190 \\ 5166 & -19142 & -6381 & -65331 & -2431 & 2127 \end{bmatrix}$$

W_{47} 44 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{47.1} : 1_{\mathrm{II}}^2 4_3^-, 1^2 3^1, 1^{-2} 17^1 \ \langle 2 \to N_{47} \rangle$ $\qquad 4_2^* 12_2^* 68_2^b 2_2^l 12_2^r 34_2^b \ (\times 2)$

$$\begin{bmatrix} -1142196 & 4488 & 4080 \\ 4488 & -16 & -17 \\ 4080 & -17 & -14 \end{bmatrix} \begin{bmatrix} 10097 & -45 & -33 \\ 1016532 & -4531 & -3322 \\ 1703196 & -7590 & -5567 \end{bmatrix} \qquad \begin{bmatrix} 13 & 37 & 191 & 7 & 7 & 5 \\ 1308 & 3726 & 19244 & 706 & 708 & 510 \\ 2194 & 6240 & 32198 & 1179 & 1176 & 833 \end{bmatrix}$$

$L_{47.2}: 1^{-2}_{6}8^{1}_{1},1^{2}3^{-},1^{-2}17^{1}\ \langle 2 \to N'_{28}\rangle$ $\qquad 8^{r}_{2}6^{l}_{2}136_{2}1^{r}_{2}24^{l}_{2}17_{2}\ (\times 2)$

$$\begin{bmatrix} -2521848 & 8568 & 4488 \\ 8568 & -26 & -17 \\ 4488 & -17 & -7 \end{bmatrix} \begin{bmatrix} 13871 & -52 & -22 \\ 2115480 & -7931 & -3355 \\ 3745440 & -14040 & -5941 \end{bmatrix} \begin{bmatrix} 17 & 25 & 263 & 5 & 11 & 5 \\ 2592 & 3813 & 40120 & 763 & 1680 & 765 \\ 4592 & 6750 & 70992 & 1349 & 2964 & 1343 \end{bmatrix}$$

$L_{47.3}: 1^{2}_{6}8^{-}_{5},1^{2}3^{-},1^{-2}17^{1}\ \langle m\rangle$ $\qquad 8^{b}_{2}6^{b}_{2}136^{*}_{2}4^{s}_{2}24^{s}_{2}68^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -6404376 & 3014712 & 21624 \\ 3014712 & -1419106 & -10179 \\ 21624 & -10179 & -73 \end{bmatrix} \begin{bmatrix} 281519 & -132526 & -943 \\ 599760 & -282339 & -2009 \\ -244800 & 115240 & 819 \end{bmatrix} \begin{bmatrix} 201 & 283 & 2903 & 105 & 101 & 63 \\ 428 & 603 & 6188 & 224 & 216 & 136 \\ -144 & -258 & -2992 & -134 & -204 & -306 \end{bmatrix}$$

W_{48} 12 lattices, $\chi = 18$ $\qquad$ 6-gon: $422422 \rtimes C_2$

$L_{48.1}: 1^{-2}_{\mathrm{II}}4^{1}_{7},1^{2}3^{1},1^{2}17^{-}\ \langle 2 \to N_{48}\rangle$ $\qquad 2^{*}_{4}4^{b}_{2}102^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -13414020 & 36108 & 52020 \\ 36108 & -94 & -145 \\ 52020 & -145 & -194 \end{bmatrix} \begin{bmatrix} 83095 & -247 & -286 \\ 16031136 & -47653 & -55176 \\ 10297512 & -30609 & -35443 \end{bmatrix} \begin{bmatrix} 6 & 33 & 235 \\ 1157 & 6366 & 45339 \\ 744 & 4090 & 29121 \end{bmatrix}$$

W_{49} 12 lattices, $\chi = 30$ $\qquad$ 8-gon: $22242224 \rtimes C_2$

$L_{49.1}: 1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}5^{1},1^{2}11^{-}\ \langle 2 \to N_{49}\rangle$ $\qquad 4^{*}_{2}20^{b}_{2}22^{s}_{2}2^{*}_{4}\ (\times 2)$

$$\begin{bmatrix} -1472020 & -81400 & 9240 \\ -81400 & -4494 & 511 \\ 9240 & 511 & -58 \end{bmatrix} \begin{bmatrix} 23099 & 1290 & -145 \\ -23100 & -1291 & 145 \\ 3474240 & 194016 & -21809 \end{bmatrix} \begin{bmatrix} 3 & 33 & 57 & 27 \\ -2 & -30 & -55 & -27 \\ 460 & 4990 & 8591 & 4061 \end{bmatrix}$$

W_{50} 12 lattices, $\chi = 24$ $\qquad$ 8-gon: $22222222 \rtimes C_2$

$L_{50.1}: 1^{-2}_{\mathrm{II}}4^{-}_{3},1^{-2}5^{-},1^{-2}11^{1}\ \langle 2 \to N_{50}\rangle$ $\qquad 2^{b}_{2}4^{b}_{2}10^{l}_{2}44^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -604340 & 110220 & 5280 \\ 110220 & -20102 & -963 \\ 5280 & -963 & -46 \end{bmatrix} \begin{bmatrix} 3079 & -562 & -26 \\ 16940 & -3092 & -143 \\ -1540 & 281 & 12 \end{bmatrix} \begin{bmatrix} 21 & 11 & 9 & -27 \\ 114 & 60 & 50 & -132 \\ 23 & 6 & -15 & -352 \end{bmatrix}$$

W_{51} 18 lattices, $\chi = 20$ $\qquad$ 6-gon: $226226 \rtimes C_2$

$L_{51.1}: 1^{-2}_{\mathrm{II}}4^{-}_{5},1^{-}3^{-}9^{-},1^{-2}19^{-}\ \langle 23 \to N_{51}, 3, 2\rangle$ $\qquad 6^{b}_{2}38^{s}_{2}18_{6}6^{b}_{2}342^{s}_{2}2_{6}$

$$\begin{bmatrix} -9620460 & 18468 & 25992 \\ 18468 & -30 & -51 \\ 25992 & -51 & -70 \end{bmatrix} \begin{bmatrix} -1 & -2 & 2 & 16 & 101 & 1 \\ -65 & -133 & 129 & 1046 & 6612 & 66 \\ -324 & -646 & 648 & 5175 & 32661 & 323 \end{bmatrix}$$

W_{52} 44 lattices, $\chi = 54$ $\qquad$ 12-gon: $222422222422 \rtimes C_2$

$L_{52.1}: 1^{2}_{\mathrm{II}}4^{1}_{1},1^{2}3^{-},1^{2}19^{1}\ \langle 2 \to N_{52}\rangle$ $\qquad 6^{l}_{2}4^{r}_{2}114^{b}_{2}2^{*}_{4}4^{*}_{2}76^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -833340 & -411084 & 2736 \\ -411084 & -202786 & 1349 \\ 2736 & 1349 & -8 \end{bmatrix} \begin{bmatrix} -684191 & -337109 & 1662 \\ 1393080 & 686387 & -3384 \\ 904020 & 445422 & -2197 \end{bmatrix} \begin{bmatrix} -1967 & -609 & -4619 & -138 & -165 & -877 \\ 4005 & 1240 & 9405 & 281 & 336 & 1786 \\ 2598 & 808 & 6156 & 186 & 226 & 1216 \end{bmatrix}$$

$L_{52.2}: 1^{2}_{2}8^{1}_{7},1^{2}3^{1},1^{2}19^{-}\ \langle 2 \to N'_{29}\rangle$ $\qquad 12^{s}_{2}8^{l}_{2}57_{2}1_{4}2^{b}_{2}152^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -1477896 & -246696 & 4560 \\ -246696 & -41179 & 761 \\ 4560 & 761 & -14 \end{bmatrix} \begin{bmatrix} 6079 & 1018 & -20 \\ -36480 & -6109 & 120 \\ -9120 & -1527 & 29 \end{bmatrix} \begin{bmatrix} 29 & 1 & -26 & -3 & -7 & -97 \\ -174 & -4 & 171 & 19 & 44 & 608 \\ -42 & 100 & 798 & 55 & 111 & 1444 \end{bmatrix}$$

$L_{52.3}: 1^{-2}_{2}8^{-}_{3},1^{2}3^{1},1^{2}19^{-}\ \langle m\rangle$ $\qquad 3^{r}_{2}8^{s}_{2}228^{*}_{2}4^{*}_{4}2^{l}_{2}152_{2}\ (\times 2)$

$$\begin{bmatrix} -508748712 & -3325152 & 93024 \\ -3325152 & -21733 & 608 \\ 93024 & 608 & -17 \end{bmatrix} \begin{bmatrix} 658615 & 4305 & -119 \\ -100768248 & -658666 & 18207 \\ -282264 & -1845 & 50 \end{bmatrix}$$

$$\begin{bmatrix} 50 & 29 & 205 & 5 & 2 & 13 \\ -7650 & -4436 & -31350 & -764 & -305 & -1976 \\ -21 & 24 & 456 & 34 & 35 & 456 \end{bmatrix}$$

W_{53} 12 lattices, $\chi = 28$ 8-gon: $22232223 \rtimes C_2$

$L_{53.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{5},1^{-2}5^{1},1^{2}13^{-}$ $\langle 2 \to N_{53} \rangle$ $2^{b}_{2}26^{l}_{2}20^{r}_{2}2^{-}_{3}$ $(\times 2)$

$$\begin{bmatrix} -527020 & 3120 & 4680 \\ 3120 & -18 & -29 \\ 4680 & -29 & -38 \end{bmatrix} \begin{bmatrix} 12349 & -75 & -105 \\ 1348620 & -8191 & -11466 \\ 489060 & -2970 & -4159 \end{bmatrix} \begin{bmatrix} -1 & -2 & 7 & 4 \\ -109 & -221 & 760 & 436 \\ -40 & -78 & 280 & 159 \end{bmatrix}$$

W_{54} 32 lattices, $\chi = 48$ 10-gon: $\infty 2222 \infty 2222 \rtimes C_2$

$L_{54.1}$: $1^{-2}_{\mathrm{II}}8^{-}_{5},1^{1}3^{1}9^{-},1^{-2}11^{-}$ $\langle 23 \to N_{54}, 3, 2 \rangle$ $66^{12,1}_{\infty b}264^{b}_{2}18^{s}_{2}22^{b}_{2}72^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} -114840 & 24552 & 9504 \\ 24552 & -5190 & -2181 \\ 9504 & -2181 & -410 \end{bmatrix} \begin{bmatrix} 375935 & -88466 & -10680 \\ 1524864 & -358835 & -43320 \\ 601920 & -141645 & -17101 \end{bmatrix} \begin{bmatrix} -1874 & -8407 & -1619 & -1581 & -991 \\ -7601 & -34100 & -6567 & -6413 & -4020 \\ -3003 & -13464 & -2592 & -2530 & -1584 \end{bmatrix}$$

W_{55} 24 lattices, $\chi = 20$ 6-gon: $622622 \rtimes C_2$

$L_{55.1}$: $1^{-2}_{\mathrm{II}}8^{-}_{5},1^{-}3^{-}9^{-},1^{2}11^{1}$ $\langle 23 \to N_{55}, 3, 2 \rangle$ $18_{6}6^{b}_{2}72^{b}_{2}2_{6}6^{b}_{2}8^{b}_{2}$

$$\begin{bmatrix} -16860888 & 101376 & -15048 \\ 101376 & -606 & 87 \\ -15048 & 87 & -10 \end{bmatrix} \begin{bmatrix} 4 & -1 & -19 & -8 & -3 & 1 \\ 783 & -196 & -3720 & -1566 & -587 & 196 \\ 792 & -201 & -3780 & -1589 & -594 & 200 \end{bmatrix}$$

W_{56} 12 lattices, $\chi = 12$ 6-gon: 222222

$L_{56.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{2}3^{1},1^{-2}23^{1}$ $\langle 2 \to N_{56} \rangle$ $12^{*}_{2}92^{b}_{2}2^{b}_{2}138^{l}_{2}4^{r}_{2}46^{b}_{2}$

$$\begin{bmatrix} -5244 & 276 & 0 \\ 276 & -2 & -5 \\ 0 & -5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 0 & -7 & -1 & -1 \\ 18 & 92 & 0 & -138 & -20 & -23 \\ 48 & 230 & -1 & -345 & -48 & -46 \end{bmatrix}$$

W_{57} 16 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{57.1}$: $1^{-2}_{\mathrm{II}}8^{1}_{7},1^{-2}5^{-},1^{2}7^{1}$ $\langle 2 \to N_{57} \rangle$ $10^{s}_{2}14^{b}_{2}2^{l}_{2}56^{r}_{2}$ $(\times 2)$

$$\begin{bmatrix} 3640 & 1400 & 0 \\ 1400 & 538 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 8 & 5 & 65 \\ 5 & -21 & -13 & -168 \\ 0 & -14 & -8 & -112 \end{bmatrix}$$

W_{58} 12 lattices, $\chi = 20$ 7-gon: 2222232

$L_{58.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{-2}7^{1},1^{2}11^{1}$ $\langle 2 \to N_{58} \rangle$ $28^{*}_{2}44^{b}_{2}14^{l}_{2}4^{r}_{2}154^{b}_{2}2^{+}_{3}2^{b}_{2}$

$$\begin{bmatrix} -19740028 & 77308 & -33264 \\ 77308 & -302 & 127 \\ -33264 & 127 & -42 \end{bmatrix} \begin{bmatrix} 11 & -29 & -41 & -137 & -912 & -5 & 4 \\ 3122 & -8228 & -11634 & -38876 & -258797 & -1419 & 1135 \\ 728 & -1914 & -2709 & -9056 & -60291 & -331 & 264 \end{bmatrix}$$

W_{59} 32 lattices, $\chi = 28$ 7-gon: $\infty 222262$

$L_{59.1}$: $1^{-2}_{\mathrm{II}}8^{-}_{3},1^{1}3^{-}9^{-},1^{2}13^{-}$ $\langle 23 \to N_{59}, 3, 2 \rangle$
$78^{12,7}_{\infty b}312^{b}_{2}18^{l}_{2}24^{r}_{2}234^{b}_{2}6_{6}18^{b}_{2}$

$$\begin{bmatrix} -15718248 & -2834208 & -2395224 \\ -2834208 & -485598 & -457431 \\ -2395224 & -457431 & -339362 \end{bmatrix} \begin{bmatrix} -26015 & 222369 & 91435 & 195783 & 443503 & 13008 & -13102 \\ 78325 & -669500 & -275289 & -589456 & -1335282 & -39164 & 39447 \\ 78039 & -667056 & -274284 & -587304 & -1330407 & -39021 & 39303 \end{bmatrix}$$

W_{60} 12 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{60.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{2}5^{-},1^{-2}17^{1}$ $\langle 2 \to N_{60} \rangle$ $34^{l}_{2}4^{r}_{2}10^{b}_{2}2^{l}_{2}68^{r}_{2}10^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 47940 & 19040 & -340 \\ 19040 & 7562 & -135 \\ -340 & -135 & 2 \end{bmatrix} \begin{bmatrix} -89761 & -35376 & -330 \\ 227120 & 89511 & 835 \\ 68000 & 26800 & 249 \end{bmatrix} \begin{bmatrix} 2197 & 215 & 83 & 0 & -27 & -2 \\ -5559 & -544 & -210 & 0 & 68 & 5 \\ -1666 & -164 & -65 & -1 & 0 & 0 \end{bmatrix}$$

W_{61} 12 lattices, $\chi = 30$ 8-gon: $22422242 \rtimes C_2$

$L_{61.1}: 1_{\mathrm{II}}^{-2} 4_{3}^{-}, 1^{2} 3^{1}, 1^{2} 29^{-}$ $\langle 2 \to N_{61} \rangle$ $12_{2}^{r} 58_{2}^{b} 4_{4}^{*} 2_{2}^{l}$ $(\times 2)$

$$\begin{bmatrix} -27083796 & 37932 & 54288 \\ 37932 & -50 & -79 \\ 54288 & -79 & -106 \end{bmatrix} \begin{bmatrix} 428735 & -660 & -803 \\ 122111808 & -187981 & -228709 \\ 128542848 & -197880 & -240755 \end{bmatrix} \begin{bmatrix} 125 & 644 & 135 & 8 \\ 35604 & 183425 & 38450 & 2278 \\ 37476 & 193082 & 40476 & 2399 \end{bmatrix}$$

W_{62} 12 lattices, $\chi = 42$ 10-gon: $4222242222 \rtimes C_2$

$L_{62.1}: 1_{\mathrm{II}}^{-2} 4_{7}^{1}, 1^{2} 7^{1}, 1^{2} 13^{-}$ $\langle 2 \to N_{62} \rangle$ $2_{4}^{*} 4_{2}^{b} 14_{2}^{s} 26_{2}^{l} 28_{2}^{r}$ $(\times 2)$

$$\begin{bmatrix} -22161412 & 108836 & 217672 \\ 108836 & -534 & -1069 \\ 217672 & -1069 & -2138 \end{bmatrix} \begin{bmatrix} 586403 & -2961 & -5760 \\ 1172808 & -5923 & -11520 \\ 59096492 & -298403 & -580481 \end{bmatrix} \begin{bmatrix} 0 & -5 & -62 & -360 & -767 \\ 2 & -2 & -105 & -689 & -1512 \\ -1 & -508 & -6258 & -36296 & -77308 \end{bmatrix}$$

W_{63} 12 lattices, $\chi = 32$ 8-gon: $62226222 \rtimes C_2$

$L_{63.1}: 1_{\mathrm{II}}^{-2} 4_{1}^{1}, 1^{2} 3^{-}, 1^{-2} 31^{-}$ $\langle 2 \to N_{63} \rangle$ $2_{6} 6_{2}^{l} 4_{2}^{r} 186_{2}^{b}$ $(\times 2)$

$$\begin{bmatrix} -2765820 & 13020 & 6324 \\ 13020 & -58 & -31 \\ 6324 & -31 & -14 \end{bmatrix} \begin{bmatrix} 43151 & -200 & -100 \\ 3980772 & -18451 & -9225 \\ 10658544 & -49400 & -24701 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 & 11 \\ 184 & -93 & -92 & 1023 \\ 495 & -246 & -248 & 2697 \end{bmatrix}$$

W_{64} 12 lattices, $\chi = 40$ 10-gon: $3222232222 \rtimes C_2$

$L_{64.1}: 1_{\mathrm{II}}^{-2} 4_{3}^{-}, 1^{-2} 5^{-}, 1^{-2} 19^{1}$ $\langle 2 \to N_{64} \rangle$ $2_{3}^{+} 2_{2}^{b} 4_{2}^{b} 10_{2}^{l} 76_{2}^{r}$ $(\times 2)$

$$\begin{bmatrix} -2676340 & -703760 & 14820 \\ -703760 & -185058 & 3897 \\ 14820 & 3897 & -82 \end{bmatrix} \begin{bmatrix} -140449 & -36927 & 759 \\ 532000 & 139874 & -2875 \\ -106400 & -27975 & 574 \end{bmatrix} \begin{bmatrix} -16 & -30 & -77 & -301 & -2329 \\ 61 & 114 & 292 & 1140 & 8816 \\ 7 & -5 & -42 & -235 & -2052 \end{bmatrix}$$

W_{65} 16 lattices, $\chi = 32$ 8-gon: $22262226 \rtimes C_2$

$L_{65.1}: 1_{\mathrm{II}}^{-2} 8_{7}^{1}, 1^{2} 3^{-}, 1^{-2} 17^{1}$ $\langle 2 \to N_{65} \rangle$ $6_{2}^{s} 34_{2}^{b} 24_{2}^{b} 2_{6}$ $(\times 2)$

$$\begin{bmatrix} -15547656 & 20808 & 31824 \\ 20808 & -26 & -45 \\ 31824 & -45 & -62 \end{bmatrix} \begin{bmatrix} 395759 & -530 & -810 \\ 136022712 & -182162 & -278397 \\ 104361912 & -139761 & -213598 \end{bmatrix} \begin{bmatrix} 272 & 514 & 115 & 2 \\ 93486 & 176664 & 39528 & 688 \\ 71727 & 135541 & 30324 & 527 \end{bmatrix}$$

W_{66} 48 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{66.1}: 1_{\mathrm{II}}^{-2} 4_{5}^{-}, 1^{1} 3^{-} 9^{-}, 1^{2} 5^{1}, 1^{2} 7^{-}$ $\langle 23 \to N_{66}, 3, 2 \rangle$ $180_{2}^{r} 42_{2}^{b} 18_{2}^{s} 70_{2}^{b} 6_{2}^{l}$ $(\times 2)$

$$\begin{bmatrix} -21420 & -3780 & 13860 \\ -3780 & -642 & 2001 \\ 13860 & 2001 & -1070 \end{bmatrix} \begin{bmatrix} -9409 & -1456 & 2464 \\ 67200 & 10399 & -17600 \\ 3780 & 585 & -991 \end{bmatrix} \begin{bmatrix} -899 & -1 & 134 & 348 & 15 \\ 6420 & 7 & -957 & -2485 & -107 \\ 360 & 0 & -54 & -140 & -6 \end{bmatrix}$$

W_{67} 88 lattices, $\chi = 27$ 8-gon: 22422222

$L_{67.1}: 1_{\mathrm{II}}^{2} 4_{1}^{1}, 1^{2} 3^{1}, 1^{2} 5^{1}, 1^{2} 7^{-}$ $\langle 2 \to N_{67} \rangle$ $20_{2}^{*} 84_{2}^{*} 4_{4}^{*} 2_{2}^{s} 30_{2}^{l} 4_{2}^{r} 70_{2}^{b} 12_{2}^{*}$

$$\begin{bmatrix} -5778780 & -1930740 & 30240 \\ -1930740 & -645076 & 10103 \\ 30240 & 10103 & -158 \end{bmatrix} \begin{bmatrix} -159 & 41 & 37 & -26 & -302 & -187 & -1886 & -631 \\ 490 & -126 & -114 & 80 & 930 & 576 & 5810 & 1944 \\ 900 & -210 & -208 & 139 & 1665 & 1040 & 10535 & 3534 \end{bmatrix}$$

$L_{67.2}: 1_{2}^{2} 8_{7}^{1}, 1^{2} 3^{-}, 1^{2} 5^{-}, 1^{2} 7^{-}$ $\langle 2 \to N'_{33} \rangle$ $10_{2}^{s} 42_{2}^{b} 2_{4} 1_{2}^{r} 60_{2}^{s} 8_{2}^{s} 140_{2}^{*} 24_{2}^{b}$

$$\begin{bmatrix} -24797640 & 31920 & 44520 \\ 31920 & -38 & -59 \\ 44520 & -59 & -79 \end{bmatrix} \begin{bmatrix} 4 & -2 & -1 & 1 & 19 & 11 & 107 & 35 \\ 875 & -441 & -219 & 220 & 4170 & 2412 & 23450 & 7668 \\ 1600 & -798 & -400 & 399 & 7590 & 4396 & 42770 & 13992 \end{bmatrix}$$

$L_{67.3}: 1_{2}^{-2} 8_{3}^{-}, 1^{2} 3^{-}, 1^{2} 5^{-}, 1^{2} 7^{-}$ $\langle m \rangle$ $10_{2}^{b} 42_{2}^{s} 2_{4}^{*} 4_{2}^{l} 15_{2}^{r} 8_{2}^{l} 35_{2} 24_{2}^{r}$

$$\begin{bmatrix} -32246760 & 36960 & 22680 \\ 36960 & -41 & -28 \\ 22680 & -28 & -13 \end{bmatrix} \begin{bmatrix} 8 & 13 & 1 & -1 & -1 & 3 & 32 & 29 \\ 4925 & 8001 & 615 & -616 & -615 & 1848 & 19705 & 17856 \\ 3345 & 5439 & 419 & -418 & -420 & 1252 & 13370 & 12120 \end{bmatrix}$$

W_{68} 176 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{68.1}: 1_{\mathrm{II}}^{2}4_{1}^{1},1^{-}3^{-}9^{1},1^{-2}5^{-},1^{2}7^{-}$ $\langle 23 \to N_{68}, 3, 2\rangle$

$36_2^{*}60_2^{b}2_2^{l}36_2^{r}42_2^{b}90_2^{s}6_2^{b}140_2^{*}$ $(\times 2)$

$$\begin{bmatrix} -15952860 & 39060 & 3780 \\ 39060 & -48 & -21 \\ 3780 & -21 & 2 \end{bmatrix} \begin{bmatrix} 145529 & -387 & -27 \\ 42656460 & -113435 & -7914 \\ 172986660 & -460014 & -32095 \end{bmatrix}$$

$$\begin{bmatrix} 325 & 363 & 27 & 47 & 31 & 26 & 2 & 11 \\ 95262 & 106400 & 7914 & 13776 & 9086 & 7620 & 586 & 3220 \\ 386316 & 431490 & 32095 & 55872 & 36855 & 30915 & 2379 & 13090 \end{bmatrix}$$

$L_{68.2}: 1_{2}^{-2}8_{3}^{-},1^{1}3^{1}9^{-},1^{-2}5^{1},1^{2}7^{-}$ $\langle 3m, 3, 2\rangle$

$18_2^{b}120_2^{*}4_2^{s}72_2^{s}84_2^{*}180_2^{l}3_2 280_2^{r}$ $(\times 2)$

$$\begin{bmatrix} 6749126720280 & -24187683240 & 83739600 \\ -24187683240 & 86684403 & -300108 \\ 83739600 & -300108 & 1039 \end{bmatrix} \begin{bmatrix} 614748959 & -2203188 & 8004 \\ 171590920200 & -614961686 & 2234105 \\ 16338543480 & -58555419 & 212726 \end{bmatrix}$$

$$\begin{bmatrix} 12529 & 27543 & 1983 & 3181 & 1805 & 1141 & 8 & 1 \\ 3497139 & 7687900 & 553502 & 887892 & 503818 & 318480 & 2233 & 280 \\ 332955 & 731940 & 52696 & 84528 & 47964 & 30330 & 216 & 280 \end{bmatrix}$$

$L_{68.3}: 1_{2}^{2}8_{7}^{1},1^{1}3^{1}9^{-},1^{-2}5^{1},1^{2}7^{-}$ $\langle 32 \to N'_{32}, 3, m\rangle$

$18_2^{l}120_2 1_2^{r}72_2^{l}21_2 45_2^{r}12_2^{*}280_2^{b}$ $(\times 2)$

$$\begin{bmatrix} 9024120 & -22680 & 0 \\ -22680 & 57 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2684639 & -6715 & -663 \\ 1071487200 & -2680076 & -264615 \\ 18476640 & -46215 & -4564 \end{bmatrix}$$

$$\begin{bmatrix} -613 & -1361 & -50 & -169 & -53 & -41 & -5 & -27 \\ -244659 & -543200 & -19956 & -67452 & -21154 & -16365 & -1996 & -10780 \\ -4221 & -9360 & -343 & -1152 & -357 & -270 & -30 & -140 \end{bmatrix}$$

W_{69} 24 lattices, $\chi = 6$ 5-gon: 22222

$L_{69.1}: 1_{\mathrm{II}}^{-2}4_{5}^{-},1^{2}3^{1},1^{-2}5^{-},1^{2}7^{-}$ $\langle 2 \to N_{69}\rangle$ $12_2^{b}10_2^{l}84_2^{r}2_2^{b}140_2^{*}$

$$\begin{bmatrix} -3898860 & 1115100 & 10080 \\ 1115100 & -318926 & -2883 \\ 10080 & -2883 & -26 \end{bmatrix} \qquad \begin{bmatrix} -19 & 13 & 121 & 2 & -101 \\ -66 & 45 & 420 & 7 & -350 \\ -48 & 50 & 336 & -1 & -350 \end{bmatrix}$$

W_{70} 24 lattices, $\chi = 12$ 6-gon: 222222

$L_{70.1}: 1_{\mathrm{II}}^{-2}4_{5}^{-},1^{2}3^{1},1^{2}5^{1},1^{-2}7^{1}$ $\langle 2 \to N_{70}\rangle$ $12_2^{*}28_2^{b}30_2^{b}14_2^{l}20_2^{r}2_2^{b}$

$$\begin{bmatrix} -713580 & -355320 & 420 \\ -355320 & -176926 & 207 \\ 420 & 207 & 2 \end{bmatrix} \qquad \begin{bmatrix} 191 & 383 & 97 & -94 & -189 & 0 \\ -384 & -770 & -195 & 189 & 380 & 0 \\ -366 & -728 & -180 & 182 & 360 & -1 \end{bmatrix}$$

W_{71} 48 lattices, $\chi = 16$ 6-gon: 222262

$L_{71.1}: 1_{\mathrm{II}}^{-2}4_{5}^{-},1^{1}3^{-}9^{-},1^{-2}5^{-},1^{-2}7^{1}$ $\langle 23 \to N_{71}, 3, 2\rangle$

$28_2^{*}60_2^{b}126_2^{s}10_2^{b}18_6 6_2^{b}$

$$\begin{bmatrix} -7158060 & -2882880 & 10080 \\ -2882880 & -1161066 & 4059 \\ 10080 & 4059 & -14 \end{bmatrix} \qquad \begin{bmatrix} -117 & -187 & -142 & 2 & 37 & -2 \\ 294 & 470 & 357 & -5 & -93 & 5 \\ 994 & 1620 & 1260 & -10 & -324 & 9 \end{bmatrix}$$

W_{72} 88 lattices, $\chi = 24$ 8-gon: 22222222

$L_{72.1}$: $1_{\mathrm{II}}^{2}4_1^{1},1^{2}3^{1},1^{-2}5^{-},1^{-2}7^{1}$ $\langle 2 \to N_{72}\rangle$ $12_2^{b}14_2^{s}2_2^{b}210_2^{l}4_2^{r}10_2^{b}28_2^{*}4_2^{*}$

$$\begin{bmatrix} -133966140 & 1288140 & -32760 \\ 1288140 & -12386 & 315 \\ -32760 & 315 & -8 \end{bmatrix} \quad \begin{bmatrix} 11 & 10 & 2 & 11 & -1 & -1 & 5 & 3 \\ 1146 & 1043 & 209 & 1155 & -104 & -105 & 518 & 312 \\ 72 & 112 & 38 & 420 & 0 & -40 & -84 & -2 \end{bmatrix}$$

$L_{72.2}$: $1_2^{2}8_7^{1},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$ $\langle 2 \to N'_{34}\rangle$ $24_2^{*}28_2^{l}1_2105_2^{r}8_2^{l}5_256_2^{r}2_2^{b}$

$$\begin{bmatrix} -34131720 & 52920 & 26040 \\ 52920 & -79 & -42 \\ 26040 & -42 & -19 \end{bmatrix} \quad \begin{bmatrix} 17 & 17 & 2 & 16 & -1 & -1 & 5 & 2 \\ 5700 & 5698 & 670 & 5355 & -336 & -335 & 1680 & 671 \\ 10692 & 10696 & 1259 & 10080 & -628 & -630 & 3136 & 1257 \end{bmatrix}$$

$L_{72.3}$: $1_2^{-2}8_3^{-},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$ $\langle m\rangle$ $24_27_2^{r}4_2^{*}420_2^{s}8_2^{s}20_2^{*}56_2^{b}2_2^{l}$

$$\begin{bmatrix} -434280 & -43680 & 2520 \\ -43680 & -4393 & 253 \\ 2520 & 253 & -14 \end{bmatrix} \quad \begin{bmatrix} -23 & -6 & 1 & 61 & 5 & -1 & -27 & -5 \\ 240 & 63 & -10 & -630 & -52 & 10 & 280 & 52 \\ 192 & 56 & -2 & -420 & -40 & 0 & 196 & 39 \end{bmatrix}$$

W_{73} 32 lattices, $\chi = 72$ 14-gon: $\infty222222\infty222222 \rtimes C_2$

$L_{73.1}$: $1_{\mathrm{II}}^{-2}8_3^{-},1^{-}5^{-}25^{-},1^{-2}11^{-}$ $\langle 25 \to N_{73}, 5, 2*\rangle$
$110_{\infty a}^{20,19}440_2^{b}50_2^{l}88_2^{r}10_2^{b}550_2^{s}2_2^{b}$ $(\times 2)$

shares genus with its 5-dual

$$\begin{bmatrix} -2584117800 & -1286540200 & 1826000 \\ -1286540200 & -640522510 & 909115 \\ 1826000 & 909115 & -1282 \end{bmatrix} \quad \begin{bmatrix} 2951517359 & 1469189856 & -2235998 \\ -5913532680 & -2943605329 & 4479949 \\ 10443879600 & 5198696160 & -7912031 \end{bmatrix}$$

$$\begin{bmatrix} -1411 & 15175 & 282 & -33161 & -5621 & -79828 & -5060 \\ 2827 & -30404 & -565 & 66440 & 11262 & 159940 & 10138 \\ -5005 & 53680 & 1000 & -117304 & -19885 & -282425 & -17903 \end{bmatrix}$$

W_{74} 12 lattices, $\chi = 38$ 8-gon: $22642264 \rtimes C_2$

$L_{74.1}$: $1_{\mathrm{II}}^{-2}4_3^{-},1^{2}3^{-},1^{2}37^{1}$ $\langle 2 \to N_{74}\rangle$ $4_2^{*}148_2^{b}6_62_4^{*}$ $(\times 2)$

$$\begin{bmatrix} -3066708 & 21312 & 20868 \\ 21312 & -146 & -145 \\ 20868 & -145 & -142 \end{bmatrix} \begin{bmatrix} 124319 & -917 & -847 \\ -124320 & 916 & 847 \\ 18381600 & -135585 & -125236 \end{bmatrix} \quad \begin{bmatrix} 37 & 327 & 1 & -1 \\ -40 & -370 & -3 & 2 \\ 5474 & 48396 & 150 & -149 \end{bmatrix}$$

W_{75} 16 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{75.1}$: $1_{\mathrm{II}}^{-2}8_5^{-},1^{-2}5^{-},1^{-2}13^{-}$ $\langle 2 \to N_{75}\rangle$ $8_2^{b}26_2^{l}40_2^{r}2_2^{l}104_2^{r}10_2^{b}$ $(\times 2)$

$$\begin{bmatrix} -305240 & -121680 & 2080 \\ -121680 & -48506 & 829 \\ 2080 & 829 & -14 \end{bmatrix} \begin{bmatrix} 18199 & 7245 & -112 \\ -46800 & -18631 & 288 \\ -70200 & -27945 & 431 \end{bmatrix} \quad \begin{bmatrix} 17 & 278 & 607 & 95 & 1661 & 41 \\ -44 & -715 & -1560 & -244 & -4264 & -105 \\ -84 & -1079 & -2280 & -347 & -5928 & -130 \end{bmatrix}$$

W_{76} 12 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{76.1}$: $1_{\mathrm{II}}^{-2}4_1^{1},1^{2}3^{1},1^{-2}47^{1}$ $\langle 2 \to N_{76}\rangle$ $12_2^{*}188_2^{b}2_2^{b}282_2^{l}4_2^{r}94_2^{b}$ $(\times 2)$

$$\begin{bmatrix} -1188348 & 1692 & 3384 \\ 1692 & -2 & -7 \\ 3384 & -7 & 2 \end{bmatrix} \begin{bmatrix} -9401 & 18 & 2 \\ -4808100 & 9206 & 1023 \\ -916500 & 1755 & 194 \end{bmatrix} \quad \begin{bmatrix} -29 & -195 & -2 & -35 & -1 & -1 \\ -14832 & -99734 & -1023 & -17907 & -512 & -517 \\ -2826 & -18988 & -194 & -3384 & -96 & -94 \end{bmatrix}$$

W_{77} 12 lattices, $\chi = 64$ 14-gon: $22222232222223 \rtimes C_2$

$L_{77.1} : 1_{\mathrm{II}}^{-2}4_7^1,1^{-2}5^-,1^{-2}31^1 \ \langle 2 \to N_{77} \rangle$ $2_2^s 310_2^b 4_2^b 62_2^s 10_2^l 124_2^r 2_3^+ \ (\times 2)$

$$\begin{bmatrix} 156860 & -31620 & -620 \\ -31620 & 6374 & 125 \\ -620 & 125 & 2 \end{bmatrix} \begin{bmatrix} 463511 & -94251 & 2670 \\ 2291520 & -465961 & 13200 \\ 425320 & -86485 & 2449 \end{bmatrix}$$

$$\begin{bmatrix} -18 & -4546 & -481 & -8208 & -7744 & -99349 & -2670 \\ -89 & -22475 & -2378 & -40579 & -38285 & -491164 & -13200 \\ -17 & -4185 & -442 & -7533 & -7105 & -91140 & -2449 \end{bmatrix}$$

W_{78} 24 lattices, $\chi = 10$ 5-gon: 26222

$L_{78.1} : 1_{\mathrm{II}}^{-2}4_1^1,1^2 3^-,1^{-2}5^1,1^2 11^- \ \langle 2 \to N_{78} \rangle$ $22_2^b 6_6 2_2^b 330_2^l 4_2^r$

$$\begin{bmatrix} -34021020 & 23100 & 36960 \\ 23100 & -14 & -27 \\ 36960 & -27 & -38 \end{bmatrix} \begin{bmatrix} -2 & -1 & 1 & 26 & 1 \\ -1221 & -609 & 610 & 15840 & 608 \\ -1078 & -540 & 539 & 14025 & 540 \end{bmatrix}$$

W_{79} 48 lattices, $\chi = 60$ 14-gon: $22222222222222 \rtimes C_2$

$L_{79.1} : 1_{\mathrm{II}}^{-2}4_1^1,1^1 3^1 9^-,1^2 5^-,1^2 11^- \ \langle 23 \to N_{79}, 3, 2 \rangle$

$18_2^s 22_2^b 12_2^b 990_2^l 4_2^r 66_2^b 10_2^b \ (\times 2)$

$$\begin{bmatrix} 24993540 & 79200 & -77220 \\ 79200 & 246 & -243 \\ -77220 & -243 & 238 \end{bmatrix} \begin{bmatrix} -121441 & -440 & 394 \\ -19005360 & -68861 & 61661 \\ -58655520 & -212520 & 190301 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 23 & 29 & 989 & 63 & 105 & 8 \\ 804 & 3630 & 4552 & 154770 & 9848 & 16379 & 1235 \\ 2439 & 11143 & 14022 & 477675 & 30416 & 50655 & 3845 \end{bmatrix}$$

W_{80} 88 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{80.1} : 1_{\mathrm{II}}^2 4_5^-,1^2 3^-,1^{-2}5^1,1^{-2}11^1 \ \langle 2 \to N_{80} \rangle$

$44_2^* 20_2^* 132_2^* 4_2^* 220_2^b 6_2^l 20_2^r 2_2^b \ (\times 2)$

$$\begin{bmatrix} 248820 & 0 & -660 \\ 0 & 4 & -1 \\ -660 & -1 & 2 \end{bmatrix} \begin{bmatrix} -551 & -2 & 2 \\ -52800 & -193 & 192 \\ -204600 & -744 & 743 \end{bmatrix} \begin{bmatrix} -21 & -9 & -17 & -1 & -29 & -2 & -1 & 0 \\ -2024 & -870 & -1650 & -98 & -2860 & -198 & -100 & 0 \\ -7810 & -3350 & -6336 & -374 & -10890 & -753 & -380 & -1 \end{bmatrix}$$

$L_{80.2} : 1_2^2 8_3^-,1^2 3^1,1^{-2}5^-,1^{-2}11^- \ \langle 2 \to N'_{36} \rangle$

$88_2^r 10_2^b 66_2^s 2_2^b 440_2^* 12_2^s 40_2^l 1_2 \ (\times 2)$

$$\begin{bmatrix} 3198360 & 531960 & -2640 \\ 531960 & 88477 & -439 \\ -2640 & -439 & 2 \end{bmatrix} \begin{bmatrix} -233531 & -38600 & -193 \\ 1408440 & 232799 & 1164 \\ 885720 & 146400 & 731 \end{bmatrix}$$

$$\begin{bmatrix} 1605 & 514 & 1543 & 193 & 21303 & 1931 & 1867 & 161 \\ -9680 & -3100 & -9306 & -1164 & -128480 & -11646 & -11260 & -971 \\ -6072 & -1945 & -5841 & -731 & -80740 & -7320 & -7080 & -611 \end{bmatrix}$$

$L_{80.3} : 1_2^{-2}8_7^1,1^2 3^1,1^{-2}5^-,1^{-2}11^- \ \langle m \rangle$ $88_2^b 10_2^s 66_2^b 2_2^l 440_2 3_2^r 40_2^s 4_2^* \ (\times 2)$

$$\begin{bmatrix} 86152440 & 195360 & 0 \\ 195360 & 443 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 862069 & 1955 & -102 \\ -380325000 & -862501 & 45000 \\ -3651120 & -8280 & 431 \end{bmatrix}$$

$$\begin{bmatrix} 107 & 37 & 119 & 16 & 1869 & 86 & 171 & 31 \\ -47212 & -16325 & -52503 & -7059 & -824560 & -37941 & -75440 & -13676 \\ -572 & -185 & -561 & -71 & -7920 & -360 & -700 & -122 \end{bmatrix}$$

W_{81} 48 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{81.1}$: $1_{\rm II}^{-2}4_1^1,1^-3^19^1,1^{-2}5^1,1^{-2}11^1$ $\langle 23 \to N_{81}, 3, 2\rangle$
$12_2^*1980_2^b2_2^l36_2^r30_2^b44_2^*$ $(\times 2)$

$$\begin{bmatrix} -7446780 & 17820 & 17820 \\ 17820 & -42 & -45 \\ 17820 & -45 & -34 \end{bmatrix} \begin{bmatrix} 29699 & -75 & -57 \\ 9751500 & -24626 & -18715 \\ 2643300 & -6675 & -5074 \end{bmatrix} \begin{bmatrix} 31 & 967 & 8 & 23 & 2 & -1 \\ 10178 & 317460 & 2626 & 7548 & 655 & -330 \\ 2760 & 86130 & 713 & 2052 & 180 & -88 \end{bmatrix}$$

W_{82} 24 lattices, $\chi = 18$ 7-gon: 2222222

$L_{82.1}$: $1_{\rm II}^{-2}4_1^1,1^23^-,1^25^-,1^{-2}11^1$ $\langle 2 \to N_{82}\rangle$ $44_2^*60_2^b2_2^l132_2^r10_2^l4_2^r6_2^b$

$$\begin{bmatrix} -13682460 & -6811200 & 14520 \\ -6811200 & -3390650 & 7227 \\ 14520 & 7227 & -14 \end{bmatrix} \begin{bmatrix} -2285 & -2773 & -326 & -4723 & 82 & 163 & -82 \\ 4598 & 5580 & 656 & 9504 & -165 & -328 & 165 \\ 3674 & 4470 & 527 & 7656 & -130 & -264 & 129 \end{bmatrix}$$

W_{83} 16 lattices, $\chi = 64$ 14-gon: $23222222322222 \rtimes C_2$

$L_{83.1}$: $1_{\rm II}^{-2}8_1^1,1^{-2}5^-,1^{-2}17^1$ $\langle 2 \to N_{83}\rangle$ $40_2^b2_3^-2_2^l136_2^r10_2^l8_2^r34_2^b$ $(\times 2)$

$$\begin{bmatrix} -156619640 & 493680 & -116960 \\ 493680 & -1538 & 349 \\ -116960 & 349 & -66 \end{bmatrix} \begin{bmatrix} 1845791 & -5510 & 1044 \\ 749136960 & -2236301 & 423720 \\ 690421680 & -2061025 & 390509 \end{bmatrix}$$

$$\begin{bmatrix} -713 & -181 & -50 & -3175 & -309 & -97 & -87 \\ -289380 & -73461 & -20293 & -1288600 & -125410 & -39368 & -35309 \\ -266700 & -67703 & -18702 & -1187552 & -115575 & -36280 & -32538 \end{bmatrix}$$

W_{84} 24 lattices, $\chi = 12$ 6-gon: 222222

$L_{84.1}$: $1_{\rm II}^{-2}4_7^1,1^23^1,1^{-2}5^-,1^{-2}13^1$ $\langle 2 \to N_{84}\rangle$ $4_2^*52_2^b10_2^l156_2^r2_2^s390_2^b$

$$\begin{bmatrix} -11003460 & -3664440 & 15600 \\ -3664440 & -1220354 & 5195 \\ 15600 & 5195 & -22 \end{bmatrix} \begin{bmatrix} 39 & 43 & -38 & -361 & -1 & 451 \\ -118 & -130 & 115 & 1092 & 3 & -1365 \\ -210 & -208 & 210 & 1872 & -1 & -2535 \end{bmatrix}$$

W_{85} 24 lattices, $\chi = 42$ 10-gon: $2224222242 \rtimes C_2$

$L_{85.1}$: $1_{\rm II}^{-2}4_7^1,1^23^1,1^25^1,1^213^-$ $\langle 2 \to N_{85}\rangle$ $20_2^b26_2^s30_2^b4_4^*2_2^b$ $(\times 2)$

$$\begin{bmatrix} -36481380 & 30420 & 42900 \\ 30420 & -22 & -37 \\ 42900 & -37 & -50 \end{bmatrix} \begin{bmatrix} 107639 & -75 & -132 \\ 26479440 & -18451 & -32472 \\ 72728760 & -50675 & -89189 \end{bmatrix} \begin{bmatrix} 11 & 31 & 47 & 43 & 14 \\ 2710 & 7631 & 11565 & 10578 & 3443 \\ 7430 & 20943 & 31755 & 29054 & 9460 \end{bmatrix}$$

W_{86} 48 lattices, $\chi = 28$ 8-gon: 22622222

$L_{86.1}$: $1_{\rm II}^{-2}4_7^1,1^13^-9^-,1^{-2}5^-,1^213^-$ $\langle 23 \to N_{86}, 3, 2\rangle$
$4_2^*2340_2^b6_6 18_2^l60_2^r234_2^b10_2^s78_2^b$

$$\begin{bmatrix} -143865540 & 159120 & 10754640 \\ 159120 & -174 & -12363 \\ 10754640 & -12363 & -693998 \end{bmatrix}$$

$$\begin{bmatrix} 3027 & 112363 & 260 & -259 & -261 & 1292 & 691 & 6225 \\ 2125402 & 78895440 & 182558 & -181857 & -183260 & 907179 & 485185 & 4370873 \\ 9046 & 335790 & 777 & -774 & -780 & 3861 & 2065 & 18603 \end{bmatrix}$$

W_{87} 12 lattices, $\chi = 84$ 18-gon: $222222222222222222 \rtimes C_2$

$L_{87.1}$: $1_{\rm II}^{-2}4_1^1,1^{-2}5^-,1^241^1$ $\langle 2 \to N_{87}\rangle$ $4_2^r10_2^b82_2^l4_2^r410_2^b2_2^b10_2^l164_2^r2_2^l$ $(\times 2)$

$$\begin{bmatrix} 63140 & -25420 & -820 \\ -25420 & 10234 & 329 \\ -820 & 329 & -38 \end{bmatrix} \begin{bmatrix} -560881 & 225378 & -10431 \\ -1394000 & 560149 & -25925 \\ 39360 & -15816 & 731 \end{bmatrix} \begin{bmatrix} 1125 & 338 & 495 & 29 & -412 & -35 & 2 & 2573 & 140 \\ 2796 & 840 & 1230 & 72 & -1025 & -87 & 5 & 6396 & 348 \\ -80 & -25 & -41 & -4 & 0 & 2 & 0 & -164 & -9 \end{bmatrix}$$

W_{88} 32 lattices, $\chi = 18$ 7-gon: 2222222

$L_{88.1} : 1_{\rm II}^{-2} 8_5^-, 1^2 3^-, 1^2 5^-, 1^2 7^- \ \langle 2 \to N_{88} \rangle$ $168_2^r 10_2^b 6_2^l 40_2^r 42_2^b 8_2^b 2_2^l$

$$\begin{bmatrix} -620760 & -248640 & 2520 \\ -248640 & -99590 & 1009 \\ 2520 & 1009 & -10 \end{bmatrix} \quad \begin{bmatrix} -265 & -2 & 13 & 79 & 25 & -11 & -13 \\ 672 & 5 & -33 & -200 & -63 & 28 & 33 \\ 1008 & 0 & -54 & -280 & -63 & 52 & 53 \end{bmatrix}$$

W_{89} 48 lattices, $\chi = 32$ 8-gon: $22262226 \rtimes C_2$

$L_{89.1} : 1_{\rm II}^{-2} 8_5^-, 1^- 3^- 9^-, 1^{-2} 5^1, 1^{-2} 7^1 \ \langle 23 \to N_{89}, 3, 2 \rangle$

$18_2^b 14_2^b 72_2^b 6_6 2_2^b 126_2^b 8_2^b 6_6$

$$\begin{bmatrix} -22538902680 & -1942920 & 19106640 \\ -1942920 & -138 & 1629 \\ 19106640 & 1629 & -16186 \end{bmatrix}$$

$$\begin{bmatrix} -397 & -78 & 35 & 16 & -17 & -214 & -97 & -208 \\ -305694 & -60060 & 26952 & 12320 & -13091 & -164787 & -74692 & -160163 \\ -499401 & -98119 & 44028 & 20127 & -21385 & -269199 & -122020 & -261651 \end{bmatrix}$$

W_{90} 12 lattices, $\chi = 37$ 9-gon: 222462222

$L_{90.1} : 1_{\rm II}^{-2} 4_7^1, 1^2 3^-, 1^2 73^1 \ \langle 2 \to N_{90} \rangle$ $4_2^* 292_2^b 6_2^b 4_4^* 2_6 6_2^s 146_2^b 2_2^s 438_2^b$

$$\begin{bmatrix} -263993988 & 88065156 & 275064 \\ 88065156 & -29377454 & -91759 \\ 275064 & -91759 & -286 \end{bmatrix} \quad \begin{bmatrix} 4225 & 79077 & 2470 & 1263 & 330 & -329 & -661 & 274 & 49870 \\ 12598 & 235790 & 7365 & 3766 & 984 & -981 & -1971 & 817 & 148701 \\ 21556 & 403398 & 12597 & 6438 & 1679 & -1680 & -3358 & 1400 & 254478 \end{bmatrix}$$

W_{91} 24 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{91.1} : 1_{\rm II}^{-2} 4_3^-, 1^2 3^1, 1^{-2} 7^-, 1^{-2} 11^- \ \langle 2 \to N_{91} \rangle$ $12_2^r 22_2^b 84_2^* 4_2^b 66_2^b 2_2^l \ (\times 2)$

$$\begin{bmatrix} -3898356 & 4620 & 7392 \\ 4620 & -2 & -9 \\ 7392 & -9 & -14 \end{bmatrix} \begin{bmatrix} -11705 & 16 & 22 \\ -403788 & 551 & 759 \\ -5933928 & 8112 & 11153 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & -1 & -1 & -17 & -6 \\ 36 & 33 & -42 & -36 & -594 & -208 \\ 504 & 506 & -504 & -506 & -8613 & -3041 \end{bmatrix}$$

W_{92} 24 lattices, $\chi = 15$ 6-gon: 222242

$L_{92.1} : 1_{\rm II}^{-2} 4_3^-, 1^2 3^1, 1^2 7^1, 1^2 11^1 \ \langle 2 \to N_{92} \rangle$ $44_2^r 14_2^l 12_2^r 154_2^b 4_4^* 2_2^l$

$$\begin{bmatrix} -3694273044 & 3152688 & 6305376 \\ 3152688 & -2690 & -5381 \\ 6305376 & -5381 & -10762 \end{bmatrix} \quad \begin{bmatrix} 19 & 5 & -17 & -101 & -9 & 0 \\ 0 & -7 & -24 & -77 & -2 & 2 \\ 11132 & 2933 & -9948 & -59136 & -5272 & -1 \end{bmatrix}$$

W_{93} 24 lattices, $\chi = 54$ 12-gon: $224222224222 \rtimes C_2$

$L_{93.1} : 1_{\rm II}^{-2} 4_3^-, 1^2 3^-, 1^2 5^1, 1^2 17^1 \ \langle 2 \to N_{93} \rangle$ $20_2^* 68_2^b 2_4^* 4_2^b 34_2^s 6_2^b \ (\times 2)$

$$\begin{bmatrix} -3813780 & 10200 & 15300 \\ 10200 & -26 & -43 \\ 15300 & -43 & -58 \end{bmatrix} \begin{bmatrix} 220319 & -568 & -920 \\ 42769620 & -110264 & -178595 \\ 26355780 & -67947 & -110056 \end{bmatrix} \quad \begin{bmatrix} 21 & 39 & 1 & -1 & -2 & 2 \\ 4080 & 7582 & 195 & -194 & -391 & 387 \\ 2510 & 4658 & 119 & -120 & -238 & 240 \end{bmatrix}$$

W_{94} 24 lattices, $\chi = 16$ 6-gon: 226222

$L_{94.1} : 1_{\rm II}^{-2} 4_3^-, 1^2 3^-, 1^{-2} 5^-, 1^{-2} 17^- \ \langle 2 \to N_{94} \rangle$ $4_2^* 340_2^b 6_6 2_2^l 204_2^r 10_2^b$

$$\begin{bmatrix} -16365070740 & -63936660 & 1970640 \\ -63936660 & -249794 & 7699 \\ 1970640 & 7699 & -226 \end{bmatrix} \quad \begin{bmatrix} 101 & -583 & -200 & -252 & -4651 & 49 \\ -25858 & 149260 & 51204 & 64517 & 1190748 & -12545 \\ -204 & 1190 & 405 & 509 & 9384 & -100 \end{bmatrix}$$

W_{95} 24 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{95.1}: 1_{\mathrm{II}}^{-2}4_5^-,1^2 3^-,1^2 7^-,1^{-2}13^1\ \langle 2 \to N_{95}\rangle$ $42_2^b 2_2^s 182_2^b 6_2^l 52_2^r\ (\times 2)$

$$\begin{bmatrix} 5460 & -1092 & -1092 \\ -1092 & 218 & 209 \\ -1092 & 209 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1092 & -209 & 1 \end{bmatrix} \qquad \begin{bmatrix} 4 & -9 & -331 & -58 & -209 \\ 21 & -47 & -1729 & -303 & -1092 \\ 0 & 2 & 0 & -6 & -52 \end{bmatrix}$$

W_{96} 24 lattices, $\chi = 56$ 12-gon: $222262222262 \rtimes C_2$

$L_{96.1}: 1_{\mathrm{II}}^{-2}4_5^-,1^2 3^-,1^{-2}7^1,1^2 13^-\ \langle 2 \to N_{96}\rangle$ $28_2^b 78_2^b 14_2^s 26_2^b 2_6 6_2^b\ (\times 2)$

$$\begin{bmatrix} -156692172 & 269724 & -125580 \\ 269724 & -434 & 177 \\ -125580 & 177 & -50 \end{bmatrix} \begin{bmatrix} 4129943 & -6851 & 2976 \\ 3749189808 & -6219383 & 2701632 \\ 2899620360 & -4810065 & 2089439 \end{bmatrix}$$

$$\begin{bmatrix} 13 & -29 & -54 & -290 & -149 & -1214 \\ 11802 & -26325 & -49021 & -263263 & -135263 & -1102077 \\ 9128 & -20358 & -37912 & -203606 & -104612 & -852345 \end{bmatrix}$$

W_{97} 24 lattices, $\chi = 60$ 14-gon: $22222222222222 \rtimes C_2$

$L_{97.1}: 1_{\mathrm{II}}^{-2}4_1^1,1^2 3^1,1^2 5^-,1^{-2}19^-\ \langle 2 \to N_{97}\rangle$

$12_2^b 38_2^s 2_2^l 228_2^r 10_2^l 4_2^r 190_2^b\ (\times 2)$

$$\begin{bmatrix} 22003140 & 6840 & -30780 \\ 6840 & -2 & -5 \\ -30780 & -5 & 38 \end{bmatrix} \begin{bmatrix} -13111 & -11 & 26 \\ -13686840 & -11485 & 27144 \\ -12402060 & -10406 & 24595 \end{bmatrix} \begin{bmatrix} 13 & 21 & 3 & 85 & 2 & 1 & 1 \\ 13578 & 21945 & 3137 & 88920 & 2095 & 1048 & 1045 \\ 12300 & 19874 & 2840 & 80484 & 1895 & 948 & 950 \end{bmatrix}$$

W_{98} 24 lattices, $\chi = 18$ 7-gon: 2222222

$L_{98.1}: 1_{\mathrm{II}}^{-2}4_1^1,1^2 3^1,1^{-2}5^1,1^2 19^1\ \langle 2 \to N_{98}\rangle$ $12_2^* 380_2^b 2_2^b 570_2^l 4_2^r 30_2^b 76_2^*$

$$\begin{bmatrix} -15644220 & 9120 & 5700 \\ 9120 & -2 & -7 \\ 5700 & -7 & 2 \end{bmatrix} \qquad \begin{bmatrix} -1 & -9 & 0 & 13 & 1 & 1 & -1 \\ -1098 & -9880 & 0 & 14250 & 1096 & 1095 & -1102 \\ -990 & -8930 & -1 & 12825 & 988 & 990 & -988 \end{bmatrix}$$

W_{99} 12 lattices, $\chi = 98$ 18-gon: $222242262222242262 \rtimes C_2$

$L_{99.1}: 1_{\mathrm{II}}^{-2}4_7^1,1^2 3^-,1^2 97^1\ \langle 2 \to N_{99}\rangle$ $4_2^* 388_2^b 6_2^s 194_2^b 2_4^* 4_2^b 582_2^s 2_6 6_2^b\ (\times 2)$

$$\begin{bmatrix} -57207108 & 148992 & 147828 \\ 148992 & -386 & -385 \\ 147828 & -385 & -382 \end{bmatrix} \begin{bmatrix} 6104015 & -16583 & -15778 \\ -5307840 & 14419 & 13720 \\ 2367031248 & -6430599 & -6118435 \end{bmatrix}$$

$$\begin{bmatrix} 125 & 3029 & 100 & 622 & 21 & 9 & 85 & -1 & 1 \\ -106 & -2522 & -81 & -485 & -15 & -4 & 0 & 2 & -3 \\ 48470 & 1174476 & 38772 & 241142 & 8140 & 3486 & 32883 & -389 & 390 \end{bmatrix}$$

W_{100} 32 lattices, $\chi = 24$ 8-gon: 22222222

$L_{100.1}: 1_{\mathrm{II}}^{-2}8_1^1,1^2 3^1,1^{-2}5^-,1^{-2}11^-\ \langle 2 \to N_{100}\rangle$

$8_2^r 110_2^b 2_2^l 264_2^r 10_2^b 22_2^b 40_2^b 66_2^l$

$$\begin{bmatrix} 703560 & -141240 & -1320 \\ -141240 & 28354 & 265 \\ -1320 & 265 & 2 \end{bmatrix} \qquad \begin{bmatrix} -299 & -641 & 0 & 53 & 1 & -42 & -213 & -683 \\ -1488 & -3190 & 0 & 264 & 5 & -209 & -1060 & -3399 \\ -176 & -385 & -1 & 0 & 0 & -22 & -120 & -396 \end{bmatrix}$$

W_{101} 24 lattices, $\chi = 22$ 7-gon: 2262222

$L_{101.1}: 1_{\mathrm{II}}^{-2}4_5^-,1^2 3^-,1^{-2}5^-,1^2 23^1\ \langle 2 \to N_{101}\rangle$ $60_2^* 92_2^b 6_6 2_2^l 276_2^r 10_2^s 46_2^b$

$$\begin{bmatrix} -579195660 & 194669700 & -401580 \\ 194669700 & -65428814 & 134783 \\ -401580 & 134783 & -178 \end{bmatrix} \qquad \begin{bmatrix} -19003 & 10303 & 4808 & -5037 & -612451 & -45867 & -61741 \\ -56760 & 30774 & 14361 & -15045 & -1829328 & -137000 & -184414 \\ -107070 & 58052 & 27090 & -28381 & -3450828 & -258435 & -347875 \end{bmatrix}$$

W_{102} 24 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{102.1} : 1^{-2}_{\mathrm{II}} 4^1_1, 1^2 3^1, 1^{-2} 7^1, 1^2 17^1$ $\langle 2 \to N_{102} \rangle$
$2^l_2 68^r_2 14^b_2 12^*_2 28^b_2 34^l_2 4^r_2 714^b_2$ $(\times 2)$

$$\begin{bmatrix} -506940 & -252756 & 1428 \\ -252756 & -126022 & 713 \\ 1428 & 713 & 178 \end{bmatrix} \begin{bmatrix} -7608385 & -3794276 & -122692 \\ 15268176 & 7614188 & 246213 \\ -359856 & -179459 & -5804 \end{bmatrix}$$

$$\begin{bmatrix} 179 & 2679 & 426 & -197 & -5155 & -26252 & -17403 & -264002 \\ -359 & -5372 & -854 & 396 & 10346 & 52683 & 34924 & 529788 \\ 2 & 0 & -7 & -30 & -280 & -1292 & -836 & -12495 \end{bmatrix}$$

W_{103} 24 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{103.1} : 1^{-2}_{\mathrm{II}} 4^1_1, 1^2 3^1, 1^2 7^-, 1^{-2} 17^-$ $\langle 2 \to N_{103} \rangle$ $12^*_2 476^b_2 2^s_2 102^l_2 4^r_2 238^b_2$ $(\times 2)$

$$\begin{bmatrix} 51517956 & 8568 & -52836 \\ 8568 & -2 & -5 \\ -52836 & -5 & 50 \end{bmatrix} \begin{bmatrix} -52837 & -24 & 71 \\ -69056652 & -31369 & 92797 \\ -62663496 & -28464 & 84205 \end{bmatrix} \begin{bmatrix} -1 & -11 & 0 & 44 & 23 & 237 \\ -1314 & -14518 & -3 & 57477 & 30056 & 309757 \\ -1188 & -13090 & -1 & 52173 & 27276 & 281078 \end{bmatrix}$$

W_{104} 24 lattices, $\chi = 60$ 14-gon: $22222222222222 \rtimes C_2$

$L_{104.1} : 1^{-2}_{\mathrm{II}} 4^-_5, 1^{-2} 5^1, 1^2 7^1, 1^2 11^1$ $\langle 2 \to N_{104} \rangle$
$28^*_2 44^b_2 2^b_2 154^l_2 20^r_2 14^b_2 220^*_2$ $(\times 2)$

$$\begin{bmatrix} -103180 & -30800 & 1540 \\ -30800 & -9194 & 461 \\ 1540 & 461 & 34 \end{bmatrix} \begin{bmatrix} -70841 & -21183 & -575 \\ 237160 & 70916 & 1925 \\ -9240 & -2763 & -76 \end{bmatrix} \begin{bmatrix} -25 & 125 & 26 & -23 & -251 & -251 & -1249 \\ 84 & -418 & -87 & 77 & 840 & 840 & 4180 \\ -14 & 0 & 2 & 0 & -20 & -21 & -110 \end{bmatrix}$$

W_{105} 32 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{105.1} : 1^{-2}_{\mathrm{II}} 8^1_7, 1^2 3^-, 1^2 5^-, 1^{-2} 13^-$ $\langle 2 \to N_{105} \rangle$
$24^b_2 26^b_2 6^b_2 10^s_2 78^b_2 2^l_2 312^r_2 10^b_2$ $(\times 2)$

$$\begin{bmatrix} -23541960 & 3623880 & 34320 \\ 3623880 & -557834 & -5283 \\ 34320 & -5283 & -50 \end{bmatrix} \begin{bmatrix} -15787201 & 2430364 & 22632 \\ -101587200 & 15638863 & 145632 \\ -103474800 & 15929451 & 148337 \end{bmatrix}$$

$$\begin{bmatrix} -13 & -107 & -172 & -1317 & -10685 & -2302 & -201073 & -12427 \\ -84 & -689 & -1107 & -8475 & -68757 & -14813 & -1293864 & -79965 \\ -48 & -650 & -1104 & -8590 & -69888 & -15074 & -1317576 & -81455 \end{bmatrix}$$

W_{106} 24 lattices, $\chi = 27$ 8-gon: 22242222

$L_{106.1} : 1^{-2}_{\mathrm{II}} 4^-_3, 1^2 3^-, 1^2 7^-, 1^2 19^1$ $\langle 2 \to N_{106} \rangle$ $6^l_2 76^r_2 42^b_2 4^*_4 2^b_2 114^b_2 4^*_2 532^b_2$

$$\begin{bmatrix} -165273780 & 78204 & 111720 \\ 78204 & -34 & -55 \\ 111720 & -55 & -74 \end{bmatrix} \begin{bmatrix} 4 & 13 & -2 & -1 & 1 & 34 & 13 & 275 \\ 2808 & 9120 & -1407 & -702 & 703 & 23883 & 9130 & 193116 \\ 3951 & 12844 & -1974 & -988 & 987 & 33573 & 12838 & 271586 \end{bmatrix}$$

W_{107} 24 lattices, $\chi = 60$ 14-gon: $22222222222222 \rtimes C_2$

$L_{107.1} : 1^{-2}_{\mathrm{II}} 4^1_1, 1^2 3^1, 1^2 11^1, 1^{-2} 13^1$ $\langle 2 \to N_{107} \rangle$
$44^*_2 156^b_2 2^b_2 858^l_2 4^r_2 286^b_2 12^*_2$ $(\times 2)$

$$\begin{bmatrix} -60078876 & 27456 & 20592 \\ 27456 & -10 & -13 \\ 20592 & -13 & -2 \end{bmatrix} \begin{bmatrix} 146431 & -60 & -60 \\ 208995072 & -85636 & -85635 \\ 148372224 & -60795 & -60796 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 7 & 2 & 511 & 31 & 571 & 89 \\ -1430 & 9984 & 2854 & 729300 & 44244 & 814957 & 127026 \\ -1012 & 7098 & 2027 & 517803 & 31412 & 578578 & 90180 \end{bmatrix}$$

W_{108} 24 lattices, $\chi = 28$ 8-gon: 22222262

$L_{108.1}$: $1\,{}^{-2}_{\mathrm{II}}4\,{}^{1}_{7},1\,{}^{2}3\,{}^{-},1\,{}^{-2}5\,{}^{1},1\,{}^{-2}29\,{}^{1}$ $\langle 2 \to N_{108}\rangle$

$20\,{}^{*}_{2}116\,{}^{b}_{2}2\,{}^{s}_{2}870\,{}^{b}_{2}4\,{}^{*}_{2}580\,{}^{b}_{2}6\,{}_{6}2\,{}^{b}_{2}$

$$\begin{bmatrix} -964563780 & 5310480 & -210540 \\ 5310480 & -29014 & 1117 \\ -210540 & 1117 & -38 \end{bmatrix} \begin{bmatrix} 1 & -335 & -52 & -2486 & -79 & -1593 & -14 & 13 \\ 230 & -77024 & -11956 & -571590 & -18164 & -366270 & -3219 & 2989 \\ 1220 & -408030 & -63337 & -3028035 & -96226 & -1940390 & -17055 & 15834 \end{bmatrix}$$

W_{109} 24 lattices, $\chi = 90$ 18-gon: $222422222222422222 \rtimes C_2$

$L_{109.1}$: $1\,{}^{-2}_{\mathrm{II}}4\,{}^{1}_{7},1\,{}^{2}3\,{}^{-},1\,{}^{2}5\,{}^{-},1\,{}^{2}29\,{}^{-}$ $\langle 2 \to N_{109}\rangle$

$10\,{}^{b}_{2}58\,{}^{l}_{2}60\,{}^{r}_{2}2\,{}^{*}_{4}4\,{}^{b}_{2}6\,{}^{s}_{2}290\,{}^{b}_{2}2\,{}^{l}_{2}348\,{}^{r}_{2}$ $(\times 2)$

$$\begin{bmatrix} -6396119940 & 2136697380 & 6986100 \\ 2136697380 & -713788318 & -2333787 \\ 6986100 & -2333787 & -7630 \end{bmatrix} \begin{bmatrix} 593000351 & -198098586 & -652050 \\ 1775113200 & -592996976 & -1951875 \\ 3069360 & -1025355 & -3376 \end{bmatrix}$$

$$\begin{bmatrix} 1236 & 5367 & 4129 & -1 & -1987 & -6871 & -563347 & -59522 & -1997591 \\ 3700 & 16066 & 12360 & -3 & -5948 & -20568 & -1686350 & -178176 & -5979684 \\ -25 & -29 & 0 & 2 & -2 & -21 & -2175 & -235 & -8004 \end{bmatrix}$$

W_{110} 24 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{110.1}$: $1\,{}^{-2}_{\mathrm{II}}4\,{}^{-}_{3},1\,{}^{-2}5\,{}^{-},1\,{}^{2}7\,{}^{-},1\,{}^{-2}13\,{}^{-}$ $\langle 2 \to N_{110}\rangle$

$26\,{}^{l}_{2}140\,{}^{r}_{2}2\,{}^{b}_{2}260\,{}^{*}_{2}4\,{}^{b}_{2}2\,{}^{l}_{2}364\,{}^{r}_{2}10\,{}^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 1334060 & -267540 & -1820 \\ -267540 & 53654 & 365 \\ -1820 & 365 & 2 \end{bmatrix} \begin{bmatrix} 6860671 & -1377120 & 4864 \\ 34190520 & -6862951 & 24240 \\ 3215940 & -645525 & 2279 \end{bmatrix}$$

$$\begin{bmatrix} -7844 & -15507 & -365 & -2739 & -61 & 0 & 73 & 1 \\ -39091 & -77280 & -1819 & -13650 & -304 & 0 & 364 & 5 \\ -3679 & -7280 & -172 & -1300 & -30 & -1 & 0 & 0 \end{bmatrix}$$

W_{111} 24 lattices, $\chi = 60$ 14-gon: $22222222222222 \rtimes C_2$

$L_{111.1}$: $1\,{}^{-2}_{\mathrm{II}}4\,{}^{-}_{5},1\,{}^{2}3\,{}^{1},1\,{}^{-2}5\,{}^{1},1\,{}^{2}31\,{}^{1}$ $\langle 2 \to N_{111}\rangle$

$12\,{}^{*}_{2}124\,{}^{b}_{2}30\,{}^{b}_{2}62\,{}^{l}_{2}20\,{}^{r}_{2}2\,{}^{b}_{2}620\,{}^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -526380 & -262260 & 0 \\ -262260 & -130666 & -3 \\ 0 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1859 & 925 & 8 \\ -3720 & -1851 & -16 \\ -1860 & -925 & -9 \end{bmatrix} \begin{bmatrix} -275 & -1081 & -127 & 170 & 299 & 146 & 6951 \\ 552 & 2170 & 255 & -341 & -600 & -293 & -13950 \\ 114 & 434 & 45 & -93 & -140 & -66 & -3100 \end{bmatrix}$$

W_{112} 24 lattices, $\chi = 66$ 14-gon: $22222242222224 \rtimes C_2$

$L_{112.1}$: $1\,{}^{-2}_{\mathrm{II}}4\,{}^{1}_{7},1\,{}^{2}3\,{}^{1},1\,{}^{2}7\,{}^{1},1\,{}^{2}23\,{}^{1}$ $\langle 2 \to N_{112}\rangle$ $2\,{}^{l}_{2}92\,{}^{r}_{2}14\,{}^{s}_{2}138\,{}^{l}_{2}28\,{}^{r}_{2}46\,{}^{b}_{2}4\,{}^{*}_{4}$ $(\times 2)$

$$\begin{bmatrix} -648168612 & 162288 & 233772 \\ 162288 & -38 & -61 \\ 233772 & -61 & -82 \end{bmatrix} \begin{bmatrix} 3894911 & -1053 & -1332 \\ 6132322560 & -1657891 & -2097160 \\ 6541288320 & -1768455 & -2237021 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 703 & 234 & 1210 & 751 & 1037 & 271 \\ 1575 & 1106852 & 368424 & 1905090 & 1182412 & 1632701 & 426674 \\ 1679 & 1180636 & 392987 & 2032119 & 1261260 & 1741583 & 455130 \end{bmatrix}$$

W_{113} 32 lattices, $\chi = 80$ 16-gon: $2226222222262222 \rtimes C_2$

$L_{113.1} : 1_{\mathrm{II}}^{-2} 8_1^1, 1^2 3^-, 1^{-2} 5^-, 1^{-2} 19^1 \ \langle 2 \to N_{113} \rangle$

$6_2^b 10_2^l 456_2^r 2_6 6_2^b 40_2^b 114_2^l 8_2^r \ (\times 2)$

$$\begin{bmatrix} -4758603960 & 3057480 & -704520 \\ 3057480 & -1946 & 429 \\ -704520 & 429 & -74 \end{bmatrix} \begin{bmatrix} 9849599 & -6408 & 1560 \\ 18693309600 & -12161584 & 2960685 \\ 14597517600 & -9496923 & 2311984 \end{bmatrix}$$

$$\begin{bmatrix} -109 & -268 & -9401 & -183 & -512 & -499 & -1199 & -277 \\ -206868 & -508630 & -17841912 & -347311 & -971712 & -947040 & -2275554 & -525712 \\ -161541 & -397185 & -13932624 & -271213 & -758805 & -739540 & -1776975 & -410528 \end{bmatrix}$$

W_{114} 24 lattices, $\chi = 80$ 16-gon: $2262222222622222 \rtimes C_2$

$L_{114.1} : 1_{\mathrm{II}}^{-2} 4_3^-, 1^2 3^-, 1^{-2} 5^1, 1^{-2} 41^1 \ \langle 2 \to N_{114} \rangle$

$20_2^* 164_2^b 2_6 6_2^s 82_2^b 4_2^* 820_2^b 6_2^b \ (\times 2)$

$$\begin{bmatrix} -1509227220 & 341940 & 496920 \\ 341940 & -74 & -117 \\ 496920 & -117 & -158 \end{bmatrix} \begin{bmatrix} 3630959 & -864 & -1143 \\ 7478567280 & -1779553 & -2354199 \\ 5881348320 & -1399488 & -1851407 \end{bmatrix}$$

$$\begin{bmatrix} 37 & 109 & 2 & -2 & -4 & 3 & 387 & 19 \\ 76210 & 224516 & 4120 & -4119 & -8241 & 6178 & 797040 & 39132 \\ 59930 & 176546 & 3239 & -3240 & -6478 & 4860 & 626890 & 30777 \end{bmatrix}$$

W_{115} 24 lattices, $\chi = 84$ 18-gon: $222222222222222222 \rtimes C_2$

$L_{115.1} : 1_{\mathrm{II}}^{-2} 4_1^1, 1^2 3^1, 1^{-2} 5^-, 1^2 43^- \ \langle 2 \to N_{115} \rangle$

$12_2^b 86_2^s 2_2^l 516_2^r 10_2^l 4_2^r 1290_2^b 2_2^b 860_2^* \ (\times 2)$

$$\begin{bmatrix} 1684740 & -330240 & -2580 \\ -330240 & 64706 & 513 \\ -2580 & 513 & 2 \end{bmatrix} \begin{bmatrix} -1307201 & 261915 & 475 \\ -6480960 & 1298546 & 2355 \\ -23818560 & 4772367 & 8654 \end{bmatrix}$$

$$\begin{bmatrix} -2113 & -4059 & -261 & -12073 & -237 & -71 & -1301 & 0 & 347 \\ -10476 & -20124 & -1294 & -59856 & -1175 & -352 & -6450 & 0 & 1720 \\ -38514 & -74003 & -4761 & -220332 & -4330 & -1300 & -23865 & -1 & 6450 \end{bmatrix}$$

W_{116} 24 lattices, $\chi = 90$ 18-gon: $224222222224222222 \rtimes C_2$

$L_{116.1} : 1_{\mathrm{II}}^{-2} 4_7^1, 1^2 3^-, 1^2 7^-, 1^2 31^1 \ \langle 2 \to N_{116} \rangle$

$6_2^s 434_2^b 2_4^* 4_2^* 868_2^b 6_2^l 124_2^r 42_2^s 62_2^b \ (\times 2)$

$$\begin{bmatrix} 2637852 & 627564 & -5208 \\ 627564 & 149302 & -1239 \\ -5208 & -1239 & 10 \end{bmatrix} \begin{bmatrix} -1702273 & -404690 & 1716 \\ 7166208 & 1703659 & -7224 \\ 1374912 & 326865 & -1387 \end{bmatrix}$$

$$\begin{bmatrix} 796 & 6134 & 81 & 19 & 309 & -5 & -59 & -5 & 81 \\ -3351 & -25823 & -341 & -80 & -1302 & 21 & 248 & 21 & -341 \\ -645 & -4991 & -67 & -18 & -434 & -3 & 0 & 0 & -62 \end{bmatrix}$$

W_{117} 24 lattices, $\chi = 104$ 20-gon: $62222222226222222222 \rtimes C_2$

$L_{117.1} : 1_{\mathrm{II}}^{-2} 4_7^1, 1^2 3^-, 1^{-2} 5^-, 1^{-2} 53^- \ \langle 2 \to N_{117} \rangle$

$2_6 6_2^b 1060_2^* 4_2^b 1590_2^s 2_2^l 636_2^r 10_2^b 106_2^l 60_2^r \ (\times 2)$

$$\begin{bmatrix} 4060860 & 810900 & -3180 \\ 810900 & 161926 & -635 \\ -3180 & -635 & 2 \end{bmatrix} \begin{bmatrix} -23937769 & -4777812 & -16605 \\ 119905080 & 23932219 & 83175 \\ 8000880 & 1596920 & 5549 \end{bmatrix}$$

$$\begin{bmatrix} 221 & 1217 & 74701 & 2103 & 525817 & 16605 & 1408231 & 48709 & 117014 & 39301 \\ -1107 & -6096 & -374180 & -10534 & -2633835 & -83175 & -7053876 & -243985 & -586127 & -196860 \\ -73 & -405 & -24910 & -702 & -175695 & -5549 & -470640 & -16280 & -39114 & -13140 \end{bmatrix}$$

W_{118} 48 lattices, $\chi = 36$ 10-gon: 2222222222

$L_{118.1}$: $1\,{}_{\mathrm{II}}^{-2}4\,{}_{7}^{1},1\,{}^{2}3\,{}^{-},1\,{}^{-2}5\,{}^{-},1\,{}^{2}7\,{}^{-},1\,{}^{-2}11\,{}^{-}$ $\langle 2 \to N_{118}\rangle$

$$60\,{}_{2}^{r}22\,{}_{2}^{b}4\,{}_{2}^{*}1540\,{}_{2}^{b}6\,{}_{2}^{b}110\,{}_{2}^{s}42\,{}_{2}^{b}10\,{}_{2}^{l}924\,{}_{2}^{r}2\,{}_{2}^{l}$$

$$\begin{bmatrix} 8560860 & -1714020 & -4620 \\ -1714020 & 343174 & 925 \\ -4620 & 925 & 2 \end{bmatrix} \quad \begin{bmatrix} -829 & -663 & -497 & -22357 & -331 & -826 & -164 & 1 & 185 & 0 \\ -4140 & -3311 & -2482 & -111650 & -1653 & -4125 & -819 & 5 & 924 & 0 \\ -240 & -187 & -138 & -6160 & -90 & -220 & -42 & 0 & 0 & -1 \end{bmatrix}$$

W_{119} 48 lattices, $\chi = 90$ 18-gon: $422222222422222222 \rtimes C_2$

$L_{119.1}$: $1\,{}_{\mathrm{II}}^{-2}4\,{}_{7}^{1},1\,{}^{2}3\,{}^{-},1\,{}^{2}5\,{}^{1},1\,{}^{2}7\,{}^{-},1\,{}^{2}11\,{}^{1}$ $\langle 2 \to N_{119}\rangle$

$$2\,{}_{4}^{*}4\,{}_{2}^{b}70\,{}_{2}^{b}132\,{}_{2}^{*}20\,{}_{2}^{b}42\,{}_{2}^{l}220\,{}_{2}^{r}6\,{}_{2}^{s}770\,{}_{2}^{b}\ (\times 2)$$

$$\begin{bmatrix} -1671382020 & 328020 & 475860 \\ 328020 & -62 & -97 \\ 475860 & -97 & -130 \end{bmatrix} \begin{bmatrix} 352351 & -72 & -96 \\ 918317400 & -187651 & -250200 \\ 604503900 & -123525 & -164701 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -2 & 25 & 17 & 85 & 453 & 28 & 368 \\ 2607 & -2606 & -5215 & 65142 & 44300 & 221508 & 1180520 & 72969 & 959035 \\ 1715 & -1716 & -3430 & 42900 & 29170 & 145845 & 777260 & 48042 & 631400 \end{bmatrix}$$

W_{120} 48 lattices, $\chi = 40$ 10-gon: 2222262222

$L_{120.1}$: $1\,{}_{\mathrm{II}}^{-2}4\,{}_{7}^{1},1\,{}^{2}3\,{}^{-},1\,{}^{-2}5\,{}^{-},1\,{}^{-2}7\,{}^{1},1\,{}^{2}11\,{}^{1}$ $\langle 2 \to N_{120}\rangle$

$$14\,{}_{2}^{l}60\,{}_{2}^{r}154\,{}_{2}^{b}10\,{}_{2}^{l}28\,{}_{2}^{r}6\,{}_{6}2\,{}_{2}^{s}2310\,{}_{2}^{b}4\,{}_{2}^{*}132\,{}_{2}^{b}$$

$$\begin{bmatrix} -929927460 & 291060 & 161700 \\ 291060 & -82 & -55 \\ 161700 & -55 & -26 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 43 & 39 & 2 & -1 & -1 & 1 & 263 & 9 & 67 \\ 17815 & 63840 & 57904 & 2970 & -1484 & -1485 & 1484 & 390390 & 13360 & 99462 \\ 36939 & 132360 & 120043 & 6155 & -3080 & -3078 & 3079 & 809655 & 27706 & 206250 \end{bmatrix}$$

W_{121} 48 lattices, $\chi = 84$ 18-gon: $222222222222222222 \rtimes C_2$

$L_{121.1}$: $1\,{}_{\mathrm{II}}^{-2}4\,{}_{1}^{1},1\,{}^{2}3\,{}^{1},1\,{}^{-2}5\,{}^{1},1\,{}^{2}7\,{}^{1},1\,{}^{2}13\,{}^{1}$ $\langle 2 \to N_{121}\rangle$

$$12\,{}_{2}^{b}14\,{}_{2}^{b}30\,{}_{2}^{l}4\,{}_{2}^{r}130\,{}_{2}^{b}28\,{}_{2}^{*}156\,{}_{2}^{b}2\,{}_{2}^{b}1820\,{}_{2}^{*}\ (\times 2)$$

$$\begin{bmatrix} -606294780 & 55970460 & 125580 \\ 55970460 & -5166946 & -11593 \\ 125580 & -11593 & -26 \end{bmatrix} \begin{bmatrix} -491401 & 45365 & 100 \\ -5307120 & 489941 & 1080 \\ -7174440 & 662329 & 1459 \end{bmatrix}$$

$$\begin{bmatrix} -95 & -24 & -7 & 7 & 108 & 75 & 209 & 11 & 589 \\ -1026 & -259 & -75 & 76 & 1170 & 812 & 2262 & 119 & 6370 \\ -1386 & -441 & -375 & -80 & -65 & 182 & 858 & 69 & 4550 \end{bmatrix}$$

W_{122} 48 lattices, $\chi = 108$ 22-gon: $2222222222222222222222 \rtimes C_2$

$L_{122.1}$: $1\,{}_{\mathrm{II}}^{-2}4\,{}_{1}^{1},1\,{}^{2}3\,{}^{1},1\,{}^{2}5\,{}^{-},1\,{}^{2}7\,{}^{1},1\,{}^{-2}13\,{}^{-}$ $\langle 2 \to N_{122}\rangle$

$$12\,{}_{2}^{b}910\,{}_{2}^{l}4\,{}_{2}^{r}210\,{}_{2}^{b}26\,{}_{2}^{s}14\,{}_{2}^{l}260\,{}_{2}^{r}2\,{}_{2}^{l}1092\,{}_{2}^{r}10\,{}_{2}^{b}28\,{}_{2}^{*}\ (\times 2)$$

$$\begin{bmatrix} -573936958140 & 420774900 & 139639500 \\ 420774900 & -308486 & -102375 \\ 139639500 & -102375 & -33974 \end{bmatrix} \begin{bmatrix} -262630369 & 192544 & 63800 \\ -358475530140 & 262811619 & 87083375 \\ 746109000 & -547000 & -181251 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -151 & -1 & 244 & 423 & 939 & 18151 & 943 & 59737 & 689 & 1039 \\ 6822 & -206115 & -1364 & 333060 & 577382 & 1281700 & 24775400 & 1287157 & 81538548 & 940455 & 1418186 \\ -6 & 455 & 0 & -735 & -1235 & -2723 & -52520 & -2726 & -172536 & -1985 & -2982 \end{bmatrix}$$

W_{123} 16 lattices, $\chi = 3$ 4-gon: 4222

$L_{123.1} : 1^{2}_{2} 8^{-}_{5}, 1^{2} 3^{1}\ \langle 2 \to N_3' \rangle$ $2_4 1^{r}_{2} 12^{*}_{2} 8^{b}_{2}$

$$\begin{bmatrix} -1176 & -552 & 120 \\ -552 & -259 & 56 \\ 120 & 56 & -11 \end{bmatrix} \qquad \begin{bmatrix} 3 & -4 & -5 & 9 \\ -7 & 9 & 12 & -20 \\ -3 & 2 & 6 & -4 \end{bmatrix}$$

$L_{123.2} : 1^{-2}_{2} 8^{1}_{1}, 1^{2} 3^{1}\ \langle m \rangle$ $2^{*}_{4} 4^{l}_{2} 3_2 8^{r}_{2}$

$$\begin{bmatrix} -2616 & 120 & 72 \\ 120 & -5 & -4 \\ 72 & -4 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 1 \\ 15 & -16 & -15 & 16 \\ 11 & -10 & -12 & 8 \end{bmatrix}$$

W_{124} 46 lattices, $\chi = 12$ 6-gon: $222222 \rtimes C_2$

$L_{124.1} : 1^{-2}_{4} 8^{1}_{1}, 1^{-2} 5^{1}\ \langle 2 \to N_4' \rangle$ $4^{l}_{2} 5_2 8_2 1^{r}_{2} 20^{*}_{2} 8^{*}_{2}$

$$\begin{bmatrix} -3320 & 120 & 120 \\ 120 & -4 & -5 \\ 120 & -5 & -3 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 3 & 1 & 3 & -1 \\ -18 & 20 & 56 & 18 & 50 & -20 \\ -10 & 5 & 24 & 9 & 30 & -8 \end{bmatrix}$$

$L_{124.2} : 1^{-2}_{6} 8^{1}_{7}, 1^{-2} 5^{1}\ \langle m \rangle$ $1_2 5^{r}_{2} 8^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} -2120 & 280 & 80 \\ 280 & -19 & -10 \\ 80 & -10 & -3 \end{bmatrix} \begin{bmatrix} 79 & -9 & -3 \\ -80 & 8 & 3 \\ 2320 & -261 & -88 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 5 \\ 1 & 0 & -4 \\ -30 & 25 & 144 \end{bmatrix}$$

$L_{124.3} : 1^{2}_{6} 8^{-}_{3}, 1^{-2} 5^{1}$ $4^{*}_{2} 20^{s}_{2} 8^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} 920 & 200 & 0 \\ 200 & 43 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 11 & 1 \\ -14 & -50 & -4 \\ -8 & -30 & -4 \end{bmatrix}$$

$L_{124.4} : [1^{1} 2^{1}]_2 16^{-}_{3}, 1^{-2} 5^{1}\ \langle 2 \rangle$ $1^{r}_{2} 80^{*}_{2} 8^{s}_{2} 16^{*}_{2} 20^{l}_{2} 2_2$

$$\begin{bmatrix} 5680 & 2720 & -80 \\ 2720 & 1302 & -38 \\ -80 & -38 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -9 & -1 & 5 & 17 & 3 \\ 0 & 20 & 2 & -12 & -40 & -7 \\ -1 & 40 & 0 & -48 & -150 & -26 \end{bmatrix}$$

$L_{124.5} : [1^{-} 2^{1}]_6 16^{1}_{7}, 1^{-2} 5^{1}\ \langle m \rangle$ $4^{*}_{2} 80^{s}_{2} 8^{*}_{2} 16^{l}_{2} 5_2 2^{r}_{2}$

$$\begin{bmatrix} 4080 & -160 & 80 \\ -160 & 6 & -2 \\ 80 & -2 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 \\ 28 & 20 & -30 & -28 & 30 & 29 \\ 6 & 0 & -8 & -8 & 5 & 6 \end{bmatrix}$$

$L_{124.6} : [1^{1} 2^{1}]_0 16^{-}_{5}, 1^{-2} 5^{1}\ \langle m \rangle$ $1_2 80_2 2^{r}_{2} 16^{s}_{2} 20^{*}_{2} 8^{l}_{2}$

$$\begin{bmatrix} -2480 & 80 & 80 \\ 80 & -2 & -4 \\ 80 & -4 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -7 & -1 & -1 & 1 & 1 \\ 0 & -160 & -23 & -24 & 20 & 22 \\ -1 & -80 & -10 & -8 & 10 & 8 \end{bmatrix}$$

$L_{124.7} : [1^{-} 2^{1}]_4 16^{1}_{1}, 1^{-2} 5^{1}$ $4^{s}_{2} 80^{l}_{2} 2_2 16_2 5^{r}_{2} 8^{*}_{2}$

$$\begin{bmatrix} -64880 & 1120 & 1120 \\ 1120 & -18 & -20 \\ 1120 & -20 & -19 \end{bmatrix} \qquad \begin{bmatrix} -1 & 7 & 2 & 5 & 2 & -1 \\ -20 & 120 & 37 & 96 & 40 & -18 \\ -38 & 280 & 78 & 192 & 75 & -40 \end{bmatrix}$$

W_{125} 20 lattices, $\chi = 18$ 6-gon: $422422 \rtimes C_2$

$L_{125.1} : 1^{-2}_{2} 8^{-}_{5}, 1^{2} 7^{-}\ \langle 2 \to N_6' \rangle$ $2^{*}_{4} 4^{*}_{2} 8^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -28056 & 392 & 448 \\ 392 & -5 & -7 \\ 448 & -7 & -6 \end{bmatrix} \begin{bmatrix} 671 & -10 & -10 \\ 27552 & -411 & -410 \\ 17472 & -260 & -261 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 \\ 40 & -42 & -40 \\ 27 & -26 & -28 \end{bmatrix}$$

$L_{125.2} : 1_2^2 16_1^1,1^2 7^- \ \langle 2, m \rangle$ $\quad 1_4 2_2^b 16_2^* 4_4^* 2_2^l 16_2$

$$\begin{bmatrix} -115696 & -57232 & 1792 \\ -57232 & -28311 & 886 \\ 1792 & 886 & -27 \end{bmatrix} \qquad \begin{bmatrix} -17 & 16 & 35 & -31 & -146 & -295 \\ 35 & -33 & -72 & 64 & 301 & 608 \\ 20 & -21 & -40 & 42 & 185 & 368 \end{bmatrix}$$

W_{126} 16 lattices, $\chi = 30$ $\quad$ 8-gon: $42224222 \rtimes C_2$

$L_{126.1} : 1_2^2 8_5^-,1^2 11^1 \ \langle 2 \to N_9' \rangle$ $\quad 2_4 1_2^r 44_2^* 8_2^b \ (\times 2)$

$$\begin{bmatrix} 616 & -176 & -88 \\ -176 & 50 & 23 \\ -88 & 23 & -3 \end{bmatrix} \begin{bmatrix} 109 & -29 & 2 \\ 440 & -117 & 8 \\ 440 & -116 & 7 \end{bmatrix} \qquad \begin{bmatrix} 3 & 4 & 35 & 1 \\ 11 & 15 & 132 & 4 \\ -6 & -3 & -22 & 0 \end{bmatrix}$$

$L_{126.2} : 1_2^{-2} 8_1^1,1^2 11^1 \ \langle m \rangle$ $\quad 2_4^* 4_2^l 11_2 8_2^r \ (\times 2)$

$$\begin{bmatrix} -13816 & 440 & 352 \\ 440 & -14 & -11 \\ 352 & -11 & -5 \end{bmatrix} \begin{bmatrix} 461 & -15 & -15 \\ 14784 & -481 & -480 \\ -616 & 20 & 19 \end{bmatrix} \qquad \begin{bmatrix} 4 & 3 & 4 & -1 \\ 129 & 98 & 132 & -32 \\ -6 & -6 & -11 & 0 \end{bmatrix}$$

W_{127} 32 lattices, $\chi = 9$ $\quad$ 5-gon: 42222

$L_{127.1} : 1_2^{-2} 8_5^-,1^2 3^-,1^2 5^1 \ \langle 2 \to N_{11}' \rangle$ $\quad 2_4^* 4_2^s 24_2^s 20_2^* 8_2^b$

$$\begin{bmatrix} -27480 & 600 & 1200 \\ 600 & -13 & -25 \\ 1200 & -25 & -38 \end{bmatrix} \qquad \begin{bmatrix} -2 & 1 & 5 & -1 & -5 \\ -110 & 54 & 276 & -50 & -272 \\ 9 & -4 & -24 & 0 & 20 \end{bmatrix}$$

$L_{127.2} : 1_2^2 8_1^1,1^2 3^-,1^2 5^1 \ \langle m \rangle$ $\quad 2_4 1_2^r 24_2^l 5_2 8_2^r$

$$\begin{bmatrix} -236280 & 2280 & 1560 \\ 2280 & -22 & -15 \\ 1560 & -15 & -7 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 7 & 2 & -1 \\ -105 & 104 & 732 & 210 & -104 \\ 2 & -1 & -12 & -5 & 0 \end{bmatrix}$$

W_{128} 64 lattices, $\chi = 12$ $\quad$ 6-gon: 222222

$L_{128.1} : 1_2^2 8_1^1,1^- 3^1 9^1,1^{-2} 5^- \ \langle 3m, 3, 2 \rangle$ $\quad 8_2^r 90_2^b 2_2^b 360_2^* 12_2^l 9_2$

$$\begin{bmatrix} -1187640 & -8640 & 6120 \\ -8640 & -51 & 42 \\ 6120 & 42 & -31 \end{bmatrix} \qquad \begin{bmatrix} 1 & 8 & 1 & 11 & -1 & -1 \\ 56 & 465 & 59 & 660 & -58 & -60 \\ 272 & 2205 & 277 & 3060 & -276 & -279 \end{bmatrix}$$

$L_{128.2} : 1_2^{-2} 8_5^-,1^1 3^1 9^-,1^{-2} 5^- \ \langle 32 \to N_{14}', 3, m \rangle$ $\quad 72_2^b 10_2^s 18_2^l 40_2 3_2^r 4_2^*$

$$\begin{bmatrix} -1281240 & -428040 & 4680 \\ -428040 & -142998 & 1563 \\ 4680 & 1563 & -17 \end{bmatrix} \qquad \begin{bmatrix} 19 & 11 & -1 & -63 & -11 & -5 \\ -60 & -35 & 3 & 200 & 35 & 16 \\ -288 & -190 & 0 & 1040 & 189 & 94 \end{bmatrix}$$

W_{129} 92 lattices, $\chi = 24$ $\quad$ 8-gon: $22222222 \rtimes C_2$

$L_{129.1} : 1_4^{-2} 8_1^1,1^2 3^1,1^{-2} 7^- \ \langle 2 \to N_{17}' \rangle$ $\quad 8_2^* 12_2^* 4_2^l 21_2 8_2 3_2 1_2^r 84_2^*$

$$\begin{bmatrix} -700728 & 2016 & 4032 \\ 2016 & -5 & -12 \\ 4032 & -12 & -23 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 1 & 8 & 3 & 2 & 1 & 5 \\ -68 & -72 & 66 & 546 & 208 & 141 & 72 & 378 \\ -140 & -138 & 140 & 1113 & 416 & 276 & 137 & 672 \end{bmatrix}$$

$L_{129.2} : 1_6^{-2} 8_7^1,1^2 3^1,1^{-2} 7^- \ \langle m \rangle$ $\quad 8_2^s 12_2^l 1_2 21_2^r \ (\times 2)$

$$\begin{bmatrix} 1848 & -504 & 0 \\ -504 & 137 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -5 & -3 & -17 \\ 4 & -18 & -11 & -63 \\ 0 & -12 & -7 & -42 \end{bmatrix}$$

$L_{129.3} : 1_6^2 8_3^-,1^2 3^1,1^{-2} 7^-$ $\quad 8_2^l 3_2^r 4_2^* 84_2^s \ (\times 2)$

$$\begin{bmatrix} 241752 & 336 & -2184 \\ 336 & -1 & -2 \\ -2184 & -2 & 19 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 3528 & 14 & -39 \\ 1176 & 5 & -14 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 76 & 84 & 86 & 84 \\ 120 & 123 & 124 & 126 \end{bmatrix}$$

$L_{129.4}$: $[1^{-}2^{1}]_{6}16^{1}_{7},1^{2}3^{1},1^{-2}7^{-}$ $\langle 2\rangle$ $\quad 2^{r}_{2}48^{s}_{2}4^{*}_{2}336^{s}_{2}8^{*}_{2}12^{*}_{2}16^{l}_{2}21_{2}$

$$\begin{bmatrix}-380688 & 2016 & 2352\\ 2016 & -10 & -14\\ 2352 & -14 & -11\end{bmatrix} \quad \begin{bmatrix}1 & 11 & 5 & 55 & 1 & -1 & -1 & 2\\ 125 & 1368 & 620 & 6804 & 122 & -126 & -124 & 252\\ 54 & 600 & 274 & 3024 & 56 & -54 & -56 & 105\end{bmatrix}$$

$L_{129.5}$: $[1^{1}2^{1}]_{2}16^{-}_{3},1^{2}3^{1},1^{-2}7^{-}$ $\langle m\rangle$ $\quad 2_{2}48_{2}1^{r}_{2}336^{*}_{2}8^{l}_{2}3^{r}_{2}16^{*}_{2}84^{l}_{2}$

$$\begin{bmatrix}-1594320 & 4368 & 9408\\ 4368 & -10 & -28\\ 9408 & -28 & -53\end{bmatrix} \quad \begin{bmatrix}2 & 19 & 4 & 83 & 1 & -1 & -1 & 11\\ 251 & 2376 & 499 & 10332 & 122 & -126 & -124 & 1386\\ 222 & 2112 & 445 & 9240 & 112 & -111 & -112 & 1218\end{bmatrix}$$

$L_{129.6}$: $[1^{-}2^{1}]_{4}16^{1}_{1},1^{2}3^{1},1^{-2}7^{-}$ $\langle m\rangle$ $\quad 8^{*}_{2}48^{*}_{2}4^{s}_{2}336^{l}_{2}2_{2}3_{2}16_{2}21^{r}_{2}$

$$\begin{bmatrix}-423024 & 2352 & 2352\\ 2352 & -2 & -14\\ 2352 & -14 & -13\end{bmatrix} \quad \begin{bmatrix}3 & -1 & -1 & 1 & 1 & 4 & 17 & 26\\ 42 & -12 & -14 & 0 & 13 & 54 & 232 & 357\\ 496 & -168 & -166 & 168 & 166 & 663 & 2816 & 4305\end{bmatrix}$$

$L_{129.7}$: $[1^{1}2^{1}]_{0}16^{-}_{5},1^{2}3^{1},1^{-2}7^{-}$ $\quad 8^{s}_{2}48^{l}_{2}1_{2}336_{2}2^{r}_{2}12^{s}_{2}16^{s}_{2}84^{*}_{2}$

$$\begin{bmatrix}-72240 & 0 & 1344\\ 0 & 2 & 0\\ 1344 & 0 & -25\end{bmatrix} \quad \begin{bmatrix}1 & 5 & 1 & 19 & 0 & -1 & -1 & 1\\ -6 & -12 & -1 & 0 & 1 & 0 & -8 & -42\\ 52 & 264 & 53 & 1008 & 0 & -54 & -56 & 42\end{bmatrix}$$

W_{130} 64 lattices, $\chi = 18$ 7-gon: 2222222

$L_{130.1}$: $1^{2}_{6}8^{-}_{3},1^{-}3^{-}9^{1},1^{2}7^{1}$ $\langle 3,2\rangle$ $\quad 6^{b}_{2}14^{l}_{2}24_{2}63^{r}_{2}8^{s}_{2}36^{*}_{2}56^{b}_{2}$

$$\begin{bmatrix}-94248 & -16128 & 1008\\ -16128 & -2757 & 171\\ 1008 & 171 & -10\end{bmatrix} \quad \begin{bmatrix}-3 & -2 & 5 & 16 & 3 & -1 & -17\\ 20 & 14 & -32 & -105 & -20 & 6 & 112\\ 39 & 35 & -48 & -189 & -40 & 0 & 196\end{bmatrix}$$

$L_{130.2}$: $1^{-2}_{6}8^{1}_{7},1^{-}3^{-}9^{1},1^{2}7^{1}$ $\langle 32 \to N'_{18}, 3m, 3, m\rangle$ $\quad 6^{s}_{2}14^{b}_{2}24^{*}_{2}252^{s}_{2}8^{l}_{2}9_{2}56^{r}_{2}$

$$\begin{bmatrix}915768 & 504 & -7560\\ 504 & -3 & -3\\ -7560 & -3 & 62\end{bmatrix} \quad \begin{bmatrix}1 & 3 & 3 & 1 & -1 & -1 & 1\\ 40 & 126 & 128 & 42 & -44 & -51 & 0\\ 123 & 371 & 372 & 126 & -124 & -126 & 112\end{bmatrix}$$

W_{131} 64 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{131.1}$: $1^{2}_{6}8^{1}_{7},1^{1}3^{-}9^{1},1^{-2}11^{-}$ $\langle 23 \to N'_{20}, 3m, 3, 2, m\rangle$

$24^{b}_{2}198^{s}_{2}6^{b}_{2}792^{*}_{2}4^{*}_{2}24^{b}_{2}22^{s}_{2}6^{b}_{2}88^{*}_{2}36^{*}_{2}$

$$\begin{bmatrix}-84744 & 0 & 792\\ 0 & 15 & -9\\ 792 & -9 & -2\end{bmatrix} \quad \begin{bmatrix}-1 & 1 & 1 & 79 & 3 & 9 & 7 & 1 & 3 & -1\\ -64 & 66 & 64 & 5016 & 190 & 568 & 440 & 62 & 176 & -66\\ -108 & 99 & 105 & 8316 & 316 & 948 & 737 & 105 & 308 & -108\end{bmatrix}$$

$L_{131.2}$: $1^{-2}_{2}16^{-}_{3},1^{-}3^{1}9^{-},1^{-2}11^{1}$ $\langle 3m, 3, m\rangle$

$48_{2}99^{r}_{2}12^{*}_{2}1584^{b}_{2}2^{b}_{2}48^{*}_{2}44^{l}_{2}3_{2}176^{r}_{2}18^{l}_{2}$

shares genus with its 3-dual

$$\begin{bmatrix}-2051280 & -1018512 & 1584\\ -1018512 & -505662 & 783\\ 1584 & 783 & -1\end{bmatrix} \quad \begin{bmatrix}31 & -16 & -31 & -2429 & -46 & -275 & -213 & -15 & -85 & 16\\ -64 & 33 & 64 & 5016 & 95 & 568 & 440 & 31 & 176 & -33\\ -1008 & 495 & 1002 & 79200 & 1504 & 9024 & 7018 & 501 & 2992 & -504\end{bmatrix}$$

W_{132} 32 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{132.1}$: $1^{2}_{2}8^{-}_{5},1^{-2}5^{-},1^{-2}7^{-}$ $\langle 2 \to N'_{23}\rangle$ $\quad 10^{b}_{2}2^{l}_{2}40_{2}1^{r}_{2}140^{*}_{2}8^{b}_{2}\ (\times 2)$

$$\begin{bmatrix}-9764440 & 82320 & 840\\ 82320 & -694 & -7\\ 840 & -7 & 1\end{bmatrix} \begin{bmatrix}-949971 & 7995 & -82\\ -112791560 & 949259 & -9736\\ 8248520 & -69420 & 711\end{bmatrix} \quad \begin{bmatrix}4 & 15 & 347 & 82 & 5763 & 1069\\ 475 & 1781 & 41200 & 9736 & 684250 & 126924\\ -30 & -128 & -3000 & -711 & -50050 & -9288\end{bmatrix}$$

$L_{132.2}$: $1^{-2}_{2}8^{1}_{1},1^{-2}5^{-},1^{-2}7^{-}$ $\langle m\rangle$ $\quad 10^{s}_{2}2^{b}_{2}40^{*}_{2}4^{l}_{2}35_{2}8^{r}_{2}\ (\times 2)$

$$\begin{bmatrix}466760 & -4480 & 0\\ -4480 & 43 & 0\\ 0 & 0 & -1\end{bmatrix} \begin{bmatrix}-50051 & 480 & -75\\ -5125120 & 49151 & -7680\\ 600600 & -5760 & 899\end{bmatrix} \quad \begin{bmatrix}-1 & 1 & 37 & 19 & 349 & 131\\ -105 & 101 & 3780 & 1944 & 35735 & 13416\\ -5 & -21 & -500 & -238 & -4200 & -1560\end{bmatrix}$$

W_{133} 32 lattices, $\chi = 54$ 12-gon: $422222422222 \rtimes C_2$

$L_{133.1} : 1^{2}_{2}8^{-}_{5},1^{2}5^{1},1^{2}7^{1}\ \langle 2 \to N'_{24}\rangle$ $2_{4}1^{r}_{2}56^{l}_{2}5^{r}_{2}28^{*}_{2}8^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} 8680 & -2240 & 0 \\ -2240 & 578 & -1 \\ 0 & -1 & -15 \end{bmatrix} \begin{bmatrix} 7349 & -1925 & -385 \\ 28560 & -7481 & -1496 \\ -2520 & 660 & 131 \end{bmatrix} \quad \begin{bmatrix} -8 & 1 & 223 & 126 & 443 & 351 \\ -31 & 4 & 868 & 490 & 1722 & 1364 \\ 2 & -1 & -84 & -45 & -154 & -120 \end{bmatrix}$$

$L_{133.2} : 1^{-2}_{2}8^{1}_{1},1^{2}5^{1},1^{2}7^{1}\ \langle m\rangle$ $2^{*}_{4}4^{s}_{2}56^{s}_{2}20^{l}_{2}7_{2}8^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -668920 & -105840 & 6440 \\ -105840 & -16745 & 1019 \\ 6440 & 1019 & -62 \end{bmatrix} \begin{bmatrix} 23309 & 3681 & -225 \\ -31080 & -4909 & 300 \\ 1906240 & 301024 & -18401 \end{bmatrix} \quad \begin{bmatrix} -1 & 3 & 73 & 73 & 60 & 91 \\ 2 & -2 & -84 & -90 & -77 & -120 \\ -71 & 278 & 6188 & 6090 & 4956 & 7464 \end{bmatrix}$$

W_{134} 32 lattices, $\chi = 21$ 7-gon: 2222224

$L_{134.1} : 1^{2}_{2}8^{1}_{1},1^{2}3^{1},1^{2}13^{-}\ \langle 2 \to N'_{27}\rangle$ $2^{b}_{2}26^{l}_{2}8^{r}_{2}2^{b}_{2}104^{*}_{2}12^{l}_{2}1_{4}$

$$\begin{bmatrix} -1962168 & 6864 & 9672 \\ 6864 & -22 & -35 \\ 9672 & -35 & -47 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 & 3 & 6 & 113 & 11 & 1 \\ -83 & -169 & 248 & 499 & 9412 & 918 & 84 \\ -144 & -286 & 432 & 862 & 16224 & 1578 & 143 \end{bmatrix}$$

$L_{134.2} : 1^{-2}_{2}8^{-}_{5},1^{2}3^{1},1^{2}13^{-}\ \langle m\rangle$ $2^{s}_{2}26^{b}_{2}8^{b}_{2}2^{l}_{2}104_{2}3^{r}_{2}4^{*}_{4}$

$$\begin{bmatrix} -8293272 & 29016 & 14664 \\ 29016 & -101 & -52 \\ 14664 & -52 & -25 \end{bmatrix} \quad \begin{bmatrix} 3 & 17 & 7 & 3 & 21 & -1 & -1 \\ 621 & 3523 & 1452 & 623 & 4368 & -207 & -208 \\ 467 & 2639 & 1084 & 463 & 3224 & -156 & -154 \end{bmatrix}$$

W_{135} 32 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{135.1} : 1^{-2}_{6}8^{1}_{7},1^{2}3^{-},1^{-2}23^{1}\ \langle 2 \to N'_{30}\rangle$ $6^{s}_{2}46^{b}_{2}24^{*}_{2}92^{s}_{2}8^{l}_{2}69_{2}1_{2}184^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -113160 & 552 & 552 \\ 552 & 5 & -5 \\ 552 & -5 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -3312 & -11 & 24 \\ -1656 & -5 & 11 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 5 & 1 & 29 & 3 & 67 \\ -48 & -46 & 48 & 230 & 44 & 1035 & 103 & 2208 \\ -159 & -161 & 156 & 782 & 156 & 4416 & 455 & 10120 \end{bmatrix}$$

$L_{135.2} : 1^{2}_{6}8^{-}_{3},1^{2}3^{-},1^{-2}23^{1}\ \langle m\rangle$ $6^{b}_{2}46^{l}_{2}24_{2}23^{r}_{2}8^{s}_{2}276^{*}_{2}4^{*}_{2}184^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} 25944 & -1104 & 0 \\ -1104 & 47 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 1104 & -48 & 7 \\ 7728 & -329 & 48 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 & 1 & -1 & -1 & -83 & -9 & -105 \\ 45 & 115 & 24 & -23 & -24 & -2208 & -242 & -2852 \\ -15 & -23 & 0 & 0 & -4 & -1794 & -212 & -2668 \end{bmatrix}$$

W_{136} 64 lattices, $\chi = 18$ 7-gon: 2222222

$L_{136.1} : 1^{-2}_{6}8^{-}_{3},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}\ \langle 2 \to N'_{31}\rangle$ $24_{2}5^{r}_{2}168^{l}_{2}1_{2}280^{r}_{2}6^{b}_{2}70^{l}_{2}$

$$\begin{bmatrix} 472920 & 157080 & -840 \\ 157080 & 52174 & -279 \\ -840 & -279 & 1 \end{bmatrix} \quad \begin{bmatrix} 247 & 83 & 251 & 0 & -93 & -1 & 244 \\ -744 & -250 & -756 & 0 & 280 & 3 & -735 \\ -72 & -25 & -84 & -1 & 0 & 0 & -70 \end{bmatrix}$$

$L_{136.2} : 1^{2}_{6}8^{1}_{7},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}\ \langle m\rangle$ $24^{*}_{2}20^{s}_{2}168^{s}_{2}4^{*}_{2}280^{b}_{2}6^{s}_{2}70^{b}_{2}$

$$\begin{bmatrix} -867720 & 288960 & 3360 \\ 288960 & -96226 & -1119 \\ 3360 & -1119 & -13 \end{bmatrix} \quad \begin{bmatrix} -13 & -7 & 1 & 3 & 51 & 1 & -13 \\ -36 & -20 & 0 & 8 & 140 & 3 & -35 \\ -264 & -90 & 252 & 86 & 1120 & 0 & -350 \end{bmatrix}$$

W_{137} 64 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{137.1} : 1^{-2}_{6}8^{-}_{3},1^{2}3^{-},1^{2}5^{-},1^{-2}7^{1}\ \langle 2 \to N'_{35}\rangle$

$6^{s}_{2}14^{l}_{2}24_{2}1^{r}_{2}40^{s}_{2}28^{*}_{2}60^{*}_{2}56^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} 8617560 & -9240 & -10920 \\ -9240 & 13 & 11 \\ -10920 & 11 & 14 \end{bmatrix} \begin{bmatrix} 41999 & -50 & -52 \\ 6384000 & -7601 & -7904 \\ 27783000 & -33075 & -34399 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -5 & -19 & -4 & -37 & -83 & -179 & -221 \\ -150 & -756 & -2880 & -607 & -5620 & -12614 & -27210 & -33600 \\ -663 & -3311 & -12576 & -2647 & -24480 & -54908 & -118410 & -146188 \end{bmatrix}$$

$L_{137.2}:\ 1_6^2 8_7^1,1^2 3^-,1^2 5^-,1^{-2} 7^1\ \langle m\rangle$ $\qquad 6_2^b 14_2^b 24_2^* 4_2^s 40_2^l 7_2 15_2 56_2^r\ (\times 2)$

$$\begin{bmatrix} 152066040 & 533400 & -47040 \\ 533400 & 1871 & -165 \\ -47040 & -165 & 14 \end{bmatrix} \begin{bmatrix} -655201 & -2298 & 156 \\ 186950400 & 655695 & -44512 \\ 2074800 & 7277 & -495 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 5 & 37 & 19 & 101 & 122 & 271 & 683 \\ 570 & -1428 & -10560 & -5422 & -28820 & -34811 & -77325 & -194880 \\ -3 & -35 & -156 & -70 & -340 & -392 & -855 & -2128 \end{bmatrix}$$

W_{138} 64 lattices, $\chi = 72$ $\qquad$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{138.1}:\ 1_2^2 8_5^-,1^2 3^-,1^{-2} 5^1,1^{-2} 13^-\ \langle 2 \to N'_{37}\rangle$

$2_2^b 26_2^b 8_2^* 780_2^l 1_2^r 312_2^l 5_2 104_2^r\ (\times 2)$

$$\begin{bmatrix} -1611480 & -106080 & 0 \\ -106080 & -6982 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 477749 & 31703 & -245 \\ -7254000 & -481369 & 3720 \\ -7059000 & -468428 & 3619 \end{bmatrix} \begin{bmatrix} 11 & 6 & -5 & -77 & 0 & 113 & 27 & 363 \\ -167 & -91 & 76 & 1170 & 0 & -1716 & -410 & -5512 \\ -160 & -78 & 80 & 1170 & -1 & -1716 & -405 & -5408 \end{bmatrix}$$

$L_{138.2}:\ 1_2^{-2} 8_1^1,1^2 3^-,1^{-2} 5^1,1^{-2} 13^-\ \langle m\rangle 2_2^s 26_2^l 8_2 195_2^r 4_2^s 312_2^s 20_2^* 104_2^b\ (\times 2)$

$$\begin{bmatrix} 69243720 & -118560 & 0 \\ -118560 & 203 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -887251 & 1519 & -112 \\ -517647000 & 886227 & -65344 \\ 8112000 & -13888 & 1023 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & 31 & 11 & 63 \\ 583 & -585 & -584 & 585 & 584 & 18096 & 6420 & 36764 \\ -15 & -13 & 0 & 0 & -2 & -156 & -70 & -468 \end{bmatrix}$$

W_{139} 64 lattices, $\chi = 108$ $\qquad$ 22-gon: $2222222222222222222222 \rtimes C_2$

$L_{139.1}:\ 1_6^{-2} 8_7^1,1^2 3^-,1^{-2} 5^-,1^2 19^-\ \langle 2 \to N'_{38}\rangle$

$1_2 285_2^r 8_2^s 60_2^* 152_2^b 6_2^s 190_2^b 24_2^b 38_2^b 6_2^l 760_2\ (\times 2)$

$$\begin{bmatrix} 3472440 & 1155960 & -2280 \\ 1155960 & 384814 & -759 \\ -2280 & -759 & 1 \end{bmatrix} \begin{bmatrix} -17255041 & -5743166 & -7095 \\ 51838080 & 17253781 & 21315 \\ 3064320 & 1019928 & 1259 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1423 & 237 & 9227 & 57957 & 9935 & 83261 & 21997 & 35714 & 25543 & 719213 \\ 0 & -4275 & -712 & -27720 & -174116 & -29847 & -250135 & -66084 & -107293 & -76737 & -2160680 \\ -1 & -285 & -44 & -1650 & -10336 & -1770 & -14820 & -3912 & -6346 & -4536 & -127680 \end{bmatrix}$$

$L_{139.2}:\ 1_6^2 8_3^-,1^2 3^-,1^{-2} 5^-,1^2 19^-\ \langle m\rangle$

$4_2^* 1140_2^s 8_2^l 15_2 152_2^r 6_2^b 190_2^l 24_2^r 38_2^s 6_2^b 760_2^*\ (\times 2)$

$$\begin{bmatrix} -40166760 & -13390440 & 34200 \\ -13390440 & -4463986 & 11401 \\ 34200 & 11401 & -29 \end{bmatrix} \begin{bmatrix} 734159 & 244846 & -665 \\ -2202480 & -734539 & 1995 \\ -419520 & -139912 & 379 \end{bmatrix}$$

$$\begin{bmatrix} -53 & -1699 & -49 & -89 & -347 & -10 & 291 & 169 & 425 & 379 & 11773 \\ 160 & 5130 & 148 & 270 & 1064 & 33 & -855 & -504 & -1273 & -1137 & -35340 \\ 398 & 13110 & 396 & 1155 & 8664 & 1110 & 6460 & 1008 & 494 & -216 & -14440 \end{bmatrix}$$

W_{140} 4 lattices, $\chi = 6$ $\qquad$ 3-gon: $\infty{\nmid}2\infty| \rtimes D_2$

$L_{140.1}:\ 1_0^2 4_1^1$ $\qquad 4_{\infty a}^{1,0} 4_2 1_{\infty}^{4,1}$

$$\begin{bmatrix} -44 & -16 & 12 \\ -16 & -5 & 4 \\ 12 & 4 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & -2 \\ 6 & -4 & -3 \end{bmatrix}$$

$L_{140.2}:\ 1_{\mathrm{II}}^2 8_1^1$ $\qquad 8_{\infty a}^{1,0} 8_2^r 2_{\infty a}^{4,1}$

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -6 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 0 \\ 4 & 0 & -4 \\ 0 & 0 & -1 \end{bmatrix}$$

W_{141} 17 lattices, $\chi = 6$ 4-gon: $\not{2}2\not{\infty}2 \rtimes D_2$

$L_{141.1} : 1^2_0 8^1_1$ $\quad$ $1^r_2 4^*_2 8^{1,0}_{\infty b} 8_2$

$$\begin{bmatrix} -696 & 72 & 80 \\ 72 & -7 & -9 \\ 80 & -9 & -8 \end{bmatrix} \qquad \begin{bmatrix} 1 & -3 & -3 & 7 \\ 5 & -18 & -16 & 40 \\ 4 & -10 & -12 & 24 \end{bmatrix}$$

$L_{141.2} : [1^- 2^1]_6 16^-_3 \ \langle 2 \rangle$ $\quad$ $16^*_2 4^l_2 2^{8,3}_\infty 8^s_2$

$$\begin{bmatrix} -848 & 160 & 32 \\ 160 & -30 & -6 \\ 32 & -6 & -1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 0 & 1 \\ -4 & -6 & -1 & 6 \\ -8 & 2 & 4 & -4 \end{bmatrix}$$

$L_{141.3} : [1^1 2^1]_2 16^1_7 \ \langle m \rangle$ $\quad$ $16^l_2 1_2 2^{8,7}_\infty 8^*_2$

$$\begin{bmatrix} -1296 & 208 & 96 \\ 208 & -30 & -16 \\ 96 & -16 & -7 \end{bmatrix} \qquad \begin{bmatrix} -3 & 1 & 2 & -3 \\ -4 & 2 & 3 & -6 \\ -32 & 9 & 20 & -28 \end{bmatrix}$$

$L_{141.4} : [1^- 2^1]_4 16^-_5 \ \langle m \rangle$ $\quad$ $16^s_2 4^*_2 8^{8,3}_{\infty z} 2^r_2$

$$\begin{bmatrix} -304 & 32 & 32 \\ 32 & -2 & -4 \\ 32 & -4 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & -4 & -2 & 3 \\ 8 & -6 & -8 & 6 \end{bmatrix}$$

$L_{141.5} : 1^1_1 8^1_7 64^1_1 \ \langle 2 \rangle$ $\quad$ $64^s_2 4^*_2 32^{16,7}_{\infty z} 8^b_2$

shares genus with $L_{141.6}$; isometric to its own 2-dual

$$\begin{bmatrix} -17344 & 512 & 512 \\ 512 & -8 & -16 \\ 512 & -16 & -15 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & -4 & -2 & 7 \\ 32 & -30 & -32 & 60 \end{bmatrix}$$

$L_{141.6} : 1^1_1 8^1_7 64^1_1$ $\quad$ $64_2 1^r_2 32^{16,15}_{\infty z} 8^l_2$

shares genus with $L_{141.5}$; isometric to its own 2-dual

$$\begin{bmatrix} -266176 & 2048 & 4096 \\ 2048 & -8 & -32 \\ 4096 & -32 & -63 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 4 \\ 0 & -4 & -2 & 15 \\ 64 & -63 & -64 & 252 \end{bmatrix}$$

W_{142} 22 lattices, $\chi = 9$ 4-gon: $4\infty22$

$L_{142.1} : 1^2_{\mathrm{II}} 4^1_1, 1^2 9^- \ \langle 2 \rangle$ $\quad$ $2^*_4 4^{3,2}_{\infty b} 4^r_2 18^b_2$

$$\begin{bmatrix} -1116 & 288 & 108 \\ 288 & -56 & -25 \\ 108 & -25 & -10 \end{bmatrix} \qquad \begin{bmatrix} -2 & -1 & 3 & 1 \\ 6 & 2 & -8 & 0 \\ -37 & -16 & 52 & 9 \end{bmatrix}$$

$L_{142.2} : 1^{-2}_2 8^-_3, 1^2 9^1 \ \langle 2 \rangle$ $\quad$ $4^*_4 2^{12,11}_{\infty a} 8^s_2 36^*_2$

$$\begin{bmatrix} -47592 & -16128 & 1728 \\ -16128 & -5465 & 585 \\ 1728 & 585 & -62 \end{bmatrix} \qquad \begin{bmatrix} 13 & -10 & -21 & 17 \\ -42 & 32 & 68 & -54 \\ -34 & 23 & 56 & -36 \end{bmatrix}$$

$L_{142.3} : 1^2_2 8^1_7, 1^2 9^1 \ \langle m \rangle$ $\quad$ $1_4 2^{12,11}_{\infty b} 8^l_2 9_2$

$$\begin{bmatrix} -38088 & 576 & 648 \\ 576 & -7 & -10 \\ 648 & -10 & -11 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & -1 & 2 \\ 6 & 1 & -8 & 9 \\ 53 & -1 & -52 & 108 \end{bmatrix}$$

W_{143} 22 lattices, $\chi = 18$ 5-gon: $2\infty\not{2}\infty2| \rtimes D_2$

$L_{143.1} : 1^2_{\mathrm{II}} 4^1_1, 1^{-2} 9^1 \ \langle 2 \rangle$ $\quad$ $2^l_2 36^{1,0}_\infty 36^*_2 4^{3,1}_{\infty a} 4^r_2$

$$\begin{bmatrix} -2556 & 900 & 144 \\ 900 & -316 & -51 \\ 144 & -51 & -8 \end{bmatrix} \qquad \begin{bmatrix} 3 & 19 & -1 & -3 & 1 \\ 5 & 36 & 0 & -6 & 0 \\ 21 & 108 & -18 & -16 & 16 \end{bmatrix}$$

$L_{143.2}$: $1^{-2}_{2}8^{-}_{3},1^{-2}9^{-}$ $\langle 2\rangle$ $\qquad 4^{s}_{2}72^{4,1}_{\infty z}18^{b}_{2}2^{12,7}_{\infty b}8^{s}_{2}$

$$\begin{bmatrix} -52200 & -25488 & 576 \\ -25488 & -12445 & 281 \\ 576 & 281 & -6 \end{bmatrix} \qquad \begin{bmatrix} 1 & -121 & -52 & -1 & 25 \\ -2 & 252 & 108 & 2 & -52 \\ 2 & 180 & 63 & -3 & -36 \end{bmatrix}$$

$L_{143.3}$: $1^{2}_{2}8^{1}_{7},1^{-2}9^{-}$ $\langle m\rangle$ $\qquad 1^{r}_{2}72^{4,3}_{\infty z}18^{s}_{2}2^{12,7}_{\infty a}8^{l}_{2}$

$$\begin{bmatrix} -334728 & 3456 & 1656 \\ 3456 & -35 & -18 \\ 1656 & -18 & -7 \end{bmatrix} \qquad \begin{bmatrix} -1 & 5 & 11 & 3 & -1 \\ -71 & 360 & 783 & 213 & -72 \\ -54 & 252 & 585 & 161 & -52 \end{bmatrix}$$

W_{144} 16 lattices, $\chi = 12$ $\qquad$ 4-gon: $2|2\infty|\infty \rtimes D_2$

$L_{144.1}$: $1^{-2}_{4}4^{1}_{7},1^{-}3^{1}9^{1}$ $\langle 3\rangle$ $\qquad 12_{2}9_{2}3^{12,7}_{\infty}12^{3,1}_{\infty b}$

$$\begin{bmatrix} -612 & -180 & -792 \\ -180 & -33 & -177 \\ -792 & -177 & -868 \end{bmatrix} \qquad \begin{bmatrix} 17 & -4 & -14 & -3 \\ 100 & -27 & -85 & -16 \\ -36 & 9 & 30 & 6 \end{bmatrix}$$

$L_{144.2}$: $1^{2}_{\mathrm{II}}8^{-}_{3},1^{1}3^{-}9^{-}$ $\langle 3\rangle$ $\qquad 24^{r}_{2}18^{b}_{2}6^{12,7}_{\infty b}24^{3,2}_{\infty a}$

$$\begin{bmatrix} -19368 & -3672 & -1944 \\ -3672 & -696 & -369 \\ -1944 & -369 & -194 \end{bmatrix} \qquad \begin{bmatrix} 13 & 2 & -7 & -5 \\ -56 & -6 & 32 & 20 \\ -24 & -9 & 9 & 12 \end{bmatrix}$$

W_{145} 2 lattices, $\chi = 6$ $\qquad$ 3-gon: ${\infty\mkern-14mu|}\;4|4 \rtimes D_2$

$L_{145.1}$: $1^{2}_{2}16^{1}_{7}$ $\qquad 1^{8,7}_{\infty}4^{*}_{4}2_{4}$

$$\begin{bmatrix} -656 & -48 & 64 \\ -48 & -3 & 5 \\ 64 & 5 & -6 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & 1 \\ -1 & 6 & -4 \\ -1 & -6 & 7 \end{bmatrix}$$

W_{146} 9 lattices, $\chi = 12$ $\qquad$ 4-gon: ${2\mkern-9mu|}\;{\infty\mkern-14mu|}\;{2\mkern-9mu|}\;{\infty\mkern-14mu|}\; \rtimes D_4$

$L_{146.1}$: $1^{-2}_{4}16^{-}_{5}$ $\qquad 16^{l}_{2}1^{8,5}_{\infty}4^{s}_{2}16^{4,1}_{\infty z}$

$$\begin{bmatrix} -3632 & 64 & 336 \\ 64 & -1 & -6 \\ 336 & -6 & -31 \end{bmatrix} \qquad \begin{bmatrix} -3 & 0 & 1 & -1 \\ -8 & 4 & 4 & -16 \\ -32 & -1 & 10 & -8 \end{bmatrix}$$

$L_{146.2}$: $1^{2}_{0}16^{1}_{1}$ $\qquad 16_{2}1^{8,1}_{\infty}4^{*}_{2}16^{1,0}_{\infty b}$

$$\begin{bmatrix} -1392 & 112 & 208 \\ 112 & -8 & -17 \\ 208 & -17 & -31 \end{bmatrix} \qquad \begin{bmatrix} 11 & 2 & -3 & -3 \\ 16 & 4 & -4 & -8 \\ 64 & 11 & -18 & -16 \end{bmatrix}$$

$L_{146.3}$: $1^{-2}_{6}16^{-}_{3}$ $\qquad 4^{l}_{2}1^{8,3}_{\infty}\,(\times 2)$

$$\begin{bmatrix} -80 & 48 & 16 \\ 48 & -15 & -8 \\ 16 & -8 & -3 \end{bmatrix} \begin{bmatrix} 15 & -7 & -3 \\ -16 & 6 & 3 \\ 112 & -49 & -22 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ -6 & -8 \end{bmatrix}$$

$L_{146.4}$: $1^{1}_{1}4^{1}_{7}16^{1}_{1}$ $\qquad 16^{s}_{2}4^{4,1}_{\infty z}1_{2}16^{4,3}_{\infty}$

$$\begin{bmatrix} -2672 & 112 & 288 \\ 112 & -4 & -12 \\ 288 & -12 & -31 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 0 & -3 \\ -4 & -2 & 2 & 8 \\ -8 & 10 & -1 & -32 \end{bmatrix}$$

$L_{146.5}$: $1^{1}_{1}4^{1}_{1}16^{1}_{7}$ $\qquad 4^{r}_{2}4^{4,3}_{\infty z}1_{2}4^{8,7}_{\infty}$

$$\begin{bmatrix} -80 & 48 & 16 \\ 48 & -12 & -8 \\ 16 & -8 & -3 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 1 & 2 \\ 1 & 0 & -1 & -1 \\ -8 & -6 & 7 & 12 \end{bmatrix}$$

W_{147} 4 lattices, $\chi = 12$ — 4-gon: $2|2\infty|\infty \rtimes D_2$

$L_{147.1}$: $1^{2}_{\mathrm{II}}16^{1}_{1}$ — $16^{r}_{2}2^{b}_{2}16^{2,1}_{\infty z}16^{1,0}_{\infty a}$

$$\begin{bmatrix}16 & 0 & 0\\ 0 & -4 & 11\\ 0 & 11 & -30\end{bmatrix} \qquad \begin{bmatrix}1 & 0 & -3 & -1\\ 0 & -8 & -16 & 24\\ 0 & -3 & -8 & 8\end{bmatrix}$$

$L_{147.2}$: $1^{1}_{7}8^{-}_{3}64^{-}_{3}$ — $32^{*}_{2}64^{s}_{2}32^{16,1}_{\infty z}8^{8,3}_{\infty b}$

$$\begin{bmatrix}-3392 & 384 & 64\\ 384 & -40 & -8\\ 64 & -8 & -1\end{bmatrix} \qquad \begin{bmatrix}-3 & -1 & 1 & 0\\ -18 & -4 & 6 & -1\\ -64 & -32 & 16 & 4\end{bmatrix}$$

W_{148} 16 lattices, $\chi = 12$ — 4-gon: $2\infty 2\infty \rtimes C_2$

$L_{148.1}$: $1^{-2}_{\mathrm{II}}8^{-}_{5},1^{-2}9^{-}$ $\langle 2\rangle$ — $18^{b}_{2}8^{6,1}_{\infty z}2^{b}_{2}72^{2,1}_{\infty z}$

$$\begin{bmatrix}936 & 288 & 0\\ 288 & 82 & 15\\ 0 & 15 & -34\end{bmatrix} \qquad \begin{bmatrix}25 & 11 & -12 & -77\\ -81 & -36 & 39 & 252\\ -36 & -16 & 17 & 108\end{bmatrix}$$

$L_{148.2}$: $1^{-2}_{\mathrm{II}}8^{-}_{5},1^{2}9^{1}$ $\langle 2\rangle$ — $2^{b}_{2}8^{6,5}_{\infty z}$ $(\times 2)$

$$\begin{bmatrix}-18648 & 432 & 216\\ 432 & -10 & -5\\ 216 & -5 & -2\end{bmatrix}\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ -216 & 5 & 1\end{bmatrix} \qquad \begin{bmatrix}0 & 1\\ 1 & 44\\ -3 & -4\end{bmatrix}$$

W_{149} 27 lattices, $\chi = 18$ — 5-gon: $\not{2}2\infty|\infty 2 \rtimes D_2$

$L_{149.1}$: $1^{-2}_{4}4^{1}_{1},1^{1}5^{1}25^{1}$ $\langle 5\rangle$ — $100_{2}1_{2}20^{10,9}_{\infty}20^{5,4}_{\infty z}5_{2}$

shares genus with its 5-dual

$$\begin{bmatrix}-11100 & -800 & -5200\\ -800 & -55 & -375\\ -5200 & -375 & -2436\end{bmatrix} \qquad \begin{bmatrix}-47 & 1 & 19 & -5 & -19\\ 0 & -1 & -4 & 4 & 3\\ 100 & -2 & -40 & 10 & 40\end{bmatrix}$$

$L_{149.2}$: $1^{2}_{\mathrm{II}}4^{-}_{5},1^{1}5^{1}25^{1}$ $\langle 2\rangle$ — $100^{*}_{2}4^{*}_{2}20^{5,4}_{\infty b}20^{5,3}_{\infty}20^{*}_{2}$

$$\begin{bmatrix}-1900 & 200 & 0\\ 200 & -20 & -5\\ 0 & -5 & 24\end{bmatrix} \qquad \begin{bmatrix}-1 & 3 & 11 & 1 & -5\\ -10 & 28 & 102 & 8 & -48\\ 0 & 6 & 20 & 0 & -10\end{bmatrix}$$

$L_{149.3}$: $1^{-2}_{2}8^{1}_{7},1^{-}5^{-}25^{-}$ $\langle 2\rangle$ — $50^{b}_{2}2^{s}_{2}10^{20,19}_{\infty b}40^{20,13}_{\infty z}10^{s}_{2}$

$$\begin{bmatrix}143800 & -3600 & -1000\\ -3600 & 90 & 25\\ -1000 & 25 & 7\end{bmatrix} \qquad \begin{bmatrix}-1 & 0 & 0 & -1 & -1\\ -25 & -1 & -3 & -24 & -23\\ -50 & 4 & 10 & -60 & -60\end{bmatrix}$$

$L_{149.4}$: $1^{2}_{2}8^{-}_{3},1^{-}5^{-}25^{-}$ $\langle m\rangle$ — $50^{s}_{2}2^{b}_{2}10^{20,19}_{\infty a}40^{20,3}_{\infty z}10^{b}_{2}$

$$\begin{bmatrix}442200 & -6400 & -2400\\ -6400 & 90 & 35\\ -2400 & 35 & 13\end{bmatrix} \qquad \begin{bmatrix}-2 & 0 & 1 & 1 & -1\\ -25 & 1 & 17 & 16 & -13\\ -300 & -2 & 140 & 140 & -150\end{bmatrix}$$

$L_{149.5}$: $1^{2}_{\mathrm{II}}8^{-}_{5},1^{-}5^{-}25^{-}$ $\langle 5\rangle$ — $200^{b}_{2}2^{l}_{2}40^{5,4}_{\infty}40^{5,2}_{\infty z}10^{b}_{2}$

shares genus with its 5-dual

$$\begin{bmatrix}-2200 & 600 & -2600\\ 600 & -110 & 425\\ -2600 & 425 & -1568\end{bmatrix} \qquad \begin{bmatrix}53 & -1 & -21 & 5 & 21\\ 1060 & -21 & -424 & 104 & 423\\ 200 & -4 & -80 & 20 & 80\end{bmatrix}$$

W_{150} 60 lattices, $\chi = 12$ 5-gon: $22|22\infty\!\!\!\!\mid\ \rtimes D_2$

$L_{150.1}$: $1^2_0 8^-_3, 1^- 3^- 9^1$ $\langle 3 \rangle$ $24_2 9^r_2 8^s_2 36^*_2 24^{3,1}_{\infty a}$

$$\begin{bmatrix} -28584 & 144 & 1080 \\ 144 & 15 & -9 \\ 1080 & -9 & -40 \end{bmatrix} \quad \begin{bmatrix} 15 & 13 & 5 & -7 & -7 \\ 88 & 75 & 28 & -42 & -40 \\ 384 & 333 & 128 & -180 & -180 \end{bmatrix}$$

$L_{150.2}$: $[1^1 2^-]_4 16^1_7, 1^- 3^- 9^1$ $\langle 3m, 3, 2 \rangle$ $6_2 9^r_2 8^*_2 144^s_2 24^{24,1}_{\infty z}$

$$\begin{bmatrix} -75024 & 4608 & 288 \\ 4608 & -282 & -18 \\ 288 & -18 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & -1 & -1 & 1 \\ -1 & -15 & -14 & -12 & 14 \\ 12 & -27 & -40 & -72 & 36 \end{bmatrix}$$

$L_{150.3}$: $[1^- 2^-]_0 16^-_3, 1^- 3^- 9^1$ $\langle 32, 3, m \rangle$ $6^r_2 36^*_2 8^s_2 144^*_2 24^{24,13}_{\infty z}$

$$\begin{bmatrix} -118224 & 5904 & 2304 \\ 5904 & -282 & -120 \\ 2304 & -120 & -43 \end{bmatrix} \quad \begin{bmatrix} 6 & 23 & 5 & -11 & -7 \\ 59 & 228 & 50 & -108 & -70 \\ 156 & 594 & 128 & -288 & -180 \end{bmatrix}$$

$L_{150.4}$: $[1^- 2^1]_6 16^1_1, 1^- 3^- 9^1$ $\langle 3m, 3, m \rangle$ $24^*_2 36^l_2 2_2 144_2 6^{24,1}_{\infty}$

$$\begin{bmatrix} -10224 & 576 & 1296 \\ 576 & -30 & -84 \\ 1296 & -84 & -115 \end{bmatrix} \quad \begin{bmatrix} -9 & -7 & 3 & 55 & 7 \\ -106 & -84 & 35 & 648 & 83 \\ -24 & -18 & 8 & 144 & 18 \end{bmatrix}$$

$L_{150.5}$: $[1^1 2^1]_2 16^-_5, 1^- 3^- 9^1$ $\langle 3 \rangle$ $24^l_2 9_2 2^r_2 144^l_2 6^{24,13}_{\infty}$

$$\begin{bmatrix} -60336 & 1440 & -13392 \\ 1440 & -30 & 336 \\ -13392 & 336 & -2911 \end{bmatrix} \quad \begin{bmatrix} 15 & 14 & -5 & -157 & -28 \\ 182 & 168 & -61 & -1896 & -337 \\ -48 & -45 & 16 & 504 & 90 \end{bmatrix}$$

W_{151} 30 lattices, $\chi = 6$ 5-gon: $2\!\!\!\mid 22|22 \rtimes D_2$

$L_{151.1}$: $1^{-2}_4 8^1_7, 1^2 3^1$ $1^r_2 4^*_2 12^s_2 8^l_2 3_2$

$$\begin{bmatrix} -456 & 48 & 0 \\ 48 & -1 & -2 \\ 0 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & -1 & 1 & 1 \\ 0 & -10 & -12 & 8 & 9 \\ -1 & -20 & -18 & 20 & 18 \end{bmatrix}$$

$L_{151.2}$: $[1^- 2^1]_4 16^1_7, 1^2 3^1$ $\langle 2 \rangle$ $16^*_2 4^s_2 48^l_2 2_2 3^r_2$

$$\begin{bmatrix} -3984 & 144 & 144 \\ 144 & -2 & -6 \\ 144 & -6 & -5 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 1 & 1 \\ -4 & -6 & 0 & 5 & 6 \\ -24 & -22 & 24 & 22 & 21 \end{bmatrix}$$

$L_{151.3}$: $[1^- 2^1]_2 16^1_1, 1^2 3^1$ $\langle m \rangle$ $16_2 1^r_2 48^*_2 8^l_2 3_2$

$$\begin{bmatrix} -6000 & 144 & 288 \\ 144 & -2 & -8 \\ 288 & -8 & -13 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 5 & -1 & -1 \\ 8 & 11 & 60 & -10 & -12 \\ 16 & 15 & 72 & -16 & -15 \end{bmatrix}$$

$L_{151.4}$: $[1^1 2^1]_0 16^-_3, 1^2 3^1$ $\langle m \rangle$ $16^l_2 1_2 48_2 2^r_2 12^*_2$

$$\begin{bmatrix} 48 & 0 & 48 \\ 0 & -2 & -12 \\ 48 & -12 & -23 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 2 & 5 \\ -4 & -6 & 0 & 11 & 30 \\ 0 & 1 & 0 & -2 & -6 \end{bmatrix}$$

$L_{151.5}$: $[1^1 2^1]_6 16^-_5, 1^2 3^1$ $16^s_2 4^*_2 48^s_2 8^*_2 12^s_2$

$$\begin{bmatrix} -816 & 48 & 48 \\ 48 & -2 & -4 \\ 48 & -4 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & -1 & -1 \\ 8 & 10 & 12 & -10 & -12 \\ 8 & 6 & 0 & -8 & -6 \end{bmatrix}$$

W_{152} 22 lattices, $\chi = 90$ 12-gon: $4\infty\infty2\infty24\infty\infty2\infty2 \rtimes C_2$

$L_{152.1}$: $1_{\mathrm{II}}^{2}4_{1}^{1},1^{2}25^{1}$ $\langle 2\rangle$ $2_{4}^{*}4_{\infty b}^{5,4}4_{\infty}^{5,3}4_{2}^{*}100_{\infty b}^{1,0}100_{2}^{r}$ $(\times 2)$

$$\begin{bmatrix} -47100 & -22000 & -700 \\ -22000 & -10276 & -327 \\ -700 & -327 & -10 \end{bmatrix} \begin{bmatrix} -3001 & -1402 & -36 \\ 6000 & 2803 & 72 \\ 16500 & 7711 & 197 \end{bmatrix} \begin{bmatrix} -15 & -1 & 15 & 37 & 263 & 387 \\ 32 & 2 & -32 & -78 & -550 & -800 \\ 3 & 4 & -4 & -46 & -500 & -1100 \end{bmatrix}$$

$L_{152.2}$: $1_{2}^{2}8_{7}^{1},1^{2}25^{-}$ $\langle 2\rangle$ $1_{4}2_{\infty b}^{20,19}8_{\infty z}^{20,3}2_{2}^{s}50_{\infty b}^{4,3}200_{2}^{l}$ $(\times 2)$

$$\begin{bmatrix} -448200 & 2000 & 2600 \\ 2000 & -7 & -12 \\ 2600 & -12 & -15 \end{bmatrix} \begin{bmatrix} 11699 & -33 & -72 \\ 335400 & -947 & -2064 \\ 1747200 & -4928 & -10753 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 4 & 54 & 267 \\ 31 & 1 & -32 & 111 & 1525 & 7600 \\ 148 & -1 & -148 & 599 & 8075 & 39900 \end{bmatrix}$$

$L_{152.3}$: $1_{2}^{-2}8_{3}^{-},1^{2}25^{-}$ $\langle m\rangle$ $4_{4}^{*}2_{\infty a}^{20,19}8_{\infty z}^{20,13}2_{2}^{b}50_{\infty a}^{4,3}200_{2}^{s}$ $(\times 2)$

$$\begin{bmatrix} -6269800 & 2600 & 60000 \\ 2600 & -1 & -25 \\ 60000 & -25 & -574 \end{bmatrix} \begin{bmatrix} 122399 & -60 & -1158 \\ 18972000 & -9301 & -179490 \\ 11954400 & -5860 & -113099 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 & 13 & 138 & 599 \\ -150 & 18 & 484 & 2028 & 21450 & 92900 \\ -98 & -1 & 292 & 1269 & 13475 & 58500 \end{bmatrix}$$

W_{153} 22 lattices, $\chi = 60$ 10-gon: $\infty222\infty\infty222\infty \rtimes C_2$

$L_{153.1}$: $1_{\mathrm{II}}^{2}4_{1}^{1},1^{-2}25^{-}$ $\langle 2\rangle$ $4_{\infty b}^{5,2}4_{2}^{r}50_{2}^{b}2_{2}^{l}4_{\infty}^{5,4}$ $(\times 2)$

$$\begin{bmatrix} 255300 & 86500 & -16400 \\ 86500 & 29308 & -5557 \\ -16400 & -5557 & 1054 \end{bmatrix} \begin{bmatrix} 179199 & 60608 & -11392 \\ -627200 & -212129 & 39872 \\ -518000 & -175195 & 32929 \end{bmatrix} \begin{bmatrix} -131 & -1763 & -9311 & -478 & -411 \\ 458 & 6172 & 32600 & 1674 & 1440 \\ 376 & 5104 & 26975 & 1387 & 1196 \end{bmatrix}$$

$L_{153.2}$: $1_{2}^{-2}8_{3}^{-},1^{-2}25^{1}$ $\langle 2\rangle$ $2_{\infty b}^{20,7}8_{2}^{s}100_{2}^{*}4_{2}^{s}8_{\infty z}^{20,9}$ $(\times 2)$

$$\begin{bmatrix} -322600 & -97800 & 8000 \\ -97800 & -29649 & 2425 \\ 8000 & 2425 & -198 \end{bmatrix} \begin{bmatrix} -1302401 & -394416 & 31856 \\ 4573200 & 1384937 & -111858 \\ 3374400 & 1021896 & -82537 \end{bmatrix} \begin{bmatrix} -20 & 17 & 43 & -13 & -149 \\ 70 & -60 & -150 & 46 & 524 \\ 49 & -48 & -100 & 38 & 396 \end{bmatrix}$$

$L_{153.3}$: $1_{2}^{2}8_{7}^{1},1^{-2}25^{1}$ $\langle m\rangle$ $2_{\infty a}^{20,7}8_{2}^{l}25_{2}1_{2}^{r}8_{\infty z}^{20,19}$ $(\times 2)$

$$\begin{bmatrix} -1566600 & -311000 & 6200 \\ -311000 & -61739 & 1230 \\ 6200 & 1230 & -23 \end{bmatrix} \begin{bmatrix} -8738101 & -1736889 & 38836 \\ 44494200 & 8844197 & -197752 \\ 23871600 & 4745004 & -106097 \end{bmatrix} \begin{bmatrix} -54 & 55 & 54 & -22 & -447 \\ 275 & -280 & -275 & 112 & 2276 \\ 149 & -148 & -150 & 59 & 1216 \end{bmatrix}$$

W_{154} 3 lattices, $\chi = 2$ 4-gon: $|\!\!\!3\,2\,|\!\!\!2\,2 \rtimes D_2$

$L_{154.1}$: $1_{\mathrm{II}}^{-2}4_{7}^{1},1^{1}3^{-}9^{1}$ $\langle 2\rangle$ $6_{3}^{+}6_{2}^{b}4_{2}^{*}36_{2}^{b}$

$$\begin{bmatrix} 252 & 72 & -36 \\ 72 & 6 & -27 \\ -36 & -27 & -14 \end{bmatrix} \begin{bmatrix} 3 & -7 & -1 & 17 \\ -7 & 17 & 2 & -42 \\ 6 & -15 & -2 & 36 \end{bmatrix}$$

W_{155} 3 lattices, $\chi = 2$ 3-gon: $6\,|\!\!\!2\,6| \rtimes D_2$

$L_{155.1}$: $1_{\mathrm{II}}^{-2}4_{7}^{1},1^{-}3^{-}9^{-}$ $\langle 2\rangle$ $6_{6}18_{2}^{b}2_{6}$

$$\begin{bmatrix} -36 & 36 & 36 \\ 36 & -30 & -27 \\ 36 & -27 & -22 \end{bmatrix} \begin{bmatrix} 2 & -2 & -1 \\ 5 & -3 & -3 \\ -3 & 0 & 2 \end{bmatrix}$$

W_{156} 6 lattices, $\chi = 18$ 6-gon: $242242 \rtimes C_2$

$L_{156.1}$: $1_{\mathrm{II}}^{-2}4_{7}^{1},1^{2}27^{-}$ $\langle 2\rangle$ $54_{2}^{b}4_{4}^{*}2_{2}^{s}$ $(\times 2)$

$$\begin{bmatrix} -1352484 & 8316 & 11556 \\ 8316 & -46 & -73 \\ 11556 & -73 & -98 \end{bmatrix} \begin{bmatrix} 38879 & -261 & -324 \\ 1356480 & -9107 & -11304 \\ 3572640 & -23983 & -29773 \end{bmatrix} \begin{bmatrix} -10 & -5 & 5 \\ -351 & -174 & 175 \\ -918 & -460 & 459 \end{bmatrix}$$

W_{157} 8 lattices, $\chi = 48$ 8-gon: $\infty 2|2\infty|\infty 2|2\infty| \rtimes D_4$

$L_{157.1} : 1^{2}_{0}4^{1}_{7},1^{-2}7^{1}$ $28^{1,0}_{\infty z}7_{2}1_{2}28^{2,1}_{\infty}\ (\times 2)$

$\begin{bmatrix} -868 & -56 & -728 \\ -56 & 0 & -25 \\ -728 & -25 & -477 \end{bmatrix} \begin{bmatrix} 377 & 18 & 279 \\ 5124 & 243 & 3782 \\ -840 & -40 & -621 \end{bmatrix}$ $\begin{bmatrix} -25 & -47 & -13 & -151 \\ -350 & -644 & -177 & -2044 \\ 56 & 105 & 29 & 336 \end{bmatrix}$

$L_{157.2} : 1^{2}_{\mathrm{II}}8^{1}_{7},1^{-2}7^{1}$ $56^{1,0}_{\infty a}56^{r}_{2}2^{s}_{2}14^{4,3}_{\infty b}\ (\times 2)$

$\begin{bmatrix} -1736 & -672 & -56 \\ -672 & -254 & -25 \\ -56 & -25 & 0 \end{bmatrix} \begin{bmatrix} 377 & 153 & 9 \\ -840 & -341 & -20 \\ -1512 & -612 & -37 \end{bmatrix}$ $\begin{bmatrix} -101 & -151 & -13 & -47 \\ 224 & 336 & 29 & 105 \\ 420 & 616 & 52 & 182 \end{bmatrix}$

W_{158} 2 lattices, $\chi = 16$ 4-gon: $3\infty 3\infty \rtimes C_2$

$L_{158.1} : 1^{-2}_{\mathrm{II}}32^{-}_{5}$ $2^{+}_{3}2^{8,5}_{\infty a}\ (\times 2)$

$\begin{bmatrix} 160 & 32 & 128 \\ 32 & 6 & 25 \\ 128 & 25 & 102 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 \\ -2 & -3 \\ -1 & 2 \end{bmatrix}$

W_{159} 2 lattices, $\chi = 12$ 4-gon: $\not 2 4 \not\infty 4 \rtimes D_2$

$L_{159.1} : 1^{2}_{2}32^{1}_{7}$ $1^{r}_{2}4^{*}_{4}2^{8,7}_{\infty a}2_{4}$

$\begin{bmatrix} -47392 & 1216 & 608 \\ 1216 & -31 & -16 \\ 608 & -16 & -7 \end{bmatrix}$ $\begin{bmatrix} -1 & -1 & 3 & 2 \\ -31 & -32 & 93 & 63 \\ -16 & -14 & 47 & 29 \end{bmatrix}$

W_{160} 10 lattices, $\chi = 24$ 5-gon: $2\not\infty 2\infty|\infty \rtimes D_2$

$L_{160.1} : 1^{2}_{\mathrm{II}}32^{1}_{1}$ $32^{b}_{2}2^{8,1}_{\infty b}2^{l}_{2}32^{1,0}_{\infty}32^{1,0}_{\infty z}$

$\begin{bmatrix} -26848 & 992 & 928 \\ 992 & -30 & -35 \\ 928 & -35 & -32 \end{bmatrix}$ $\begin{bmatrix} -5 & -4 & 5 & 67 & 29 \\ -16 & -11 & 15 & 192 & 80 \\ -128 & -104 & 128 & 1728 & 752 \end{bmatrix}$

$L_{160.2} : 1^{-}_{3}4^{1}_{1}64^{-}_{5}$ $16^{s}_{2}64^{8,1}_{\infty z}64^{*}_{2}16^{16,9}_{\infty z}4^{16,5}_{\infty}$

$\begin{bmatrix} -14528 & -1344 & -1472 \\ -1344 & -124 & -136 \\ -1472 & -136 & -149 \end{bmatrix}$ $\begin{bmatrix} -1 & 1 & 7 & 5 & 0 \\ 2 & 24 & -8 & -30 & -9 \\ 8 & -32 & -64 & -24 & 8 \end{bmatrix}$

$L_{160.3} : 1^{-}_{5}4^{1}_{1}64^{-}_{3}$ $16^{*}_{2}4^{8,3}_{\infty z}1^{r}_{2}16^{16,15}_{\infty z}4^{16,11}_{\infty}$

$\begin{bmatrix} -21824 & 576 & 576 \\ 576 & -12 & -16 \\ 576 & -16 & -15 \end{bmatrix}$ $\begin{bmatrix} -1 & -1 & 1 & 11 & 2 \\ -6 & -8 & 6 & 74 & 15 \\ -32 & -30 & 31 & 336 & 60 \end{bmatrix}$

$L_{160.4} : 1^{1}_{1}4^{1}_{1}64^{1}_{7}$ $4^{r}_{2}4^{8,7}_{\infty z}1_{2}4^{16,15}_{\infty}16^{16,3}_{\infty z}$

$\begin{bmatrix} -13376 & 960 & 448 \\ 960 & -60 & -32 \\ 448 & -32 & -15 \end{bmatrix}$ $\begin{bmatrix} -1 & -1 & 2 & 9 & 5 \\ 1 & 0 & -2 & -7 & -2 \\ -32 & -30 & 63 & 280 & 152 \end{bmatrix}$

$L_{160.5} : 1^{1}_{7}4^{1}_{1}64^{1}_{1}$ $4^{r}_{2}64^{2,1}_{\infty a}64_{2}4^{16,1}_{\infty}16^{16,5}_{\infty z}$

$\begin{bmatrix} 64 & 0 & 0 \\ 0 & -1020 & -32 \\ 0 & -32 & -1 \end{bmatrix}$ $\begin{bmatrix} -1 & -1 & 1 & 0 & -1 \\ -7 & -16 & 0 & 1 & -2 \\ 208 & 480 & 0 & -32 & 56 \end{bmatrix}$

W_{161} 12 lattices, $\chi = 24$ 6-gon: $\not\infty 2|2\not\infty 2|2 \rtimes D_4$

$L_{161.1} : 1^{2}_{0}32^{1}_{1}$ $32^{1,0}_{\infty b}32_{2}1^{r}_{2}32^{4,3}_{\infty z}32^{s}_{2}4^{*}_{2}$

$\begin{bmatrix} -63712 & 256 & 3968 \\ 256 & -1 & -16 \\ 3968 & -16 & -247 \end{bmatrix}$ $\begin{bmatrix} -3 & 3 & 1 & 7 & 1 & -1 \\ -16 & 224 & 44 & 240 & 0 & -32 \\ -48 & 32 & 13 & 96 & 16 & -14 \end{bmatrix}$

$L_{161.2}$: $[1^{-}2^{1}]_{2}32^{-}_{3}$ $\qquad$ $4^{8,3}_{\infty z}1^{r}_{2}8^{*}_{2}\,(\times 2)$

$$\begin{bmatrix}-3488 & 160 & 160\\ 160 & -6 & -8\\ 160 & -8 & -7\end{bmatrix}\begin{bmatrix}127 & -6 & -6\\ 896 & -43 & -42\\ 1792 & -84 & -85\end{bmatrix}\qquad\begin{bmatrix}5 & 1 & -1\\ 36 & 8 & -6\\ 70 & 13 & -16\end{bmatrix}$$

$L_{161.3}$: $[1^{1}2^{1}]_{2}64^{1}_{7}$ $\langle m\rangle$ $\qquad$ $2^{16,7}_{\infty}8^{l}_{2}1_{2}2^{16,15}_{\infty}8^{*}_{2}4^{l}_{2}$

$$\begin{bmatrix}-32320 & 704 & 704\\ 704 & -14 & -16\\ 704 & -16 & -15\end{bmatrix}\qquad\begin{bmatrix}1 & 7 & 2 & 2 & -1 & -1\\ 13 & 102 & 30 & 31 & -14 & -16\\ 32 & 216 & 61 & 60 & -32 & -30\end{bmatrix}$$

$L_{161.4}$: $1^{-}_{3}4^{1}_{7}32^{-}_{3}$ $\qquad$ $32^{8,5}_{\infty z}32^{s}_{2}16^{*}_{2}\,(\times 2)$

$$\begin{bmatrix}-4000 & 992 & 64\\ 992 & -244 & -16\\ 64 & -16 & -1\end{bmatrix}\begin{bmatrix}-1 & 0 & 0\\ -112 & 25 & 2\\ 1344 & -312 & -25\end{bmatrix}\qquad\begin{bmatrix}-3 & -1 & 1\\ -12 & -12 & -2\\ -16 & 96 & 80\end{bmatrix}$$

$L_{161.5}$: $1^{1}_{7}4^{1}_{1}32^{1}_{1}$ $\qquad$ $32^{2,1}_{\infty a}32_{2}4^{r}_{2}\,(\times 2)$

$$\begin{bmatrix}32 & 0 & 0\\ 0 & -252 & -16\\ 0 & -16 & -1\end{bmatrix}\begin{bmatrix}-5 & 15 & 1\\ -8 & 29 & 2\\ 96 & -360 & -25\end{bmatrix}\qquad\begin{bmatrix}-3 & -5 & -1\\ 0 & -8 & -3\\ -16 & 96 & 40\end{bmatrix}$$

W_{162} 16 lattices, $\chi = 36$ $\qquad$ 6-gon: $\not{2}\infty|\infty\not{2}\infty|\infty \rtimes D_4$

$L_{162.1}$: $1^{2}_{0}4^{1}_{1},1^{-2}9^{1}$ $\qquad$ $9_{2}4^{6,1}_{\infty}4^{3,1}_{\infty z}1_{2}36^{2,1}_{\infty}36^{1,0}_{\infty z}$

$$\begin{bmatrix}-298476 & -100440 & -33696\\ -100440 & -33799 & -11339\\ -33696 & -11339 & -3804\end{bmatrix}\qquad\begin{bmatrix}-1 & -13 & 15 & 55 & 367 & 119\\ 9 & 44 & -56 & -191 & -1260 & -396\\ -18 & -16 & 34 & 82 & 504 & 126\end{bmatrix}$$

$L_{162.2}$: $1^{2}_{0}4^{1}_{1},1^{2}9^{-}$ $\qquad$ $4_{2}1^{12,5}_{\infty}4^{3,2}_{\infty a}\,(\times 2)$

$$\begin{bmatrix}-348012 & -147888 & -3492\\ -147888 & -62845 & -1484\\ -3492 & -1484 & -35\end{bmatrix}\begin{bmatrix}61559 & 26182 & 608\\ -139320 & -59255 & -1376\\ -233280 & -99216 & -2305\end{bmatrix}\qquad\begin{bmatrix}-159 & -84 & -47\\ 360 & 190 & 106\\ 596 & 323 & 194\end{bmatrix}$$

$L_{162.3}$: $1^{2}_{\mathrm{II}}8^{1}_{1},1^{-2}9^{-}$ $\qquad$ $8^{r}_{2}18^{4,1}_{\infty a}72^{1,0}_{\infty a}72^{r}_{2}2^{12,1}_{\infty a}8^{3,2}_{\infty a}$

$$\begin{bmatrix}-741240 & 333072 & -99000\\ 333072 & -149664 & 44485\\ -99000 & 44485 & -13222\end{bmatrix}\qquad\begin{bmatrix}-41 & -121 & 233 & 1531 & 275 & 157\\ -96 & -288 & 540 & 3600 & 648 & 372\\ -16 & -63 & 72 & 648 & 121 & 76\end{bmatrix}$$

$L_{162.4}$: $1^{2}_{\mathrm{II}}8^{1}_{1},1^{2}9^{1}$ $\qquad$ $2^{l}_{2}8^{3,2}_{\infty}8^{3,1}_{\infty z}\,(\times 2)$

$$\begin{bmatrix}-1289592 & -282096 & -9504\\ -282096 & -61708 & -2079\\ -9504 & -2079 & -70\end{bmatrix}\begin{bmatrix}83447 & 18259 & 608\\ -377712 & -82647 & -2752\\ -109800 & -24025 & -801\end{bmatrix}\qquad\begin{bmatrix}8 & 7 & -47\\ -36 & -32 & 212\\ -17 & 0 & 84\end{bmatrix}$$

W_{163} 49 lattices, $\chi = 36$ $\qquad$ 8-gon: $\not{2}2\not{\propto}2\not{2}2\not{\propto}2 \rtimes D_4$

$L_{163.1}$: $1^{2}_{0}8^{-}_{5},1^{1}5^{-}25^{1}$ $\langle 5\rangle$ $\qquad$ $25^{r}_{2}4^{*}_{2}40^{5,4}_{\infty b}40_{2}1^{r}_{2}100^{*}_{2}40^{5,1}_{\infty a}40_{2}$

$$\begin{bmatrix}-733400 & 1000 & 8600\\ 1000 & 35 & -25\\ 8600 & -25 & -96\end{bmatrix}\qquad\begin{bmatrix}37 & 25 & 125 & 91 & 4 & -11 & -11 & 23\\ 1105 & 746 & 3728 & 2712 & 119 & -330 & -328 & 688\\ 3025 & 2044 & 10220 & 7440 & 327 & -900 & -900 & 1880\end{bmatrix}$$

$L_{163.2}$: $[1^{1}2^{1}]_{6}16^{-}_{3},1^{1}5^{-}25^{1}$ $\langle 5m,5,2\rangle$

$400^{*}_{2}4^{l}_{2}10^{40,19}_{\infty}40^{s}_{2}16^{*}_{2}100^{l}_{2}10^{40,11}_{\infty}40^{s}_{2}$

$$\begin{bmatrix}-587600 & 21600 & 800\\ 21600 & -790 & -30\\ 800 & -30 & -1\end{bmatrix}\qquad\begin{bmatrix}-1 & -1 & -4 & -9 & -3 & -3 & 0 & 1\\ -20 & -22 & -89 & -202 & -68 & -70 & -1 & 22\\ -200 & -146 & -560 & -1220 & -392 & -350 & 20 & 140\end{bmatrix}$$

$L_{163.3}:\ [1^{-}2^{1}]_{2}16^{1}_{7},1^{1}5^{-}25^{1}\ \langle 52,5,m\rangle 400^{l}_{2}1_{2}10^{40,39}_{\infty}40^{*}_{2}16^{l}_{2}25_{2}10^{40,31}_{\infty}40^{*}_{2}$

$\begin{bmatrix} -931600 & 27600 & 10400 \\ 27600 & -790 & -320 \\ 10400 & -320 & -111 \end{bmatrix}$ $\begin{bmatrix} -19 & 4 & 44 & 119 & 47 & 34 & 10 & -11 \\ -340 & 72 & 791 & 2138 & 844 & 610 & 179 & -198 \\ -800 & 167 & 1840 & 4980 & 1968 & 1425 & 420 & -460 \end{bmatrix}$

$L_{163.4}:\ [1^{-}2^{-}]_{0}16^{-}_{5},1^{1}5^{-}25^{1}\ \langle 5m,5,m\rangle$

$400^{s}_{2}4^{*}_{2}40^{40,19}_{\infty z}10^{r}_{2}16^{s}_{2}100^{*}_{2}40^{40,11}_{\infty z}10^{r}_{2}$

$\begin{bmatrix} -676400 & 8800 & 201600 \\ 8800 & -90 & -2820 \\ 201600 & -2820 & -58499 \end{bmatrix}$ $\begin{bmatrix} 1853 & 315 & 1579 & 576 & 203 & -141 & -145 & 141 \\ 37040 & 6296 & 31558 & 11511 & 4056 & -2820 & -2898 & 2819 \\ 4600 & 782 & 3920 & 1430 & 504 & -350 & -360 & 350 \end{bmatrix}$

$L_{163.5}:\ 1^{1}_{1}8^{-}_{3}64^{1}_{1},1^{1}5^{-}25^{1}\ \langle 25,5,2\rangle$

$1600_{2}1^{r}_{2}160^{80,79}_{\infty z}40^{b}_{2}64^{s}_{2}100^{*}_{2}160^{80,71}_{\infty z}40^{l}_{2}$

shares genus with its 2-dual ≅ 5-dual; isometric to its own 2.5-dual

$\begin{bmatrix} -958400 & 19200 & 233600 \\ 19200 & -360 & -4880 \\ 233600 & -4880 & -55311 \end{bmatrix}$ $\begin{bmatrix} 1301 & 50 & 943 & 317 & 91 & -61 & -65 & 122 \\ 26000 & 999 & 18838 & 6331 & 1816 & -1220 & -1298 & 2439 \\ 3200 & 123 & 2320 & 780 & 224 & -150 & -160 & 300 \end{bmatrix}$

W_{164} 44 lattices, $\chi = 54$ 10-gon: $42\infty2242\infty22 \rtimes C_2$

$L_{164.1}:\ 1^{2}_{\mathrm{II}}4^{-}_{5},1^{2}9^{1},1^{2}5^{1}\ \langle 2\rangle$ $4^{*}_{4}2^{l}_{2}20^{3,2}_{\infty}20^{*}_{2}36^{*}_{2}\ (\times 2)$

$\begin{bmatrix} -6278220 & 29700 & 59400 \\ 29700 & -140 & -281 \\ 59400 & -281 & -562 \end{bmatrix}$ $\begin{bmatrix} -18181 & 91 & 172 \\ 181800 & -911 & -1720 \\ -2017980 & 10101 & 19091 \end{bmatrix}$ $\begin{bmatrix} 9 & 10 & 13 & -3 & -11 \\ -58 & -102 & -200 & -70 & -54 \\ 982 & 1111 & 1480 & -280 & -1134 \end{bmatrix}$

$L_{164.2}:\ 1^{-2}_{2}8^{1}_{7},1^{2}9^{-},1^{2}5^{-}\ \langle 2\rangle$ $2^{*}_{4}4^{s}_{2}40^{12,5}_{\infty z}10^{s}_{2}18^{b}_{2}\ (\times 2)$

$\begin{bmatrix} -6977160 & 40320 & 23040 \\ 40320 & -233 & -133 \\ 23040 & -133 & -70 \end{bmatrix}$ $\begin{bmatrix} -230401 & 1328 & 624 \\ -40406400 & 232897 & 109434 \\ 921600 & -5312 & -2497 \end{bmatrix}$ $\begin{bmatrix} -44 & -163 & -329 & -63 & -55 \\ -7716 & -28586 & -57700 & -11050 & -9648 \\ 175 & 652 & 1320 & 255 & 225 \end{bmatrix}$

$L_{164.3}:\ 1^{2}_{2}8^{-}_{3},1^{2}9^{-},1^{2}5^{-}\ \langle m\rangle$ $2_{4}1^{r}_{2}40^{12,11}_{\infty z}10^{b}_{2}18^{s}_{2}\ (\times 2)$

$\begin{bmatrix} -133937640 & 26822160 & 347040 \\ 26822160 & -5371367 & -69498 \\ 347040 & -69498 & -899 \end{bmatrix}$ $\begin{bmatrix} 39008339 & -7810964 & -101675 \\ 191483280 & -38342289 & -499100 \\ 255534840 & -51167864 & -666051 \end{bmatrix}$ $\begin{bmatrix} 76 & -55 & -53 & 824 & 3577 \\ 373 & -270 & -260 & 4045 & 17559 \\ 503 & -359 & -360 & 5385 & 23409 \end{bmatrix}$

W_{165} 12 lattices, $\chi = 6$ 5-gon: 22222

$L_{165.1}:\ 1^{-2}_{\mathrm{II}}4^{1}_{1},1^{2}9^{1},1^{-2}5^{-}\ \langle 2\rangle$ $90^{l}_{2}4^{r}_{2}10^{l}_{2}36^{r}_{2}2^{b}_{2}$

$\begin{bmatrix} -400860 & -197640 & 1080 \\ -197640 & -97442 & 531 \\ 1080 & 531 & -2 \end{bmatrix}$ $\begin{bmatrix} -44 & 43 & 22 & -299 & -43 \\ 90 & -88 & -45 & 612 & 88 \\ 135 & -144 & -70 & 1008 & 143 \end{bmatrix}$

W_{166} 12 lattices, $\chi = 24$ 8-gon: $22|22|22|22| \rtimes D_4$

$L_{166.1}:\ 1^{-2}_{\mathrm{II}}4^{1}_{1},1^{-2}9^{-},1^{-2}5^{-}\ \langle 2\rangle$ $4^{r}_{2}18^{b}_{2}10^{b}_{2}2^{l}_{2}\ (\times 2)$

$\begin{bmatrix} 2340 & -1080 & 0 \\ -1080 & 498 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ $\begin{bmatrix} 11 & 4 & -7 & -6 \\ 24 & 9 & -15 & -13 \\ 8 & 0 & -10 & -8 \end{bmatrix}$

W_{167} 4 lattices, $\chi = 6$ 4-gon: $4|42|2 \rtimes D_2$

$L_{167.1}:\ 1^{2}_{2}16^{-}_{5},1^{2}3^{-}$ $1_{4}2^{*}_{4}4^{s}_{2}16^{l}_{2}$

$\begin{bmatrix} -2352 & -672 & 240 \\ -672 & -191 & 67 \\ 240 & 67 & -22 \end{bmatrix}$ $\begin{bmatrix} -1 & -4 & 3 & 9 \\ 5 & 18 & -14 & -40 \\ 4 & 11 & -10 & -24 \end{bmatrix}$

W_{168} 6 lattices, $\chi = 8$ 4-gon: $6262 \rtimes C_2$

$L_{168.1}$: $1_{\mathrm{II}}^{-2}16_7^1,1^-3^-9^-$ $\langle 3\rangle$ $6_6 18_2^b 6_6 2_2^b$

$$\begin{bmatrix} -48528 & 1296 & 720 \\ 1296 & -30 & -21 \\ 720 & -21 & -10 \end{bmatrix} \qquad \begin{bmatrix} 2 & -2 & -1 & 1 \\ 31 & -30 & -16 & 15 \\ 78 & -81 & -39 & 40 \end{bmatrix}$$

W_{169} 36 lattices, $\chi = 24$ 6-gon: $2|2\not\infty 2|2\not\infty \rtimes D_4$

$L_{169.1}$: $1_6^2 16_5^-,1^-3^1 9^1$ $\langle 3\rangle$ $3_2^r 144_2^s 12_{\infty z}^{24,1}$ $(\times 2)$

$$\begin{bmatrix} -305712 & -9360 & -9936 \\ -9360 & -285 & -303 \\ -9936 & -303 & -322 \end{bmatrix} \begin{bmatrix} -449 & -13 & -14 \\ -32256 & -937 & -1008 \\ 44352 & 1287 & 1385 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & -1 \\ 85 & 120 & -82 \\ -111 & -144 & 108 \end{bmatrix}$$

$L_{169.2}$: $1_4^{-2}16_7^1,1^-3^1 9^1$ $\langle 3\rangle$ $12_2^* 36_2^s 48_{\infty z}^{12,1} 48_2^l 9_2 3_{\infty}^{24,7}$

$$\begin{bmatrix} -30096 & 4032 & -6336 \\ 4032 & -492 & 891 \\ -6336 & 891 & -1297 \end{bmatrix} \qquad \begin{bmatrix} 57 & 65 & -165 & -527 & -239 & -62 \\ 152 & 174 & -440 & -1408 & -639 & -166 \\ -174 & -198 & 504 & 1608 & 729 & 189 \end{bmatrix}$$

$L_{169.3}$: $1_0^2 16_3^-,1^-3^1 9^1$ $\langle 3\rangle$ $12_2^l 9_2 48_{\infty}^{6,1} 48_2^* 36_2^l 3_{\infty}^{24,19}$

$$\begin{bmatrix} -100944 & -20592 & -11808 \\ -20592 & -4128 & -2487 \\ -11808 & -2487 & -1297 \end{bmatrix} \qquad \begin{bmatrix} -95 & -106 & 275 & 1293 & 1315 & 207 \\ 304 & 339 & -880 & -4136 & -4206 & -662 \\ 282 & 315 & -816 & -3840 & -3906 & -615 \end{bmatrix}$$

$L_{169.4}$: $1_3^- 4_7^1 16_1^1,1^-3^1 9^1$ $\langle 3\rangle$ $12_2^s 144_2^l 12_{\infty}^{24,13} 12_2 144_2 3_{\infty}^{12,1}$

$$\begin{bmatrix} -227952 & -35280 & 9072 \\ -35280 & -5460 & 1404 \\ 9072 & 1404 & -361 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -2 & -3 & -7 & 0 \\ -8 & -12 & 13 & 35 & 120 & 7 \\ -6 & -72 & 0 & 60 & 288 & 27 \end{bmatrix}$$

$L_{169.5}$: $1_3^- 4_1^1 16_7^1,1^-3^1 9^1$ $\langle 3\rangle$ $12_2^l 36_2 48_{\infty}^{12,1} 48_2^l 36_2 3_{\infty}^{12,7}$

$$\begin{bmatrix} -30096 & 15984 & -6336 \\ 15984 & -7980 & 3228 \\ -6336 & 3228 & -1297 \end{bmatrix} \qquad \begin{bmatrix} -19 & -22 & 55 & 177 & 161 & 21 \\ 76 & 87 & -220 & -704 & -639 & -83 \\ 282 & 324 & -816 & -2616 & -2376 & -309 \end{bmatrix}$$

W_{170} 8 lattices, $\chi = 12$ 5-gon: $22|22\not\infty \rtimes D_2$

$L_{170.1}$: $1_2^{-2}16_1^1,1^-3^1 9^1$ $\langle 3\rangle$ $12_2^* 144_2^b 2_2^l 144_2 3_{\infty}^{24,1}$

$$\begin{bmatrix} -1105776 & 18288 & 7200 \\ 18288 & -285 & -123 \\ 7200 & -123 & -46 \end{bmatrix} \qquad \begin{bmatrix} -7 & -11 & 3 & 103 & 6 \\ -154 & -240 & 66 & 2256 & 131 \\ -684 & -1080 & 293 & 10080 & 588 \end{bmatrix}$$

W_{171} 8 lattices, $\chi = 48$ 8-gon: $\infty 2|2\infty|\infty 2|2\infty| \rtimes D_4$

$L_{171.1}$: $1_{\mathrm{II}}^2 16_3^-,1^1 3^1 9^-$ $\langle 3\rangle$ $48_{\infty a}^{3,2} 48_2^r 18_2^b 48_{\infty z}^{6,1}$ $(\times 2)$

$$\begin{bmatrix} -8784 & 1152 & 2304 \\ 1152 & -132 & -327 \\ 2304 & -327 & -572 \end{bmatrix} \begin{bmatrix} 10943 & -1862 & -2318 \\ 33120 & -5636 & -7015 \\ 25056 & -4263 & -5308 \end{bmatrix} \qquad \begin{bmatrix} 179 & 629 & 263 & 575 \\ 544 & 1904 & 795 & 1736 \\ 408 & 1440 & 603 & 1320 \end{bmatrix}$$

W_{172} 16 lattices, $\chi = 6$ 5-gon: $2|22\not|2 \rtimes D_2$

$L_{172.1}$: $1_2^2 8_5^-,1^-3^1 9^-$ $\langle 2, m\rangle$ $8_2^* 12_2^* 72_2^b 2_2^s 18_2^b$

$$\begin{bmatrix} -5976 & 216 & 216 \\ 216 & -6 & -9 \\ 216 & -9 & -7 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 5 & 1 & 1 \\ -12 & -10 & 60 & 11 & 9 \\ -16 & -18 & 72 & 16 & 18 \end{bmatrix}$$

$L_{172.2} : 1^{-2}_{6} 16^{1}_{1}, 1^{1} 3^{-} 9^{1}\ \langle 3, m\rangle$ $\quad 16^{b}_{2} 6^{l}_{2} 144_{2} 1^{r}_{2} 36^{*}_{2}$

shares genus with its 3-dual

$\begin{bmatrix} 13968 & -288 & -144 \\ -288 & 6 & 3 \\ -144 & 3 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & 0 & 1 & 0 & -1 \\ -40 & 1 & 48 & 0 & -42 \\ -16 & 0 & 0 & -1 & -18 \end{bmatrix}$

W_{173} 8 lattices, $\chi = 40$ 6-gon: $3\infty\infty3\infty\infty \rtimes C_2$

$L_{173.1} : 1^{-2}_{\mathrm{II}} 8^{-}_{5}, 1^{-2} 25^{1}\ \langle 2\rangle$ $\quad 2^{-}_{3} 2^{20,17}_{\infty b} 8^{10,7}_{\infty z}\ (\times 2)$

$\begin{bmatrix} -45400 & 600 & 800 \\ 600 & -6 & -11 \\ 800 & -11 & -14 \end{bmatrix} \begin{bmatrix} 799 & -7 & -15 \\ 7200 & -64 & -135 \\ 39200 & -343 & -736 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & 8 & 5 \\ 16 & 71 & 48 \\ 99 & 393 & 244 \end{bmatrix}$

W_{174} 8 lattices, $\chi = 30$ 6-gon: $2\infty2\infty2\infty \rtimes C_3$

$L_{174.1} : 1^{-2}_{\mathrm{II}} 8^{-}_{5}, 1^{2} 25^{-}\ \langle 2\rangle$ $\quad 50^{b}_{2} 8^{10,1}_{\infty z} 2^{b}_{2} 8^{10,9}_{\infty z} 2^{b}_{2} 200^{2,1}_{\infty z}$

$\begin{bmatrix} 12200 & -5000 & 200 \\ -5000 & 2042 & -55 \\ 200 & -55 & -98 \end{bmatrix}$ $\quad \begin{bmatrix} -152 & -73 & 75 & 459 & 176 & 1379 \\ -375 & -180 & 185 & 1132 & 434 & 3400 \\ -100 & -48 & 49 & 300 & 115 & 900 \end{bmatrix}$

W_{175} 27 lattices, $\chi = 12$ 6-gon: $\not{2}2|2\not{2}2|2 \rtimes D_4$

$L_{175.1} : 1^{-2}_{4} 8^{1}_{7}, 1^{1} 3^{1} 9^{1}\ \langle 2\rangle$ $\quad 36^{l}_{2} 1_{2} 3_{2} 9^{r}_{2} 4^{*}_{2} 12^{*}_{2}$

$\begin{bmatrix} -1800 & 288 & -72 \\ 288 & -15 & -6 \\ -72 & -6 & 7 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & 1 & 2 & 2 & -1 & -3 \\ -12 & 11 & 22 & 21 & -12 & -34 \\ -18 & 20 & 39 & 36 & -22 & -60 \end{bmatrix}$

$L_{175.2} : 1^{-2}_{2} 8^{1}_{1}, 1^{1} 3^{1} 9^{1}\ \langle m\rangle$ $\quad 36^{*}_{2} 4^{l}_{2} 3^{r}_{2}\ (\times 2)$

$\begin{bmatrix} 8136 & -576 & 288 \\ -576 & 39 & -18 \\ 288 & -18 & 7 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -96 & 4 & -1 \\ -288 & 15 & -4 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & 1 & 1 \\ -24 & 22 & 22 \\ -18 & 16 & 15 \end{bmatrix}$

$L_{175.3} : 1^{2}_{2} 8^{-}_{5}, 1^{1} 3^{1} 9^{1}$ $\quad 9_{2} 1^{r}_{2} 12^{l}_{2}\ (\times 2)$

$\begin{bmatrix} -1368 & 144 & 360 \\ 144 & -15 & -36 \\ 360 & -36 & -71 \end{bmatrix} \begin{bmatrix} 143 & -15 & -33 \\ 1680 & -176 & -385 \\ -144 & 15 & 32 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & -1 & -1 \\ 9 & -12 & -10 \\ 0 & 1 & 0 \end{bmatrix}$

$L_{175.4} : [1^{-} 2^{1}]_{4} 16^{1}_{7}, 1^{1} 3^{1} 9^{1}\ \langle 2\rangle$ $\quad 4^{*}_{2} 144^{l}_{2} 3^{r}_{2} 16^{*}_{2} 36^{s}_{2} 48^{s}_{2}$

$\begin{bmatrix} -5904 & 864 & 288 \\ 864 & -78 & -30 \\ 288 & -30 & -11 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & -5 & 0 & 1 & 1 & -1 \\ 20 & 108 & 1 & -20 & -24 & 16 \\ -82 & -432 & -3 & 80 & 90 & -72 \end{bmatrix}$

$L_{175.5} : [1^{1} 2^{1}]_{0} 16^{-}_{3}, 1^{1} 3^{1} 9^{1}\ \langle m\rangle$ $\quad 1^{r}_{2} 144^{*}_{2} 12^{*}_{2} 16^{l}_{2} 9_{2} 48_{2}$

$\begin{bmatrix} -90576 & -6048 & 2448 \\ -6048 & -402 & 162 \\ 2448 & 162 & -65 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 5 & -1 & -1 & 1 & 5 \\ -29 & -132 & 32 & 28 & -33 & -152 \\ -35 & -144 & 42 & 32 & -45 & -192 \end{bmatrix}$

$L_{175.6} : [1^{1} 2^{1}]_{6} 16^{-}_{5}, 1^{1} 3^{1} 9^{1}\ \langle m\rangle$ $\quad 4^{s}_{2} 144^{s}_{2} 12^{s}_{2} 16^{s}_{2} 36^{*}_{2} 48^{*}_{2}$

$\begin{bmatrix} 8784 & 432 & -288 \\ 432 & -6 & -6 \\ -288 & -6 & 7 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & -1 & 1 & 1 & -1 & -3 \\ -18 & -24 & 16 & 16 & -18 & -52 \\ -58 & -72 & 54 & 56 & -54 & -168 \end{bmatrix}$

$L_{175.7} : 1^{1}_{1} 8^{-}_{5} 64^{1}_{1}, 1^{1} 3^{1} 9^{1}\ \langle 3, 2\rangle$ $\quad 64^{s}_{2} 36^{*}_{2} 192^{l}_{2} 1_{2} 576^{r}_{2} 12^{b}_{2}$

shares genus with its 2-dual $\cong$ 3-dual; isometric to its own 2.3-dual

$\begin{bmatrix} 318528 & 576 & -576 \\ 576 & -24 & 0 \\ -576 & 0 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & -1 & -1 & 0 & 1 & 0 \\ -16 & -18 & -20 & 0 & 24 & 1 \\ -544 & -558 & -576 & -1 & 576 & 6 \end{bmatrix}$

W_{176} 34 lattices, $\chi = 24$ $\qquad$ 7-gon: $222|222\infty\!\!\!\!\!\mid\;\; \rtimes D_2$

$L_{176.1}: 1^2_0 8^1_7,1^{-2}7^1$ $\qquad$ $56_2 1_2 7^r_2 8^s_2 28^*_2 4^*_2 56^{1,0}_{\infty b}$

$$\begin{bmatrix} -740040 & 12600 & 12992 \\ 12600 & -211 & -225 \\ 12992 & -225 & -224 \end{bmatrix} \qquad \begin{bmatrix} 211 & 49 & 181 & 41 & 23 & -15 & -15 \\ 6328 & 1469 & 5425 & 1228 & 686 & -450 & -448 \\ 5880 & 1366 & 5047 & 1144 & 644 & -418 & -420 \end{bmatrix}$$

$L_{176.2}: [1^{-}2^{1}]_4 16^-_3,1^{-2}7^1 \ \langle 2, m\rangle$ $\qquad$ $14^r_2 4^s_2 112^l_2 2^r_2 28^*_2 16^*_2 56^{8,5}_{\infty z}$

$$\begin{bmatrix} -345296 & 7840 & 8624 \\ 7840 & -178 & -196 \\ 8624 & -196 & -211 \end{bmatrix} \qquad \begin{bmatrix} -3 & 5 & 73 & 5 & 23 & 1 & -15 \\ -133 & 204 & 3024 & 209 & 966 & 44 & -630 \\ 0 & 14 & 168 & 10 & 42 & 0 & -28 \end{bmatrix}$$

$L_{176.3}: [1^{1}2^{1}]_6 16^1_1,1^{-2}7^1 \ \langle m\rangle$ $\qquad$ $56^*_2 4^*_2 112^s_2 8^l_2 7_2 16_2 14^{8,1}_{\infty}$

$$\begin{bmatrix} -655984 & -185808 & 11312 \\ -185808 & -52630 & 3204 \\ 11312 & 3204 & -195 \end{bmatrix} \qquad \begin{bmatrix} 3 & 7 & -5 & -15 & -74 & -83 & -47 \\ -14 & -28 & 28 & 62 & 301 & 336 & 189 \\ -56 & -54 & 168 & 148 & 651 & 704 & 378 \end{bmatrix}$$

$L_{176.4}: [1^{-}2^{1}]_2 16^-_5,1^{-2}7^1$ $\qquad$ $56^l_2 1^r_2 112^*_2 8^*_2 28^s_2 16^l_2 14^{8,5}_{\infty}$

$$\begin{bmatrix} -560 & -112 & 784 \\ -112 & -22 & 168 \\ 784 & 168 & -783 \end{bmatrix} \qquad \begin{bmatrix} 3 & 7 & -5 & -29 & -295 & -167 & -96 \\ -14 & -28 & 28 & 118 & 1190 & 672 & 385 \\ 0 & 1 & 0 & -4 & -42 & -24 & -14 \end{bmatrix}$$

$L_{176.5}: 1^1_7 8^1_1 64^1_7,1^{-2}7^1 \ \langle 2\rangle$ $\qquad$ $56^s_2 4^b_2 448^l_2 8^r_2 28^*_2 64^s_2 224^{16,1}_{\infty z}$

shares genus with $L_{176.6}$; isometric to its own 2-dual

$$\begin{bmatrix} -189504 & -9856 & 448 \\ -9856 & -504 & 24 \\ 448 & 24 & -1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 11 & 0 & -1 & -1 & 1 \\ -35 & -11 & -112 & 1 & 14 & 12 & -14 \\ 476 & 170 & 2016 & 16 & -126 & -160 & 112 \end{bmatrix}$$

$L_{176.6}: 1^1_7 8^1_1 64^1_7,1^{-2}7^1$ $\qquad$ $56^b_2 4^l_2 448_2 8_2 7^r_2 64^*_2 224^{16,9}_{\infty z}$

shares genus with $L_{176.5}$; isometric to its own 2-dual

$$\begin{bmatrix} -3777088 & 165312 & -54656 \\ 165312 & -7224 & 2384 \\ -54656 & 2384 & -785 \end{bmatrix} \qquad \begin{bmatrix} -3 & 6 & 157 & 10 & 11 & 1 & -29 \\ -133 & 249 & 6552 & 419 & 462 & 44 & -1218 \\ -196 & 338 & 8960 & 576 & 637 & 64 & -1680 \end{bmatrix}$$

W_{177} 44 lattices, $\chi = 24$ $\qquad$ 8-gon: $2|22|22|22|2 \rtimes D_4$

$L_{177.1}: 1^{-2}_4 16^-_3,1^2 3^1,1^{-2}5^1 \ \langle 2\rangle$ $\qquad$ $48^*_2 4^*_2 12^l_2 5_2 48_2 1_2 3^r_2 20^*_2$

$$\begin{bmatrix} -96720 & 720 & 240 \\ 720 & -4 & -5 \\ 240 & -5 & 7 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 & 1 & 7 & 1 & 2 & 3 \\ -120 & -118 & -120 & 115 & 816 & 117 & 234 & 350 \\ -48 & -50 & -54 & 45 & 336 & 49 & 99 & 150 \end{bmatrix}$$

$L_{177.2}: 1^2_0 16^1_7,1^2 3^1,1^{-2}5^1 \ \langle m\rangle$ $\qquad$ $48^l_2 1^r_2 12^*_2 20^s_2 48^s_2 4^l_2 3_2 5^r_2$

$$\begin{bmatrix} 205680 & 1680 & -1200 \\ 1680 & -19 & -10 \\ -1200 & -10 & 7 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 & 7 & 19 & 5 & 5 & 4 \\ 0 & 1 & 0 & -10 & -24 & -6 & -6 & -5 \\ -168 & -170 & -174 & 1180 & 3216 & 848 & 849 & 680 \end{bmatrix}$$

$L_{177.3}: 1^1_7 4^1_1 16^1_7,1^2 3^1,1^{-2}5^1$ $\qquad$ $48^l_2 4^r_2 12^l_2 20_2 48_2 4_2 3_2 20^r_2$

$$\begin{bmatrix} 205680 & 4080 & -1200 \\ 4080 & 52 & -24 \\ -1200 & -24 & 7 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 & 2 & 7 & 2 & 2 & 3 \\ 0 & 1 & 0 & -5 & -12 & -3 & -3 & -5 \\ -168 & -168 & -174 & 320 & 1152 & 332 & 333 & 500 \end{bmatrix}$$

$L_{177.4}: 1^1_1 4^1_7 16^1_7,1^2 3^1,1^{-2}5^1$ $\qquad$ $12_2 1_2 48_2 5_2 12^r_2 4^s_2 48^s_2 20^l_2$

$$\begin{bmatrix} -30480 & 1440 & 0 \\ 1440 & -4 & -8 \\ 0 & -8 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & -5 & -3 & -4 & -1 & -1 & 1 \\ 21 & 0 & -108 & -65 & -87 & -22 & -24 & 20 \\ 168 & -1 & -864 & -515 & -684 & -170 & -168 & 170 \end{bmatrix}$$

W_{178} 12 lattices, $\chi = 18$ 6-gon: $422422 \rtimes C_2$

$L_{178.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}9^{-},1^{2}7^{-}$ $\langle 2\rangle$ $2^{*}_{4}4^{b}_{2}18^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} -506772 & 2268 & 4032 \\ 2268 & -10 & -19 \\ 4032 & -19 & -26 \end{bmatrix} \begin{bmatrix} 5039 & -22 & -44 \\ 876960 & -3829 & -7656 \\ 138600 & -605 & -1211 \end{bmatrix} \begin{bmatrix} 1 & 21 & 20 \\ 173 & 3654 & 3483 \\ 28 & 578 & 549 \end{bmatrix}$$

W_{179} 44 lattices, $\chi = 72$ 14-gon: $222\infty222222\infty222 \rtimes C_2$

$L_{179.1}$: $1^{2}_{\mathrm{II}}4^{1}_{7},1^{2}9^{-},1^{-2}7^{1}$ $\langle 2\rangle$ $2^{s}_{2}126^{b}_{2}4^{*}_{2}28^{3,2}_{\infty b}28^{r}_{2}18^{s}_{2}14^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} -642852 & 2772 & 1512 \\ 2772 & -8 & -9 \\ 1512 & -9 & -2 \end{bmatrix} \begin{bmatrix} 109493 & -341 & -341 \\ 13517532 & -42099 & -42098 \\ 21639996 & -67394 & -67395 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & -1 & 15 & 34 & 54 \\ 124 & 126 & -124 & -126 & 1848 & 4194 & 6664 \\ 197 & 189 & -198 & -196 & 2968 & 6723 & 10675 \end{bmatrix}$$

$L_{179.2}$: $1^{-2}_{6}8^{-}_{5},1^{2}9^{1},1^{-2}7^{1}$ $\langle 2\rangle$ $1^{r}_{2}252^{*}_{2}8^{b}_{2}14^{12,5}_{\infty a}56^{l}_{2}9^{r}_{2}28^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -5026392 & -29232 & -504 \\ -29232 & -170 & -3 \\ -504 & -3 & 1 \end{bmatrix} \begin{bmatrix} 266111 & 1552 & -24 \\ -45671472 & -266363 & 4119 \\ -2794176 & -16296 & 251 \end{bmatrix}$$

$$\begin{bmatrix} 24 & 3035 & 449 & 222 & 193 & 43 & 27 \\ -4119 & -520884 & -77060 & -38101 & -33124 & -7380 & -4634 \\ -251 & -31878 & -4720 & -2338 & -2044 & -459 & -294 \end{bmatrix}$$

$L_{179.3}$: $1^{2}_{6}8^{1}_{1},1^{2}9^{1},1^{-2}7^{1}$ $\langle m\rangle$ $4^{l}_{2}63_{2}8^{r}_{2}14^{12,5}_{\infty b}56^{s}_{2}36^{l}_{2}7^{r}_{2}$ $(\times 2)$

$$\begin{bmatrix} 289800 & 4536 & 0 \\ 4536 & 71 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 62495 & 980 & -124 \\ -4046616 & -63456 & 8029 \\ -484344 & -7595 & 960 \end{bmatrix} \begin{bmatrix} 23 & 755 & 225 & 113 & 103 & 49 & 9 \\ -1490 & -48888 & -14568 & -7315 & -6664 & -3168 & -581 \\ -184 & -5859 & -1736 & -861 & -756 & -342 & -56 \end{bmatrix}$$

W_{180} 2 lattices, $\chi = 24$ 5-gon: $\rlap{|}2\infty4|4\infty \rtimes D_2$

$L_{180.1}$: $1^{2}_{2}64^{1}_{7}$ $1^{r}_{2}4^{16,15}_{\infty z}1_{4}2^{*}_{4}4^{16,7}_{\infty z}$

$$\begin{bmatrix} -16960 & -896 & 320 \\ -896 & -47 & 17 \\ 320 & 17 & -6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 & -1 \\ -1 & -10 & -7 & 2 & 10 \\ -3 & 24 & 32 & 5 & -26 \end{bmatrix}$$

W_{181} 14 lattices, $\chi = 12$ 5-gon: $2|22\rlap{|}\infty2 \rtimes D_2$

$L_{181.1}$: $[1^{1}2^{-}]_{4}32^{-}_{5}$ $32^{s}_{2}8^{*}_{2}32^{l}_{2}1^{8,5}_{\infty}4^{*}_{2}$

$$\begin{bmatrix} -13664 & -416 & 928 \\ -416 & -10 & 28 \\ 928 & 28 & -63 \end{bmatrix} \begin{bmatrix} -3 & 5 & 25 & 2 & -3 \\ -8 & 6 & 40 & 4 & -4 \\ -48 & 76 & 384 & 31 & -46 \end{bmatrix}$$

$L_{181.2}$: $[1^{1}2^{1}]_{0}64^{1}_{1}$ $\langle m\rangle$ $64^{*}_{2}4^{s}_{2}64^{l}_{2}2^{16,9}_{\infty}8^{s}_{2}$

shares genus with $L_{181.3}$

$$\begin{bmatrix} -59840 & -2752 & -3008 \\ -2752 & -126 & -138 \\ -3008 & -138 & -151 \end{bmatrix} \begin{bmatrix} 1 & 1 & 5 & 0 & -1 \\ 48 & 24 & 64 & -9 & -22 \\ -64 & -42 & -160 & 8 & 40 \end{bmatrix}$$

$L_{181.3}$: $[1^{1}2^{1}]_{0}64^{1}_{1}$ $64^{l}_{2}1_{2}64_{2}2^{16,1}_{\infty}8^{*}_{2}$

shares genus with $L_{181.2}$

$$\begin{bmatrix} -342464 & 6656 & 2880 \\ 6656 & -126 & -58 \\ 2880 & -58 & -23 \end{bmatrix} \begin{bmatrix} -7 & 2 & 45 & 4 & -5 \\ -208 & 60 & 1344 & 119 & -150 \\ -352 & 99 & 2240 & 200 & -248 \end{bmatrix}$$

$L_{181.4}$: $1^{1}_{1}4^{1}_{7}32^{1}_{1}$ $4^{*}_{2}16^{l}_{2}1_{2}32^{4,3}_{\infty}32^{s}_{2}$

$$\begin{bmatrix} -55776 & 480 & 3712 \\ 480 & -4 & -32 \\ 3712 & -32 & -247 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & 7 & 1 \\ -8 & 6 & 14 & 64 & -8 \\ -14 & -16 & 13 & 96 & 16 \end{bmatrix}$$

$L_{181.5}: 1^1_1 4^1_1 32^1_7$ $\qquad$ $4^l_2 4_2 1^r_2 32^{8,7}_{\infty z} 32^*_2$

$$\begin{bmatrix} -46112 & 2112 & 2144 \\ 2112 & -92 & -100 \\ 2144 & -100 & -99 \end{bmatrix} \qquad \begin{bmatrix} -5 & 2 & 9 & 41 & -5 \\ -30 & 11 & 53 & 244 & -28 \\ -78 & 32 & 141 & 640 & -80 \end{bmatrix}$$

$L_{181.6}: 1^1_7 8^1_1 64^1_1$ $\qquad$ $64^r_2 4^b_2 64^l_2 8^{16,9}_{\infty} 8_2$

$$\begin{bmatrix} 64 & 0 & 0 \\ 0 & -56 & -8 \\ 0 & -8 & -1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 3 & 1 & 0 \\ 0 & -1 & -8 & -1 & 1 \\ 0 & 6 & 32 & 0 & -8 \end{bmatrix}$$

W_{182} 6 lattices, $\chi = 48$ $\qquad$ 8-gon: $\infty\!\!\!\!|\,2\!\!\!|\,\infty\!\!\!\!|\,2\!\!\!|\,\infty\!\!\!\!|\,2\!\!\!|\,\infty\!\!\!\!|\,2\!\!\!| \rtimes D_8$

$L_{182.1}: 1^2_0 64^1_1$ $\qquad$ $64^{4,3}_{\infty z} 64^s_2 4^{16,1}_{\infty z} 1_2 64^{2,1}_{\infty} 64^*_2 4^{16,9}_{\infty z} 1^r_2$

$$\begin{bmatrix} -181184 & 448 & 4800 \\ 448 & -1 & -12 \\ 4800 & -12 & -127 \end{bmatrix} \qquad \begin{bmatrix} -15 & -1 & 1 & 0 & -9 & -23 & -7 & -3 \\ -608 & -64 & 40 & 8 & -256 & -800 & -256 & -116 \\ -512 & -32 & 34 & -1 & -320 & -800 & -242 & -103 \end{bmatrix}$$

$L_{182.2}: 1^1_1 4^1_7 64^1_1$ $\qquad$ $64^{2,1}_{\infty b} 64_2 1^{8,1}_{\infty} 4^s_2\ (\times 2)$

$$\begin{bmatrix} -153536 & 832 & 4416 \\ 832 & -4 & -24 \\ 4416 & -24 & -127 \end{bmatrix} \begin{bmatrix} -1369 & 9 & 39 \\ -2736 & 17 & 78 \\ -47424 & 312 & 1351 \end{bmatrix} \qquad \begin{bmatrix} -23 & -9 & 0 & 1 \\ -32 & 16 & 4 & 4 \\ -800 & -320 & -1 & 34 \end{bmatrix}$$

$L_{182.3}: 1^-_5 4^1_7 64^-_5$ $\qquad$ $64^{8,3}_{\infty z} 64^l_2 1^{8,5}_{\infty} 4^*_2\ (\times 2)$

$$\begin{bmatrix} -120000 & -1728 & 3904 \\ -1728 & -20 & 56 \\ 3904 & 56 & -127 \end{bmatrix} \begin{bmatrix} 4799 & 60 & -156 \\ 7200 & 89 & -234 \\ 150400 & 1880 & -4889 \end{bmatrix} \qquad \begin{bmatrix} 99 & 53 & 2 & -3 \\ 152 & 88 & 4 & -4 \\ 3104 & 1664 & 63 & -94 \end{bmatrix}$$

W_{183} 34 lattices, $\chi = 18$ $\qquad$ 6-gon: $\infty\!\!\!\!|\,22\,2\!\!\!|\,22 \rtimes D_2$

$L_{183.1}: 1^2_0 8^1_1, 1^2 9^1$ $\qquad$ $8^{6,1}_{\infty} 8^*_2 36^l_2 1^r_2 4^l_2 9_2$

$$\begin{bmatrix} -444600 & 4680 & 4392 \\ 4680 & -47 & -49 \\ 4392 & -49 & -40 \end{bmatrix} \qquad \begin{bmatrix} 15 & -7 & -7 & 3 & 17 & 46 \\ 808 & -376 & -378 & 161 & 914 & 2475 \\ 656 & -308 & -306 & 132 & 746 & 2016 \end{bmatrix}$$

$L_{183.2}: [1^- 2^1]_6 16^-_3, 1^2 9^1\ \langle 2 \rangle$ $\qquad$ $8^{24,1}_{\infty z} 2^r_2 36^*_2 16^*_2 4^*_2 144^s_2$

$$\begin{bmatrix} -463824 & -11520 & -12240 \\ -11520 & -286 & -304 \\ -12240 & -304 & -323 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 5 & 3 & 1 & 1 \\ 2 & -13 & -108 & -28 & 0 & 36 \\ 36 & 12 & -90 & -88 & -38 & -72 \end{bmatrix}$$

$L_{183.3}: [1^1 2^1]_2 16^1_7, 1^2 9^1\ \langle m \rangle$ $\qquad$ $8^{24,13}_{\infty z} 2_2 9^r_2 16^l_2 1^r_2 144^*_2$

$$\begin{bmatrix} -123408 & 6048 & 2448 \\ 6048 & -286 & -124 \\ 2448 & -124 & -47 \end{bmatrix} \qquad \begin{bmatrix} -7 & 6 & 40 & 31 & 3 & -11 \\ -70 & 59 & 396 & 308 & 30 & -108 \\ -180 & 156 & 1035 & 800 & 77 & -288 \end{bmatrix}$$

$L_{183.4}: [1^- 2^1]_4 16^-_5, 1^2 9^1\ \langle m \rangle$ $\qquad$ $2^{24,13}_{\infty} 8^*_2 36^s_2 16^s_2 4^s_2 144^l_2$

$$\begin{bmatrix} -60336 & 17712 & 2880 \\ 17712 & -5198 & -848 \\ 2880 & -848 & -133 \end{bmatrix} \qquad \begin{bmatrix} -28 & 25 & 29 & -41 & -61 & -671 \\ -87 & 78 & 90 & -128 & -190 & -2088 \\ -52 & 44 & 54 & -72 & -110 & -1224 \end{bmatrix}$$

$L_{183.5}: 1^1_1 8^1_7 64^1_1, 1^2 9^1\ \langle 2 \rangle$ $\qquad$ $8^{24,1}_{\infty b} 32^*_2 36^s_2 64^s_2 4^s_2 576^b_2$

shares genus with $L_{183.6}$; isometric to its own 2-dual

$$\begin{bmatrix} -607680 & 9216 & 95040 \\ 9216 & -136 & -1488 \\ 95040 & -1488 & -14287 \end{bmatrix} \qquad \begin{bmatrix} 62 & -33 & -31 & 55 & 53 & 1289 \\ 2231 & -1186 & -1116 & 1976 & 1906 & 46368 \\ 180 & -96 & -90 & 160 & 154 & 3744 \end{bmatrix}$$

$L_{183.6}$: $1^1_1 8^1_7 64^1_1,1^2 9^1$ $\qquad$ $8^{24,1}_{\infty a} 32^{\,l}_2 9_2 64_2 1_2 576^{\,r}_2$

shares genus with $L_{183.5}$; isometric to its own 2-dual

$$\begin{bmatrix} -42328512 & 78336 & -13117824 \\ 78336 & -136 & 24384 \\ -13117824 & 24384 & -4063999 \end{bmatrix} \quad \begin{bmatrix} -1016 & 255 & 254 & -425 & -339 & -18551 \\ -36577 & 9182 & 9144 & -15304 & -12205 & -667872 \\ 3060 & -768 & -765 & 1280 & 1021 & 55872 \end{bmatrix}$$

W_{184} 34 lattices, $\chi = 72$ $\qquad$ 12-gon: $\not\infty 2|2\not\infty 2|2\not\infty 2|2\not\infty 2|2 \rtimes D_8$

$L_{184.1}$: $1^2_0 8^1_1,1^{-2}9^-$ $\qquad$ $72^{2,1}_{\infty} 72^*_2 4^*_2 8^{3,2}_{\infty b} 8_2 1_2$ $(\times 2)$

$$\begin{bmatrix} -639288 & 7128 & 7200 \\ 7128 & -79 & -81 \\ 7200 & -81 & -80 \end{bmatrix} \begin{bmatrix} 41381 & -456 & -475 \\ 2221560 & -24481 & -25500 \\ 1472328 & -16224 & -16901 \end{bmatrix} \quad \begin{bmatrix} 121 & 23 & -3 & -3 & 19 & 15 \\ 6480 & 1224 & -162 & -160 & 1024 & 807 \\ 4320 & 828 & -106 & -108 & 672 & 532 \end{bmatrix}$$

$L_{184.2}$: $[1^-2^1]_6 16^-_3,1^{-2}9^-$ $\langle 2\rangle$ $\qquad$ $72^{8,1}_{\infty z} 18^r_2 4^l_2 2^{24,11}_{\infty} 8^s_2 16^s_2$ $(\times 2)$

$$\begin{bmatrix} -32976 & -8208 & -3456 \\ -8208 & -2042 & -856 \\ -3456 & -856 & -345 \end{bmatrix} \begin{bmatrix} 20987 & 5141 & 1855 \\ -94248 & -23087 & -8330 \\ 23760 & 5820 & 2099 \end{bmatrix} \quad \begin{bmatrix} 757 & 202 & 35 & -3 & -29 & 1 \\ -3402 & -909 & -158 & 13 & 130 & -4 \\ 864 & 234 & 42 & -2 & -32 & 0 \end{bmatrix}$$

$L_{184.3}$: $[1^1 2^1]_2 16^1_7,1^{-2}9^-$ $\langle m\rangle$ $\qquad$ $72^{8,5}_{\infty z} 18_2 1_2 2^{24,23}_{\infty} 8^*_2 16^*_2$ $(\times 2)$

$$\begin{bmatrix} -65808 & 1440 & 2304 \\ 1440 & -30 & -52 \\ 2304 & -52 & -79 \end{bmatrix} \begin{bmatrix} 3059 & -55 & -120 \\ 52632 & -947 & -2064 \\ 53856 & -968 & -2113 \end{bmatrix} \quad \begin{bmatrix} 67 & 16 & 1 & -1 & -3 & 1 \\ 1134 & 261 & 14 & -21 & -54 & 20 \\ 1188 & 288 & 19 & -16 & -52 & 16 \end{bmatrix}$$

$L_{184.4}$: $[1^-2^1]_4 16^-_5,1^{-2}9^-$ $\langle m\rangle$ $\qquad$ $18^{8,5}_{\infty} 72^*_2 4^*_2 8^{24,11}_{\infty z} 2^r_2 16^l_2$ $(\times 2)$

$$\begin{bmatrix} -396720 & 3744 & 3744 \\ 3744 & -34 & -36 \\ 3744 & -36 & -35 \end{bmatrix} \begin{bmatrix} 9935 & -90 & -96 \\ 347760 & -3151 & -3360 \\ 702144 & -6360 & -6785 \end{bmatrix} \quad \begin{bmatrix} 19 & 7 & -1 & -1 & 3 & 19 \\ 657 & 234 & -36 & -34 & 107 & 672 \\ 1350 & 504 & -70 & -72 & 210 & 1336 \end{bmatrix}$$

$L_{184.5}$: $1^1_1 8^1_7 64^1_1,1^{-2}9^-$ $\langle 2\rangle$ $\qquad$ $72^{8,1}_{\infty b} 288^*_2 4^*_2 32^{48,23}_{\infty z} 8^b_2 64^b_2$ $(\times 2)$

shares genus with $L_{184.6}$; isometric to its own 2-dual

$$\begin{bmatrix} -14837184 & 46080 & 46080 \\ 46080 & -136 & -144 \\ 46080 & -144 & -143 \end{bmatrix} \begin{bmatrix} 53999 & -150 & -170 \\ 1911600 & -5311 & -6018 \\ 15465600 & -42960 & -48689 \end{bmatrix} \quad \begin{bmatrix} 62 & 173 & 21 & 61 & 24 & 45 \\ 2205 & 6138 & 744 & 2158 & 847 & 1584 \\ 17748 & 49536 & 6014 & 17472 & 6876 & 12896 \end{bmatrix}$$

$L_{184.6}$: $1^1_1 8^1_7 64^1_1,1^{-2}9^-$ $\qquad$ $72^{8,1}_{\infty a} 288^l_2 1^r_2 32^{48,47}_{\infty z} 8^l_2 64^r_2$ $(\times 2)$

shares genus with $L_{184.5}$; isometric to its own 2-dual

$$\begin{bmatrix} -202207680 & 170496 & 340992 \\ 170496 & -136 & -288 \\ 340992 & -288 & -575 \end{bmatrix} \begin{bmatrix} 371951 & -270 & -630 \\ 13266288 & -9631 & -22470 \\ 213913728 & -155280 & -362321 \end{bmatrix}$$

$$\begin{bmatrix} 116 & 317 & 19 & 109 & 42 & 77 \\ 4149 & 11322 & 678 & 3886 & 1495 & 2736 \\ 66708 & 182304 & 10927 & 62688 & 24156 & 44288 \end{bmatrix}$$

W_{185} 12 lattices, $\chi = 30$ $\qquad$ 8-gon: $22242224 \rtimes C_2$

$L_{185.1}$: $1^{-2}_{\rm II} 4^1_7,1^2 3^-,1^2 25^-$ $\langle 2\rangle$ $\qquad$ $2^b_2 50^s_2 6^b_2 4^*_4$ $(\times 2)$

$$\begin{bmatrix} -221700 & 73800 & 1800 \\ 73800 & -24566 & -601 \\ 1800 & -601 & -10 \end{bmatrix} \begin{bmatrix} 61499 & -20541 & -328 \\ 183000 & -61123 & -976 \\ 70500 & -23547 & -377 \end{bmatrix} \quad \begin{bmatrix} 1 & 294 & 124 & 287 \\ 3 & 875 & 369 & 854 \\ -1 & 325 & 141 & 330 \end{bmatrix}$$

W_{186} 12 lattices, $\chi = 10$ $\qquad$ 5-gon: 22226

$L_{186.1}$: $1^{-2}_{\rm II} 4^1_7,1^2 3^-,1^{-2} 25^1$ $\langle 2\rangle$ $\qquad$ $6^b_2 100^*_2 4^b_2 150^s_2 2_6$

$$\begin{bmatrix} -2716400100 & 11466900 & -790200 \\ 11466900 & -48386 & 3309 \\ -790200 & 3309 & -194 \end{bmatrix} \quad \begin{bmatrix} 161 & 343 & -137 & -2656 & -168 \\ 40203 & 85650 & -34210 & -663225 & -41951 \\ 29946 & 63800 & -25482 & -494025 & -31249 \end{bmatrix}$$

W_{187} 4 lattices, $\chi = 8$ 5-gon: 22232

$L_{187.1} : 1^{-2}_{\mathrm{II}} 16^{1}_{1}, 1^{-2} 5^{-}$ $\quad 16^{r}_{2} 10^{b}_{2} 16^{b}_{2} 2^{-}_{3} 2^{l}_{2}$

$$\begin{bmatrix} -57840 & -880 & 1040 \\ -880 & -2 & 13 \\ 1040 & 13 & -18 \end{bmatrix} \qquad \begin{bmatrix} -15 & -1 & 5 & 1 & -2 \\ -256 & -15 & 88 & 17 & -35 \\ -1056 & -70 & 352 & 70 & -141 \end{bmatrix}$$

W_{188} 4 lattices, $\chi = 18$ 5-gon: $42\infty\!|\,24| \rtimes D_2$

$L_{188.1} : 1^{2}_{2} 16^{-}_{3}, 1^{2} 5^{1}$ $\quad 2^{*}_{4} 4^{l}_{2} 5^{8,3}_{\infty} 20^{l}_{2} 1_{4}$

$$\begin{bmatrix} -16720 & 480 & 240 \\ 480 & -11 & -8 \\ 240 & -8 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 7 & 4 \\ 15 & -16 & -15 & 110 & 62 \\ 39 & -38 & -40 & 260 & 151 \end{bmatrix}$$

W_{189} 36 lattices, $\chi = 36$ 8-gon: $2|22|\infty 2|22|\infty \rtimes D_4$

$L_{189.1} : 1^{2}_{0} 16^{-}_{5}, 1^{1} 5^{1} 25^{1}$ $\langle 5 \rangle$ $\quad 20^{s}_{2} 400^{s}_{2} 4^{*}_{2} 80^{5,4}_{\infty b} 80_{2} 1^{r}_{2} 400^{l}_{2} 5^{40,21}_{\infty}$

shares genus with its 5-dual

$$\begin{bmatrix} -2375600 & -43600 & -45200 \\ -43600 & -795 & -825 \\ -45200 & -825 & -856 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 1 & 7 & 9 & 1 & 11 & 0 \\ -298 & 360 & 302 & 2064 & 2576 & 279 & 2920 & -21 \\ 340 & -400 & -344 & -2360 & -2960 & -322 & -3400 & 20 \end{bmatrix}$$

$L_{189.2} : 1^{-2}_{4} 16^{1}_{1}, 1^{1} 5^{1} 25^{1}$ $\langle 5 \rangle$ $\quad 20^{*}_{2} 400^{*}_{2} 4^{s}_{2} 80^{20,9}_{\infty z} 80^{l}_{2} 1_{2} 400_{2} 5^{40,1}_{\infty}$

shares genus with its 5-dual

$$\begin{bmatrix} -13903600 & 108800 & 40400 \\ 108800 & -795 & -325 \\ 40400 & -325 & -116 \end{bmatrix} \qquad \begin{bmatrix} -11 & -19 & 9 & 87 & 149 & 20 & 291 & 10 \\ -418 & -720 & 342 & 3304 & 5656 & 759 & 11040 & 379 \\ -2660 & -4600 & 2176 & 21040 & 36040 & 4838 & 70400 & 2420 \end{bmatrix}$$

$L_{189.3} : 1^{-2}_{6} 16^{1}_{7}, 1^{1} 5^{1} 25^{1}$ $\langle 5 \rangle$ $\quad 20^{*}_{2} 100^{l}_{2} 1_{2} 5^{40,39}_{\infty} 20^{*}_{2} 4^{l}_{2} 25_{2} 5^{40,31}_{\infty}$

$$\begin{bmatrix} -1072400 & 288400 & -71200 \\ 288400 & -75055 & 20390 \\ -71200 & 20390 & -4111 \end{bmatrix} \qquad \begin{bmatrix} 957 & 1019 & -377 & -1916 & -6769 & -3691 & -6833 & -990 \\ 2376 & 2530 & -936 & -4757 & -16806 & -9164 & -16965 & -2458 \\ -4790 & -5100 & 1887 & 9590 & 33880 & 18474 & 34200 & 4955 \end{bmatrix}$$

$L_{189.4} : 1^{2}_{2} 16^{-}_{3}, 1^{1} 5^{1} 25^{1}$ $\quad 20^{l}_{2} 25_{2} 1^{r}_{2} 20^{40,39}_{\infty z} 5^{r}_{2} 4^{*}_{2} 100^{l}_{2} 5^{40,11}_{\infty}$

$$\begin{bmatrix} -475600 & -47600 & -44800 \\ -47600 & -4670 & -4585 \\ -44800 & -4585 & -4111 \end{bmatrix} \qquad \begin{bmatrix} -99 & -106 & 39 & 503 & 510 & 595 & 2373 & 209 \\ 528 & 565 & -208 & -2682 & -2719 & -3172 & -12650 & -1114 \\ 490 & 525 & -193 & -2490 & -2525 & -2946 & -11750 & -1035 \end{bmatrix}$$

$L_{189.5} : 1^{-}_{5} 4^{1}_{7} 16^{1}_{1}, 1^{1} 5^{1} 25^{1}$ $\langle 5 \rangle$ $\quad 20^{s}_{2} 400^{s}_{2} 4^{s}_{2} 80^{10,9}_{\infty b} 80_{2} 1_{2} 400_{2} 5^{20,1}_{\infty}$

shares genus with its 2-dual $\cong$ 5-dual; isometric to its own 2.5-dual

$$\begin{bmatrix} -1775600 & 164400 & 40400 \\ 164400 & -15220 & -3740 \\ 40400 & -3740 & -919 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & -7 & -9 & -1 & -11 & 0 \\ 28 & -60 & -30 & -184 & -196 & -18 & -120 & 11 \\ -70 & 200 & 78 & 440 & 400 & 29 & 0 & -45 \end{bmatrix}$$

$L_{189.6} : 1^{-}_{5} 4^{1}_{1} 16^{1}_{7}, 1^{1} 5^{1} 25^{1}$ $\langle 5 \rangle$ $\quad 20^{l}_{2} 100_{2} 1_{2} 20^{40,39}_{\infty} 20^{r}_{2} 4^{l}_{2} 100_{2} 5^{20,11}_{\infty}$

shares genus with its 5-dual

$$\begin{bmatrix} -456400 & -268000 & -44800 \\ -268000 & -152620 & -25140 \\ -44800 & -25140 & -4111 \end{bmatrix} \qquad \begin{bmatrix} -99 & -106 & 39 & 397 & 702 & 383 & 1419 & 103 \\ 528 & 565 & -208 & -2117 & -3743 & -2042 & -7565 & -549 \\ -2150 & -2300 & 847 & 8620 & 15240 & 8314 & 30800 & 2235 \end{bmatrix}$$

W_{190} 6 lattices, $\chi = 6$ 4-gon: 6223

$L_{190.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{5}, 1^{-} 3^{-} 27^{-}$ $\langle 2 \rangle$ $\quad 6_{6} 2^{s}_{2} 54^{b}_{2} 6^{-}_{3}$

$$\begin{bmatrix} -124524 & 2052 & -29592 \\ 2052 & -30 & 507 \\ -29592 & 507 & -6934 \end{bmatrix} \qquad \begin{bmatrix} -26 & -18 & 52 & 27 \\ -410 & -285 & 819 & 427 \\ 81 & 56 & -162 & -84 \end{bmatrix}$$

W_{191} 22 lattices, $\chi = 36$ — 8-gon: $2\infty 222\infty 22 \rtimes C_2$

$L_{191.1}$: $1^{2}_{\mathrm{II}}4^{1}_{1},1^{1}3^{1}27^{-}$ $\langle 2\rangle$ — $12^{*}_{2}4^{3,2}_{\infty b}4^{r}_{2}54^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 9739332 & -38988 & -110160 \\ -38988 & 156 & 441 \\ -110160 & 441 & 1246 \end{bmatrix} \begin{bmatrix} -9217 & 38 & 104 \\ -230400 & 949 & 2600 \\ -732672 & 3021 & 8267 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 3 & 25 \\ 4 & -18 & 60 & 522 \\ -90 & -82 & 244 & 2025 \end{bmatrix}$$

$L_{191.2}$: $1^{2}_{2}8^{1}_{7},1^{-}3^{-}27^{1}$ $\langle 2\rangle$ — $24^{b}_{2}2^{12,5}_{\infty a}8^{s}_{2}108^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -4970376 & -835272 & 32616 \\ -835272 & -140367 & 5481 \\ 32616 & 5481 & -214 \end{bmatrix} \begin{bmatrix} -76321 & -12820 & 500 \\ 541872 & 91021 & -3550 \\ 2243808 & 376908 & -14701 \end{bmatrix} \qquad \begin{bmatrix} -11 & -14 & -85 & -373 \\ 80 & 100 & 604 & 2646 \\ 372 & 427 & 2512 & 10908 \end{bmatrix}$$

$L_{191.3}$: $1^{-2}_{2}8^{-}_{3},1^{-}3^{-}27^{1}$ $\langle m\rangle$ — $24^{r}_{2}2^{12,5}_{\infty b}8^{l}_{2}27_{2}$ $(\times 2)$

$$\begin{bmatrix} -758376 & -250128 & 4752 \\ -250128 & -82497 & 1566 \\ 4752 & 1566 & -25 \end{bmatrix} \begin{bmatrix} -338005 & -111523 & 2290 \\ 1030248 & 339925 & -6980 \\ 283392 & 93504 & -1921 \end{bmatrix} \qquad \begin{bmatrix} -147 & -167 & -979 & -2123 \\ 448 & 509 & 2984 & 6471 \\ 120 & 139 & 820 & 1782 \end{bmatrix}$$

W_{192} 22 lattices, $\chi = 36$ — 8-gon: $2\infty 222\infty 22 \rtimes C_2$

$L_{192.1}$: $1^{2}_{\mathrm{II}}4^{1}_{1},1^{1}3^{-}27^{1}$ $\langle 2\rangle$ — $108^{*}_{2}4^{3,1}_{\infty a}4^{r}_{2}6^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} -1255068 & -800172 & 84348 \\ -800172 & -510132 & 53769 \\ 84348 & 53769 & -5666 \end{bmatrix} \begin{bmatrix} -31051 & -19688 & 2047 \\ 67500 & 42799 & -4450 \\ 178200 & 112992 & -11749 \end{bmatrix} \qquad \begin{bmatrix} 2819 & 243 & -11 & -121 \\ -6192 & -534 & 24 & 266 \\ -16794 & -1450 & 64 & 723 \end{bmatrix}$$

$L_{192.2}$: $1^{2}_{2}8^{1}_{7},1^{-}3^{1}27^{-}$ $\langle 2\rangle$ — $216^{b}_{2}2^{12,1}_{\infty b}8^{s}_{2}12^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -603720 & -301536 & 3672 \\ -301536 & -150603 & 1833 \\ 3672 & 1833 & -22 \end{bmatrix} \begin{bmatrix} -25201 & -12575 & 150 \\ 52416 & 26155 & -312 \\ 160272 & 79977 & -955 \end{bmatrix} \qquad \begin{bmatrix} -1453 & -99 & -125 & -51 \\ 3024 & 206 & 260 & 106 \\ 9396 & 637 & 796 & 318 \end{bmatrix}$$

$L_{192.3}$: $1^{-2}_{2}8^{-}_{3},1^{-}3^{1}27^{-}$ $\langle m\rangle$ — $216^{r}_{2}2^{12,1}_{\infty a}8^{l}_{2}3_{2}$ $(\times 2)$

$$\begin{bmatrix} -1015848 & 10368 & 5184 \\ 10368 & -105 & -54 \\ 5184 & -54 & -25 \end{bmatrix} \begin{bmatrix} 3167 & -34 & -14 \\ 226512 & -2432 & -1001 \\ 166320 & -1785 & -736 \end{bmatrix} \qquad \begin{bmatrix} 59 & 5 & 9 & 3 \\ 4248 & 359 & 644 & 214 \\ 3024 & 259 & 472 & 159 \end{bmatrix}$$

W_{193} 6 lattices, $\chi = 12$ — 6-gon: $222222 \rtimes C_2$

$L_{193.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{5},1^{-}3^{1}27^{1}$ $\langle 2\rangle$ — $12^{*}_{2}108^{b}_{2}2^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 756 & 324 & -108 \\ 324 & 138 & -51 \\ -108 & -51 & -10 \end{bmatrix} \begin{bmatrix} 179 & 85 & 15 \\ -396 & -188 & -33 \\ 108 & 51 & 8 \end{bmatrix} \qquad \begin{bmatrix} -1 & -115 & -10 \\ 2 & 252 & 22 \\ 0 & -54 & -5 \end{bmatrix}$$

W_{194} 32 lattices, $\chi = 18$ — 6-gon: $2\infty 2222$

$L_{194.1}$: $1^{-2}_{\mathrm{II}}8^{1}_{1},1^{2}9^{-},1^{-}5^{-}25^{-}$ $\langle 25, 5, 2*\rangle$ — $72^{r}_{2}10^{60,49}_{\infty a}40^{b}_{2}450^{l}_{2}8^{r}_{2}50^{l}_{2}$

shares genus with its 5-dual

$$\begin{bmatrix} -8854200 & 1479600 & 145800 \\ 1479600 & -239810 & -20665 \\ 145800 & -20665 & -562 \end{bmatrix} \qquad \begin{bmatrix} -12367 & -1020 & 1007 & 2068 & -731 & -2689 \\ -92304 & -7613 & 7516 & 15435 & -5456 & -20070 \\ 185688 & 15315 & -15120 & -31050 & 10976 & 40375 \end{bmatrix}$$

W_{195} 32 lattices, $\chi = 72$ 12-gon: $2\infty 2|2\infty 2|2\infty 2|2\infty 2| \rtimes D_4$

$L_{195.1}$: $1_{\mathrm{II}}^{-2}8_1^1,1^{-2}9^1,1^-5^-25^-$ $\langle 25,5,2*\rangle 8_2^r10_{\infty b}^{60,29}40_2^b50_2^b360_{\infty z}^{10,1}90_2^l$ $(\times 2)$
shares genus with its 5-dual

$$\begin{bmatrix} -12431315950200 & -85733217000 & -591012000 \\ -85733217000 & -591263590 & -4075945 \\ -591012000 & -4075945 & -28098 \end{bmatrix} \begin{bmatrix} -726032161 & -5007114 & -34524 \\ 104802166080 & 722772931 & 4983512 \\ 68540893200 & 472695405 & 3259229 \end{bmatrix}$$

$$\begin{bmatrix} -1839 & -1134 & -1639 & -631 & -2147 & -632 \\ 265456 & 163691 & 236588 & 91085 & 309924 & 91233 \\ 173928 & 107200 & 154780 & 59475 & 201780 & 59040 \end{bmatrix}$$

W_{196} 92 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{196.1}$: $1_4^{-2}8_1^1,1^29^-,1^{-2}5^1$ $\langle 2\rangle$ $45_2^r4_2^*72_2^*20_2^*8_2^*180_2^l1_272_25_28_2$

$$\begin{bmatrix} -34713720 & -239400 & -104400 \\ -239400 & -1651 & -720 \\ -104400 & -720 & -313 \end{bmatrix} \begin{bmatrix} 107 & 25 & 85 & 17 & -1 & -79 & -5 & -31 & -1 & 7 \\ -15435 & -3608 & -12276 & -2460 & 140 & 11340 & 719 & 4464 & 145 & -1008 \\ -180 & -38 & -108 & -10 & 12 & 270 & 14 & 72 & 0 & -16 \end{bmatrix}$$

$L_{196.2}$: $1_6^{-2}8_7^1,1^29^-,1^{-2}5^1$ $\langle m\rangle$ $45_21_2^r72_2^l5_2^r8_2^l$ $(\times 2)$

$$\begin{bmatrix} -388942920 & -2682360 & 52200 \\ -2682360 & -18499 & 360 \\ 52200 & 360 & -7 \end{bmatrix} \begin{bmatrix} 52919 & 365 & -7 \\ -7673400 & -52926 & 1015 \\ -52920 & -365 & 6 \end{bmatrix} \begin{bmatrix} 17 & 3 & 31 & 6 & 5 \\ -2475 & -436 & -4500 & -870 & -724 \\ -540 & -55 & -288 & -5 & 48 \end{bmatrix}$$

$L_{196.3}$: $1_6^28_3^-,1^29^-,1^{-2}5^1$ $180_2^*4_2^s72_2^s20_2^s8_2^s$ $(\times 2)$

$$\begin{bmatrix} -2359080 & 1049040 & 8280 \\ 1049040 & -466489 & -3682 \\ 8280 & -3682 & -29 \end{bmatrix} \begin{bmatrix} 12959 & -5764 & -44 \\ 29160 & -12970 & -99 \\ -3240 & 1441 & 10 \end{bmatrix} \begin{bmatrix} -403 & -35 & -49 & 31 & 41 \\ -900 & -78 & -108 & 70 & 92 \\ -810 & -92 & -288 & -40 & 24 \end{bmatrix}$$

$L_{196.4}$: $[1^12^1]_216_3^-,1^29^-,1^{-2}5^1$ $\langle 2\rangle$ $180_2^*16_2^s72_2^*80_2^*8_2^*720_2^l1_218_2^r20_2^l2_2^r$

$$\begin{bmatrix} 229680 & 56880 & 21600 \\ 56880 & 14086 & 5346 \\ 21600 & 5346 & 1993 \end{bmatrix} \begin{bmatrix} 259 & -1 & -635 & -1291 & -657 & -30623 & -836 & -2693 & -1027 & -6 \\ -1080 & 4 & 2646 & 5380 & 2738 & 127620 & 3484 & 11223 & 4280 & 25 \\ 90 & 0 & -216 & -440 & -224 & -10440 & -285 & -918 & -350 & -2 \end{bmatrix}$$

$L_{196.5}$: $[1^-2^1]_616_7^1,1^29^-,1^{-2}5^1$ $\langle m\rangle$ $45_2^r16_2^*72_2^s80_2^s8_2^s720_2^*4_2^l18_25_22_2$

$$\begin{bmatrix} -452880 & 152640 & 720 \\ 152640 & -51434 & -244 \\ 720 & -244 & -1 \end{bmatrix} \begin{bmatrix} 16 & -7 & -19 & 7 & 19 & 1331 & 81 & 149 & 37 & 6 \\ 45 & -20 & -54 & 20 & 54 & 3780 & 230 & 423 & 105 & 17 \\ 495 & -168 & -504 & 160 & 496 & 35280 & 2154 & 3978 & 995 & 166 \end{bmatrix}$$

$L_{196.6}$: $[1^12^1]_016_5^-,1^29^-,1^{-2}5^1$ $\langle m\rangle$ $180_2^s16_2^l18_280_22_2720_21_2^r72_2^*20_2^*8_2^*$

$$\begin{bmatrix} -2159280 & 8640 & 8640 \\ 8640 & -34 & -36 \\ 8640 & -36 & -31 \end{bmatrix} \begin{bmatrix} 17 & 7 & 10 & 17 & 2 & 101 & 2 & 7 & -1 & -1 \\ 3060 & 1256 & 1791 & 3040 & 357 & 18000 & 356 & 1242 & -180 & -178 \\ 1170 & 488 & 702 & 1200 & 142 & 7200 & 143 & 504 & -70 & -72 \end{bmatrix}$$

$L_{196.7}$: $[1^-2^1]_416_1^1,1^29^-,1^{-2}5^1$ $45_216_218_2^r80_2^l2_2^r720_2^s4_2^*72_2^l5_2^r8_2^l$

$$\begin{bmatrix} -2720880 & 7200 & 7200 \\ 7200 & -2 & -20 \\ 7200 & -20 & -19 \end{bmatrix} \begin{bmatrix} 101 & 47 & 41 & 37 & 1 & 1 & -1 & -1 & 2 & 7 \\ 1980 & 920 & 801 & 720 & 19 & 0 & -20 & -18 & 40 & 138 \\ 36135 & 16816 & 14670 & 13240 & 358 & 360 & -358 & -360 & 715 & 2504 \end{bmatrix}$$

W_{197} 4 lattices, $\chi = 24$ 6-gon: $4|4\rlap{|}24|4\rlap{|}2 \rtimes D_4$

$L_{197.1}$: $1_2^232_5^-,1^23^1$ $1_42_4^*4_2^l$ $(\times 2)$

$$\begin{bmatrix} -1347936 & 11328 & 5664 \\ 11328 & -95 & -48 \\ 5664 & -48 & -23 \end{bmatrix} \begin{bmatrix} 7039 & -60 & -28 \\ 670560 & -5716 & -2667 \\ 332640 & -2835 & -1324 \end{bmatrix} \begin{bmatrix} -1 & 3 & 9 \\ -95 & 287 & 858 \\ -48 & 139 & 424 \end{bmatrix}$$

W_{198} 8 lattices, $\chi = 32$ — 6-gon: $62\infty62\infty \rtimes C_2$

$L_{198.1}$: $1^{-2}_{\rm II}32^{1}_{7},1^{1}3^{-}9^{-}$ $\langle 3\rangle$ — $6_6\,18^{b}_{2}\,6^{24,7}_{\infty a}$ $(\times 2)$

$$\begin{bmatrix}-3322656 & -643968 & -453600\\ -643968 & -124806 & -87915\\ -453600 & -87915 & -61922\end{bmatrix} \begin{bmatrix}-250561 & -48546 & -34220\\ 760320 & 147311 & 103840\\ 756000 & 146475 & 103249\end{bmatrix} \quad \begin{bmatrix}-150 & 110 & 43\\ 457 & -333 & -131\\ 450 & -333 & -129\end{bmatrix}$$

W_{199} 8 lattices, $\chi = 12$ — 6-gon: $222|222| \rtimes D_2$

$L_{199.1}$: $1^{-2}_{2}32^{1}_{1},1^{-}3^{1}9^{-}$ $\langle 3\rangle$ — $32^{l}_{2}\,3_2\,288^{r}_{2}\,2^{b}_{2}\,288^{*}_{2}\,12^{s}_{2}$

shares genus with its 3-dual

$$\begin{bmatrix}152352 & 75456 & -576\\ 75456 & 37371 & -285\\ -576 & -285 & 2\end{bmatrix} \quad \begin{bmatrix}39 & 20 & 281 & 0 & -47 & -1\\ -80 & -41 & -576 & 0 & 96 & 2\\ -160 & -81 & -1152 & -1 & 144 & 0\end{bmatrix}$$

W_{200} 8 lattices, $\chi = 24$ — 7-gon: $2\infty\!\!\!\!\!\;|\;\,222|22 \rtimes D_2$

$L_{200.1}$: $1^{-2}_{6}32^{1}_{1},1^{-}3^{-}9^{1}$ $\langle 3\rangle$ — $32^{b}_{2}\,6^{24,17}_{\infty b}\,6^{l}_{2}\,32_2\,9^{r}_{2}\,32^{s}_{2}\,36^{*}_{2}$

$$\begin{bmatrix}-202464 & -34272 & 1440\\ -34272 & -5799 & 243\\ 1440 & 243 & -10\end{bmatrix} \quad \begin{bmatrix}-19 & -5 & -2 & 5 & 4 & 7 & -1\\ 128 & 34 & 14 & -32 & -27 & -48 & 6\\ 368 & 105 & 51 & -64 & -81 & -160 & 0\end{bmatrix}$$

W_{201} 8 lattices, $\chi = 112$ — 14-gon: $3\infty2\infty2\infty\infty3\infty2\infty2\infty\infty \rtimes C_2$

$L_{201.1}$: $1^{-2}_{\rm II}8^{-}_{5},1^{-2}49^{1}$ $\langle 2\rangle$ — $2^{+}_{3}\,2^{28,25}_{\infty b}\,8^{b}_{2}\,98^{4,1}_{\infty b}\,392^{b}_{2}\,2^{28,1}_{\infty b}\,8^{14,11}_{\infty z}$ $(\times 2)$

$$\begin{bmatrix}-446488 & 1176 & 2744\\ 1176 & -2 & -9\\ 2744 & -9 & -14\end{bmatrix} \begin{bmatrix}148175 & -423 & -864\\ 22950816 & -65519 & -133824\\ 14175504 & -40467 & -82657\end{bmatrix} \quad \begin{bmatrix}1 & 0 & -1 & 1 & 47 & 5 & 27\\ 156 & 1 & -156 & 147 & 7252 & 773 & 4180\\ 95 & -1 & -96 & 98 & 4508 & 479 & 2584\end{bmatrix}$$

W_{202} 12 lattices, $\chi = 15$ — 6-gon: 222224

$L_{202.1}$: $1^{-2}_{\rm II}4^{1}_{7},1^{2}9^{1},1^{2}11^{-}$ $\langle 2\rangle$ — $2^{s}_{2}\,198^{b}_{2}\,4^{*}_{2}\,36^{b}_{2}\,22^{b}_{2}\,4^{*}_{4}$

$$\begin{bmatrix}-900900 & 3960 & 5544\\ 3960 & -14 & -25\\ 5544 & -25 & -34\end{bmatrix} \quad \begin{bmatrix}1 & 49 & 5 & 5 & -2 & -1\\ 27 & 1287 & 130 & 126 & -55 & -26\\ 143 & 7029 & 718 & 720 & -286 & -144\end{bmatrix}$$

W_{203} 4 lattices, $\chi = 48$ — 10-gon: $\infty\!\!\!\!\!\;|\;\,22|22\infty\!\!\!\!\!\;|\;\,22|22 \rtimes D_4$

$L_{203.1}$: $1^{2}_{6}16^{1}_{1},1^{-2}7^{1}$ — $28^{8,5}_{\infty z}\,7_2\,16^{r}_{2}\,14^{b}_{2}\,16^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix}15120 & 4256 & -224\\ 4256 & 1198 & -63\\ -224 & -63 & 3\end{bmatrix} \begin{bmatrix}-1009 & -282 & 9\\ 3696 & 1033 & -33\\ 2688 & 752 & -25\end{bmatrix} \quad \begin{bmatrix}27 & 25 & 57 & 44 & 37\\ -98 & -91 & -208 & -161 & -136\\ -42 & -49 & -128 & -112 & -112\end{bmatrix}$$

W_{204} 12 lattices, $\chi = 36$ — 10-gon: $2222222222 \rtimes C_2$

$L_{204.1}$: $1^{-2}_{\rm II}4^{1}_{1},1^{2}9^{-},1^{-2}13^{-}$ $\langle 2\rangle$ — $234^{l}_{2}\,4^{r}_{2}\,18^{b}_{2}\,26^{b}_{2}\,2^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix}-53820 & -26676 & 468\\ -26676 & -13222 & 233\\ 468 & 233 & 58\end{bmatrix} \begin{bmatrix}419795 & 208311 & 10557\\ -845676 & -419642 & -21267\\ -6084 & -3019 & -154\end{bmatrix} \quad \begin{bmatrix}-58 & -117 & -232 & -219 & 10\\ 117 & 236 & 468 & 442 & -20\\ 0 & -4 & -9 & -13 & -3\end{bmatrix}$$

W_{205} 60 lattices, $\chi = 12$ — 6-gon: $22|222|2 \rtimes D_2$

$L_{205.1}$: $1^{2}_{0}8^{1}_{7},1^{2}3^{-},1^{-2}5^{-}$ — $60^{*}_{2}\,4^{s}_{2}\,40^{l}_{2}\,1_2\,15^{r}_{2}\,8^{s}_{2}$

$$\begin{bmatrix}347640 & 6240 & 840\\ 6240 & 112 & 15\\ 840 & 15 & 1\end{bmatrix} \quad \begin{bmatrix}-17 & -5 & -11 & 0 & 4 & 1\\ 960 & 282 & 620 & 0 & -225 & -56\\ -90 & -26 & -60 & -1 & 15 & 4\end{bmatrix}$$

$L_{205.2}$: $[1^{1}2^{1}]_{0}16^{1}_{7},1^{2}3^{-},1^{-2}5^{-}$ $\langle 2\rangle$ $\quad 15^{r}_{2}16^{*}_{2}40^{l}_{2}1_{2}240_{2}2_{2}$

$$\begin{bmatrix} -161040 & -960 & 1680 \\ -960 & 2 & 8 \\ 1680 & 8 & -17 \end{bmatrix} \qquad \begin{bmatrix} 2 & -1 & -3 & 0 & 13 & 1 \\ 60 & -28 & -90 & -1 & 360 & 29 \\ 225 & -112 & -340 & -1 & 1440 & 112 \end{bmatrix}$$

$L_{205.3}$: $[1^{-}2^{1}]_{4}16^{-}_{3},1^{2}3^{-},1^{-2}5^{-}$ $\langle m\rangle$ $\quad 60^{*}_{2}16^{s}_{2}40^{*}_{2}4^{s}_{2}240^{l}_{2}2^{r}_{2}$

$$\begin{bmatrix} 26160 & -6720 & 2880 \\ -6720 & 1726 & -740 \\ 2880 & -740 & 317 \end{bmatrix} \qquad \begin{bmatrix} -11 & 1 & 37 & 19 & 163 & 1 \\ -30 & 4 & 110 & 56 & 480 & 3 \\ 30 & 0 & -80 & -42 & -360 & -2 \end{bmatrix}$$

$L_{205.4}$: $[1^{-}2^{1}]_{2}16^{-}_{5},1^{2}3^{-},1^{-2}5^{-}$ $\langle m\rangle$ $\quad 60^{s}_{2}16^{l}_{2}10_{2}1^{r}_{2}240^{*}_{2}8^{*}_{2}$

$$\begin{bmatrix} -24240 & 960 & 0 \\ 960 & -2 & -6 \\ 0 & -6 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 1 & 0 & -7 & -1 \\ -30 & 24 & 25 & 0 & -180 & -26 \\ -150 & 152 & 150 & -1 & -1080 & -152 \end{bmatrix}$$

$L_{205.5}$: $[1^{1}2^{1}]_{6}16^{1}_{1},1^{2}3^{-},1^{-2}5^{-}$ $\quad 15_{2}16_{2}10^{r}_{2}4^{*}_{2}240^{s}_{2}8^{l}_{2}$

$$\begin{bmatrix} 151440 & 3120 & 2880 \\ 3120 & 58 & 56 \\ 2880 & 56 & 53 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -2 & -1 & -1 & 1 \\ 75 & -56 & -125 & -64 & -60 & 66 \\ -135 & 112 & 240 & 122 & 120 & -124 \end{bmatrix}$$

W_{206} 120 lattices, $\chi = 36$ $\quad$ 9-gon: $2\infty\!\!\!|\;2222|222 \rtimes D_2$

$L_{206.1}$: $1^{2}_{0}8^{1}_{7},1^{-}3^{1}9^{1},1^{2}5^{1}$ $\langle 3\rangle$ $\quad 9_{2}120^{6,1}_{\infty}120^{*}_{2}36^{l}_{2}5_{2}3^{r}_{2}8^{s}_{2}12^{*}_{2}20^{l}_{2}$

$$\begin{bmatrix} -330120 & -1800 & 2520 \\ -1800 & 3 & 12 \\ 2520 & 12 & -19 \end{bmatrix} \qquad \begin{bmatrix} -5 & -13 & -7 & -1 & 1 & 1 & 1 & -1 & -7 \\ -114 & -280 & -140 & -18 & 20 & 19 & 16 & -32 & -170 \\ -747 & -1920 & -1020 & -144 & 145 & 144 & 140 & -162 & -1060 \end{bmatrix}$$

$L_{206.2}$: $[1^{-}2^{1}]_{4}16^{-}_{3},1^{-}3^{1}9^{1},1^{2}5^{1}$ $\langle 3,2\rangle$

$36^{l}_{2}30^{24,19}_{\infty}120^{*}_{2}144^{l}_{2}5_{2}48_{2}2_{2}3^{r}_{2}80^{*}_{2}$

$$\begin{bmatrix} 223920 & 14400 & -1440 \\ 14400 & 174 & -36 \\ -1440 & -36 & 5 \end{bmatrix} \qquad \begin{bmatrix} -1 & -2 & -1 & 7 & 3 & 7 & 1 & 1 & 3 \\ -48 & -95 & -50 & 324 & 140 & 328 & 47 & 47 & 140 \\ -630 & -1260 & -660 & 4320 & 1865 & 4368 & 626 & 627 & 1880 \end{bmatrix}$$

$L_{206.3}$: $[1^{1}2^{1}]_{0}16^{1}_{7},1^{1}3^{1}9^{-},1^{2}5^{1}$ $\langle 32,3m,3,m\rangle$

$16^{s}_{2}120^{24,17}_{\infty z}30_{2}1^{r}_{2}720^{*}_{2}12^{l}_{2}18^{r}_{2}48^{s}_{2}180^{*}_{2}$

$$\begin{bmatrix} -712080 & 0 & 3600 \\ 0 & 30 & -6 \\ 3600 & -6 & -17 \end{bmatrix} \qquad \begin{bmatrix} 11 & 21 & 7 & 1 & 11 & -1 & -1 & 5 & 37 \\ 436 & 830 & 275 & 39 & 420 & -40 & -39 & 200 & 1470 \\ 2168 & 4140 & 1380 & 197 & 2160 & -198 & -198 & 984 & 7290 \end{bmatrix}$$

$L_{206.4}$: $[1^{-}2^{1}]_{2}16^{-}_{5},1^{-}3^{1}9^{1},1^{2}5^{1}$ $\langle 3m,3,m\rangle$

$9^{r}_{2}120^{24,7}_{\infty z}30^{r}_{2}144^{s}_{2}20^{*}_{2}48^{s}_{2}8^{l}_{2}3_{2}80_{2}$

$$\begin{bmatrix} 261856080 & -6681600 & 141840 \\ -6681600 & 170490 & -3618 \\ 141840 & -3618 & 83 \end{bmatrix} \qquad \begin{bmatrix} -1 & 277 & 764 & 3749 & 1897 & 1415 & 203 & 41 & 41 \\ -39 & 10810 & 29815 & 146304 & 74030 & 55220 & 7922 & 1600 & 1600 \\ 9 & -2160 & -5970 & -29304 & -14830 & -11064 & -1588 & -321 & -320 \end{bmatrix}$$

$L_{206.5}$: $[1^{1}2^{1}]_{6}16^{1}_{1},1^{-}3^{1}9^{1},1^{2}5^{1}$ $\langle 3\rangle$ $\quad 36^{*}_{2}120^{24,19}_{\infty z}30_{2}144_{2}5^{r}_{2}48^{*}_{2}8^{*}_{2}12^{s}_{2}80^{s}_{2}$

$$\begin{bmatrix} -238320 & 2880 & 0 \\ 2880 & -6 & -12 \\ 0 & -12 & 5 \end{bmatrix} \qquad \begin{bmatrix} -1 & -3 & -1 & 1 & 1 & 3 & 1 & 1 & 1 \\ -84 & -250 & -85 & 72 & 80 & 244 & 82 & 82 & 80 \\ -198 & -600 & -210 & 144 & 185 & 576 & 196 & 198 & 200 \end{bmatrix}$$

W_{207} 60 lattices, $\chi = 18$ $\quad$ 7-gon: $22|222\,2\!\!\!|\;2 \rtimes D_2$

$L_{207.1}$: $1^{-2}_{4}8^{-}_{3},1^{2}3^{-},1^{2}5^{1}$ $\quad 24_{2}5^{r}_{2}8^{s}_{2}20^{*}_{2}24^{*}_{2}4^{l}_{2}1_{2}$

$$\begin{bmatrix} -39720 & 480 & 0 \\ 480 & -4 & -3 \\ 0 & -3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 5 & 1 & -1 & -3 & -1 & 1 & 1 \\ 408 & 80 & -84 & -250 & -84 & 82 & 82 \\ 240 & 45 & -52 & -150 & -48 & 50 & 49 \end{bmatrix}$$

$L_{207.2} : [1^{-}2^{1}]_{6}16^{-}_{5},1^{2}3^{-},1^{2}5^{1}\ \langle 2\rangle$ $\qquad 24^{l}_{2}5_{2}2_{2}80_{2}6^{r}_{2}16^{s}_{2}4^{*}_{2}$

$$\begin{bmatrix} -364080 & 2640 & 2640 \\ 2640 & -14 & -20 \\ 2640 & -20 & -19 \end{bmatrix} \qquad \begin{bmatrix} -1 & 2 & 2 & 23 & 4 & 1 & -1 \\ -18 & 40 & 39 & 440 & 75 & 16 & -20 \\ -120 & 235 & 236 & 2720 & 474 & 120 & -118 \end{bmatrix}$$

$L_{207.3} : [1^{1}2^{1}]_{2}16^{1}_{1},1^{2}3^{-},1^{2}5^{1}\ \langle m\rangle$ $\qquad 24^{*}_{2}20^{l}_{2}2^{r}_{2}80^{l}_{2}6_{2}16_{2}1^{r}_{2}$

$$\begin{bmatrix} -4080 & 240 & 720 \\ 240 & -14 & -40 \\ 720 & -40 & -79 \end{bmatrix} \qquad \begin{bmatrix} -1 & 9 & 4 & 43 & 7 & 1 & -1 \\ -18 & 180 & 79 & 840 & 135 & 16 & -20 \\ 0 & -10 & -4 & -40 & -6 & 0 & 1 \end{bmatrix}$$

$L_{207.4} : [1^{1}2^{-}]_{4}16^{-}_{3},1^{2}3^{-},1^{2}5^{1}\ \langle m\rangle$ $\qquad 6^{r}_{2}20^{*}_{2}8^{*}_{2}80^{s}_{2}24^{*}_{2}16^{l}_{2}1_{2}$

$$\begin{bmatrix} -169680 & 1440 & 1200 \\ 1440 & -10 & -12 \\ 1200 & -12 & -7 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 3 & 5 & 5 & 1 \\ 57 & -60 & -58 & 180 & 294 & 292 & 58 \\ 72 & -70 & -72 & 200 & 348 & 352 & 71 \end{bmatrix}$$

$L_{207.5} : [1^{-}2^{-}]_{0}16^{1}_{7},1^{2}3^{-},1^{2}5^{1}$ $\qquad 6_{2}5^{r}_{2}8^{s}_{2}80^{*}_{2}24^{s}_{2}16^{*}_{2}4^{l}_{2}$

$$\begin{bmatrix} 478320 & 11280 & 1200 \\ 11280 & 266 & 28 \\ 1200 & 28 & -5 \end{bmatrix} \qquad \begin{bmatrix} -5 & 2 & 7 & 23 & -1 & -11 & -7 \\ 213 & -85 & -298 & -980 & 42 & 468 & 298 \\ -6 & 5 & 12 & 40 & 0 & -16 & -10 \end{bmatrix}$$

W_{208} 120 lattices, $\chi = 48$ $\qquad$ 12-gon: $222|222|222|222| \rtimes D_4$

$L_{208.1} : 1^{-2}_{4}8^{-}_{3},1^{-}3^{1}9^{1},1^{-2}5^{-}\ \langle 3\rangle$ $\qquad 8^{s}_{2}36^{*}_{2}12^{s}_{2}360^{l}_{2}3_{2}9^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -85179240 & 2184120 & -28440 \\ 2184120 & -56004 & 729 \\ -28440 & 729 & -7 \end{bmatrix} \begin{bmatrix} -1634491 & 41899 & -429 \\ -63825120 & 1636111 & -16752 \\ -6172200 & 158220 & -1621 \end{bmatrix}$$

$$\begin{bmatrix} 265 & 919 & 593 & 3869 & 205 & 185 \\ 10348 & 35886 & 23156 & 151080 & 8005 & 7224 \\ 1004 & 3474 & 2238 & 14580 & 771 & 693 \end{bmatrix}$$

$L_{208.2} : [1^{-}2^{-}]_{0}16^{1}_{7},1^{-}3^{1}9^{1},1^{-2}5^{-}\ \langle 3, 2\rangle$ $\qquad 8^{*}_{2}144^{l}_{2}3_{2}90^{r}_{2}48^{s}_{2}36^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -182160 & 26640 & 5040 \\ 26640 & -3894 & -738 \\ 5040 & -738 & -139 \end{bmatrix} \begin{bmatrix} 4799 & -710 & -130 \\ 24000 & -3551 & -650 \\ 46080 & -6816 & -1249 \end{bmatrix} \qquad \begin{bmatrix} 1 & 49 & 14 & 134 & 83 & 65 \\ 6 & 252 & 71 & 675 & 416 & 324 \\ 4 & 432 & 129 & 1260 & 792 & 630 \end{bmatrix}$$

$L_{208.3} : [1^{1}2^{1}]_{2}16^{1}_{1},1^{-}3^{1}9^{1},1^{-2}5^{-}\ \langle 32, 3, m\rangle$ $\qquad 2_{2}144_{2}3^{r}_{2}360^{*}_{2}48^{l}_{2}9_{2}\ (\times 2)$

$$\begin{bmatrix} -10658160 & 30960 & -3222720 \\ 30960 & -78 & 9480 \\ -3222720 & 9480 & -973279 \end{bmatrix} \begin{bmatrix} -3654421 & 8701 & -1124011 \\ -109651080 & 261073 & -33726014 \\ 11032560 & -26268 & 3393347 \end{bmatrix}$$

$$\begin{bmatrix} -53 & -3959 & -1108 & -21047 & -6487 & -2531 \\ -1591 & -118800 & -33247 & -631530 & -194644 & -75942 \\ 160 & 11952 & 3345 & 63540 & 19584 & 7641 \end{bmatrix}$$

$L_{208.4} : [1^{-}2^{1}]_{6}16^{-}_{5},1^{-}3^{1}9^{1},1^{-2}5^{-}\ \langle 3m, 3\rangle$ $\qquad 2^{r}_{2}144^{s}_{2}12^{*}_{2}360^{s}_{2}48^{*}_{2}36^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} -91440 & -4320 & 2160 \\ -4320 & -42 & 12 \\ 2160 & 12 & -1 \end{bmatrix} \begin{bmatrix} -301 & -5 & 2 \\ 64200 & 1069 & -428 \\ 115200 & 1920 & -769 \end{bmatrix} \qquad \begin{bmatrix} 0 & -5 & -3 & -29 & -9 & -7 \\ 1 & 1080 & 644 & 6210 & 1924 & 1494 \\ 2 & 1944 & 1158 & 11160 & 3456 & 2682 \end{bmatrix}$$

$L_{208.5} : [1^{1}2^{-}]_{4}16^{-}_{3},1^{-}3^{1}9^{1},1^{-2}5^{-}\ \langle 3m, 3, m\rangle$ $\qquad 8^{s}_{2}144^{*}_{2}12^{l}_{2}90_{2}48_{2}9^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -1625040 & 101520 & 30240 \\ 101520 & -6342 & -1890 \\ 30240 & -1890 & -559 \end{bmatrix} \begin{bmatrix} -21601 & 1356 & 372 \\ -320400 & 20113 & 5518 \\ -86400 & 5424 & 1487 \end{bmatrix} \qquad \begin{bmatrix} -7 & 5 & 19 & 127 & 95 & 44 \\ -106 & 60 & 278 & 1875 & 1408 & 654 \\ -20 & 72 & 90 & 540 & 384 & 171 \end{bmatrix}$$

W_{209} 11 lattices, $\chi = 18$ — 5-gon: $\not{2}2\infty|\infty 2 \rtimes D_2$

$L_{209.1}$: $1^{2}_{\mathrm{II}}4^{-}_{5},1^{-}5^{1}25^{-}$ $\langle 2\rangle$ — $50^{b}_{2}2^{l}_{2}20^{5,4}_{\infty}20^{5,2}_{\infty b}20^{r}_{2}$

$$\begin{bmatrix} 9300 & 4800 & -1300 \\ 4800 & 2480 & -675 \\ -1300 & -675 & 188 \end{bmatrix} \qquad \begin{bmatrix} -54 & -9 & -39 & 1 & -57 \\ 125 & 21 & 92 & -2 & 132 \\ 75 & 13 & 60 & 0 & 80 \end{bmatrix}$$

$L_{209.2}$: $1^{-2}_{2}8^{1}_{7},1^{1}5^{-}25^{1}$ $\langle 2\rangle$ — $100^{*}_{2}4^{s}_{2}40^{20,9}_{\infty z}10^{20,7}_{\infty a}40^{s}_{2}$

$$\begin{bmatrix} -302600 & -148800 & -52000 \\ -148800 & -73165 & -25555 \\ -52000 & -25555 & -8894 \end{bmatrix} \qquad \begin{bmatrix} -407 & 85 & 1453 & 204 & -211 \\ 950 & -198 & -3388 & -476 & 492 \\ -350 & 72 & 1240 & 175 & -180 \end{bmatrix}$$

$L_{209.3}$: $1^{2}_{2}8^{-}_{3},1^{1}5^{-}25^{1}$ $\langle m\rangle$ — $25_{2}1^{r}_{2}40^{20,19}_{\infty z}10^{20,7}_{\infty b}40^{l}_{2}$

$$\begin{bmatrix} -74600 & -3800 & -10600 \\ -3800 & -190 & -545 \\ -10600 & -545 & -1499 \end{bmatrix} \qquad \begin{bmatrix} -16 & 2 & 91 & 15 & -13 \\ 105 & -14 & -612 & -99 & 88 \\ 75 & -9 & -420 & -70 & 60 \end{bmatrix}$$

W_{210} 6 lattices, $\chi = 12$ — 6-gon: $222222 \rtimes C_2$

$L_{210.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{1}5^{-}25^{-}$ $\langle 2\rangle$ — $4^{r}_{2}50^{b}_{2}10^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} 5700 & 2400 & 200 \\ 2400 & 1010 & 85 \\ 200 & 85 & 6 \end{bmatrix} \begin{bmatrix} 19 & 9 & 0 \\ -40 & -19 & 0 \\ -100 & -45 & -1 \end{bmatrix} \qquad \begin{bmatrix} 11 & 7 & -4 \\ -24 & -15 & 9 \\ -28 & -25 & 5 \end{bmatrix}$$

W_{211} 22 lattices, $\chi = 12$ — 6-gon: $222|222| \rtimes D_2$

$L_{211.1}$: $1^{2}_{\mathrm{II}}4^{1}_{7},1^{-}3^{1}9^{-},1^{-2}5^{1}$ $\langle 2\rangle$ — $12^{*}_{2}20^{b}_{2}18^{s}_{2}30^{s}_{2}2^{b}_{2}180^{*}_{2}$

$$\begin{bmatrix} 3420 & -1260 & 0 \\ -1260 & 462 & 3 \\ 0 & 3 & -4 \end{bmatrix} \qquad \begin{bmatrix} 3 & 11 & 1 & -13 & -7 & -77 \\ 8 & 30 & 3 & -35 & -19 & -210 \\ 6 & 20 & 0 & -30 & -16 & -180 \end{bmatrix}$$

$L_{211.2}$: $1^{2}_{6}8^{1}_{1},1^{1}3^{-}9^{1},1^{-2}5^{-}$ $\langle 2\rangle$ — $6^{b}_{2}40^{*}_{2}36^{l}_{2}15^{r}_{2}4^{*}_{2}360^{b}_{2}$

$$\begin{bmatrix} -1490040 & -2880 & 5040 \\ -2880 & 6 & 9 \\ 5040 & 9 & -17 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & -1 & -2 & -1 & -1 \\ 19 & 60 & -18 & -40 & -22 & -60 \\ 306 & 920 & -306 & -615 & -310 & -360 \end{bmatrix}$$

$L_{211.3}$: $1^{-2}_{6}8^{-}_{5},1^{1}3^{-}9^{1},1^{-2}5^{-}$ $\langle m\rangle$ — $6^{l}_{2}40_{2}9^{r}_{2}60^{l}_{2}1_{2}360^{r}_{2}$

$$\begin{bmatrix} 85251240 & -2185920 & 56160 \\ -2185920 & 56049 & -1440 \\ 56160 & -1440 & 37 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & -1 & 7 & 4 & 133 \\ 79 & 40 & -39 & 280 & 159 & 5280 \\ 39 & 40 & 0 & 270 & 116 & 3600 \end{bmatrix}$$

W_{212} 9 lattices, $\chi = 12$ — 6-gon: $2\not{2}2|2\not{2}2| \rtimes D_4$

$L_{212.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{3},1^{-}3^{1}9^{-},1^{-}5^{-}25^{-}$ $\langle 25,5,2\rangle$ — $12^{r}_{2}50^{b}_{2}18^{l}_{2}300^{r}_{2}2^{b}_{2}450^{l}_{2}$

$$\begin{bmatrix} 398700 & -126000 & -900 \\ -126000 & 39810 & 285 \\ -900 & 285 & 2 \end{bmatrix} \qquad \begin{bmatrix} -3 & -2 & 1 & 7 & 0 & -11 \\ -8 & -5 & 3 & 20 & 0 & -30 \\ -204 & -175 & 27 & 300 & -1 & -675 \end{bmatrix}$$

W_{213} 6 lattices, $\chi = 6$ — 5-gon: $2|22\not{2}2 \rtimes D_2$

$L_{213.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{3},1^{1}3^{1}9^{1},1^{2}5^{-}$ $\langle 2\rangle$ — $90^{l}_{2}12^{r}_{2}10^{b}_{2}36^{*}_{2}4^{b}_{2}$

$$\begin{bmatrix} 47340 & 900 & -720 \\ 900 & -6 & -9 \\ -720 & -9 & 10 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 2 & -1 & -1 \\ 15 & 52 & 35 & -18 & -18 \\ 90 & 264 & 175 & -90 & -88 \end{bmatrix}$$

W_{214} 26 lattices, $\chi = 24$ 8-gon: $22|22|22|22| \rtimes D_4$

$L_{214.1}$: $1_{\mathrm{II}}^{2}4_{7}^{1},1^{1}3^{1}9^{1},1^{-2}5^{1}$ $\langle 2\rangle$ $12_{2}^{*}4_{2}^{b}30_{2}^{b}36_{2}^{*}$ $(\times 2)$

$$\begin{bmatrix} 23580 & -540 & 180 \\ -540 & 12 & -3 \\ 180 & -3 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 360 & -11 & 8 \\ 540 & -15 & 11 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 & 1 \\ -50 & -56 & -70 & 24 \\ -18 & -26 & -45 & -18 \end{bmatrix}$$

$L_{214.2}$: $1_{6}^{-2}8_{5}^{-},1^{-}3^{-}9^{-},1^{-2}5^{-}$ $\langle 2\rangle$ $6_{2}^{b}8_{2}^{*}60_{2}^{*}72_{2}^{b}$ $(\times 2)$

$$\begin{bmatrix} 230760 & 4320 & -1800 \\ 4320 & 69 & -33 \\ -1800 & -33 & 14 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 720 & 19 & -6 \\ 2160 & 60 & -19 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & 5 \\ -10 & -16 & -10 & 24 \\ -153 & -172 & 90 & 684 \end{bmatrix}$$

$L_{214.3}$: $1_{6}^{2}16_{1}^{1},1^{1}3^{1}9^{1},1^{-2}5^{1}$ $\langle 2, m\rangle$ $3_{2}16_{2}^{r}30_{2}^{b}144_{2}^{*}12_{2}^{*}16_{2}^{b}30_{2}^{l}144_{2}$

$$\begin{bmatrix} -929520 & -4320 & -5760 \\ -4320 & 3 & -6 \\ -5760 & -6 & -17 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 & 1 & 1 & 5 & 6 & 11 \\ -1 & -448 & -445 & 456 & 448 & 2232 & 2675 & 4896 \\ 0 & 496 & 495 & -504 & -498 & -2488 & -2985 & -5472 \end{bmatrix}$$

W_{215} 34 lattices, $\chi = 108$ 20-gon: $22\nmid 22222\not\infty 2222\nmid 22222\not\infty 22 \rtimes D_4$

$L_{215.1}$: $1_{0}^{2}8_{1}^{1},1^{2}17^{1}$ $68_{2}^{*}8_{2}^{*}4_{2}^{l}1_{2}8_{2}17_{2}^{r}4_{2}^{*}136_{\infty b}^{1,0}136_{2}1_{2}^{r}$ $(\times 2)$

$$\begin{bmatrix} -163064 & 1632 & 1632 \\ 1632 & -16 & -17 \\ 1632 & -17 & -15 \end{bmatrix} \begin{bmatrix} 9757 & -105 & -84 \\ 652392 & -7021 & -5616 \\ 317832 & -3420 & -2737 \end{bmatrix}$$

$$\begin{bmatrix} 463 & 71 & 29 & 13 & 53 & 155 & 31 & 95 & 41 & 2 \\ 30974 & 4748 & 1938 & 868 & 3536 & 10336 & 2066 & 6324 & 2720 & 132 \\ 15062 & 2312 & 946 & 425 & 1736 & 5083 & 1018 & 3128 & 1360 & 67 \end{bmatrix}$$

$L_{215.2}$: $[1^{1}2^{1}]_{2}16_{7}^{1},1^{2}17^{1}$ $\langle 2\rangle$ $17_{2}2_{2}1_{2}^{r}16_{2}^{*}8_{2}^{*}272_{2}^{l}1_{2}34_{\infty}^{8,7}136_{2}^{*}16_{2}^{l}$ $(\times 2)$

$$\begin{bmatrix} -59024 & -1360 & 1360 \\ -1360 & 2 & 8 \\ 1360 & 8 & -15 \end{bmatrix} \begin{bmatrix} 475 & 21 & -18 \\ 44744 & 1973 & -1692 \\ 64736 & 2856 & -2449 \end{bmatrix} \begin{bmatrix} 32 & 4 & 1 & 1 & -1 & -25 & -1 & -5 & -7 & -1 \\ 2992 & 369 & 88 & 60 & -138 & -2924 & -110 & -527 & -714 & -100 \\ 4335 & 536 & 129 & 96 & -188 & -4080 & -155 & -748 & -1020 & -144 \end{bmatrix}$$

$L_{215.3}$: $[1^{-}2^{1}]_{6}16_{3}^{-},1^{2}17^{1}$ $\langle m\rangle$ $68_{2}^{l}2_{2}^{r}4_{2}^{*}16_{2}^{s}8_{2}^{s}272_{2}^{*}4_{2}^{l}34_{\infty}^{8,3}136_{2}^{s}16_{2}^{*}$ $(\times 2)$

$$\begin{bmatrix} 33456 & 8160 & 3536 \\ 8160 & 1990 & 862 \\ 3536 & 862 & 373 \end{bmatrix} \begin{bmatrix} -29581 & -7743 & -4350 \\ 148920 & 38981 & 21900 \\ -63920 & -16732 & -9401 \end{bmatrix}$$

$$\begin{bmatrix} -5929 & -434 & -325 & -519 & -475 & -5177 & -237 & -301 & -129 & -1 \\ 29852 & 2185 & 1636 & 2612 & 2390 & 26044 & 1192 & 1513 & 646 & 4 \\ -12818 & -938 & -702 & -1120 & -1024 & -11152 & -510 & -646 & -272 & 0 \end{bmatrix}$$

$L_{215.4}$: $[1^{-}2^{1}]_{4}16_{5}^{-},1^{2}17^{1}$ $\langle m\rangle$ $68_{2}^{*}8_{2}^{*}4_{2}^{s}16_{2}^{l}2_{2}^{r}272_{2}^{s}4_{2}^{*}136_{\infty z}^{8,3}34_{2}^{r}16_{2}^{s}$ $(\times 2)$

$$\begin{bmatrix} -92208 & 1632 & 544 \\ 1632 & -2 & -12 \\ 544 & -12 & -3 \end{bmatrix} \begin{bmatrix} 5711 & -78 & -36 \\ 66640 & -911 & -420 \\ 761600 & -10400 & -4801 \end{bmatrix}$$

$$\begin{bmatrix} 127 & 25 & 15 & 41 & 29 & 815 & 49 & 207 & 92 & 75 \\ 1496 & 294 & 176 & 480 & 339 & 9520 & 572 & 2414 & 1071 & 872 \\ 16898 & 3328 & 1998 & 5464 & 3866 & 108664 & 6534 & 27608 & 12274 & 10008 \end{bmatrix}$$

$L_{215.5}$: $1_{1}^{1}8_{7}^{1}64_{1}^{1},1^{2}17^{1}$ $\langle 2\rangle$ $17_{2}^{r}32_{2}^{*}4_{2}^{s}64_{2}^{b}8_{2}^{l}1088_{2}1_{2}^{r}544_{\infty z}^{16,15}136_{2}^{l}64_{2}$ $(\times 2)$
shares genus with $L_{215.6}$; isometric to its own 2-dual

$$\begin{bmatrix} 1036864 & 1088 & -1088 \\ 1088 & -8 & 0 \\ -1088 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1089 & -26 & 4 \\ -133824 & -3199 & 492 \\ -1166336 & -27872 & 4287 \end{bmatrix}$$

$$\begin{bmatrix} -19 & -9 & -1 & -1 & 0 & 1 & 0 & -3 & -7 & -11 \\ -2329 & -1102 & -122 & -120 & 1 & 136 & 0 & -374 & -867 & -1360 \\ -20315 & -9616 & -1066 & -1056 & 4 & 1088 & -1 & -3264 & -7548 & -11840 \end{bmatrix}$$

$L_{215.6}$: $1^{1}_{1}8^{1}_{7}64^{1}_{1}{,}1^{2}17^{1}$ $\quad 68^{*}_{2}32^{l}_{2}1_{2}64^{r}_{2}8^{b}_{2}1088^{s}_{2}4^{*}_{2}544^{16,7}_{\infty z}136^{b}_{2}64^{s}_{2}\ (\times 2)$
shares genus with $L_{215.5}$; isometric to its own 2-dual

$$\begin{bmatrix} -294848 & 16320 & 8704 \\ 16320 & 312 & 128 \\ 8704 & 128 & 49 \end{bmatrix} \begin{bmatrix} 20671 & 120 & 24 \\ -5917360 & -34351 & -6870 \\ 11783040 & 68400 & 13679 \end{bmatrix}$$

$$\begin{bmatrix} 115 & 25 & 1 & -1 & -1 & 1 & 1 & 31 & 44 & 61 \\ -32912 & -7154 & -286 & 288 & 287 & -272 & -286 & -8874 & -12597 & -17464 \\ 65518 & 14240 & 569 & -576 & -572 & 544 & 570 & 17680 & 25092 & 34784 \end{bmatrix}$$

W_{216} 4 lattices, $\chi = 18$ 5-gon: $24\infty\!\!\!\!|\ 42| \rtimes D_2$

$L_{216.1}$: $1^{2}_{2}16^{1}_{7}{,}1^{2}9^{-}$ $\quad 18^{b}_{2}2_{4}1^{24,23}_{\infty}4^{*}_{4}2^{s}_{2}$

$$\begin{bmatrix} -262800 & 2016 & 2880 \\ 2016 & -14 & -23 \\ 2880 & -23 & -31 \end{bmatrix} \quad \begin{bmatrix} -2 & -1 & 1 & 5 & 2 \\ -81 & -39 & 40 & 196 & 77 \\ -126 & -64 & 63 & 318 & 128 \end{bmatrix}$$

W_{217} 12 lattices, $\chi = 42$ 10-gon: $2224222242 \rtimes C_2$

$L_{217.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{7}{,}1^{2}3^{-}{,}1^{2}49^{1}\ \langle 2\rangle$ $\quad 196^{b}_{2}6^{s}_{2}98^{b}_{2}2^{*}_{4}4^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -11857562724 & 60807432 & -344568 \\ 60807432 & -311830 & 1767 \\ -344568 & 1767 & -10 \end{bmatrix} \begin{bmatrix} -4842181 & 24831 & -144 \\ -943687080 & 4839285 & -28064 \\ 97381620 & -499379 & 2895 \end{bmatrix}$$

$$\begin{bmatrix} -87 & -1 & 43 & 8 & 151 \\ -16954 & -195 & 8379 & 1559 & 29428 \\ 1960 & 0 & -1078 & -180 & -3074 \end{bmatrix}$$

W_{218} 32 lattices, $\chi = 80$ 12-gon: $62\infty22\infty62\infty22\infty \rtimes C_2$

$L_{218.1}$: $1^{-2}_{\mathrm{II}}8^{1}_{7}{,}1^{-}3^{-}9^{1}{,}1^{-2}25^{-}\ \langle 23,3,2\rangle$ $\quad 6_{6}2^{b}_{2}24^{30,29}_{\infty z}6^{s}_{2}50^{b}_{2}24^{30,11}_{\infty z}\ (\times 2)$

$$\begin{bmatrix} -217800 & 0 & -3600 \\ 0 & 6 & 9 \\ -3600 & 9 & -46 \end{bmatrix} \begin{bmatrix} -28001 & 100 & -310 \\ -2528400 & 9029 & -27993 \\ 1713600 & -6120 & 18971 \end{bmatrix} \quad \begin{bmatrix} 100 & 9 & 19 & 2 & 2 & -1 \\ 9029 & 812 & 1712 & 179 & 175 & -92 \\ -6120 & -551 & -1164 & -123 & -125 & 60 \end{bmatrix}$$

W_{219} 32 lattices, $\chi = 60$ 14-gon: $22222222222222 \rtimes C_2$

$L_{219.1}$: $1^{2}_{2}8^{-}_{5}{,}1^{2}3^{1}{,}1^{-2}25^{-}\ \langle 2\rangle$ $\quad 2^{b}_{2}200^{*}_{2}12^{l}_{2}1^{r}_{2}300^{*}_{2}8^{b}_{2}50^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -11650200 & 23400 & 49800 \\ 23400 & -47 & -100 \\ 49800 & -100 & -199 \end{bmatrix} \begin{bmatrix} 142349 & -286 & -611 \\ 71131200 & -142913 & -305312 \\ -131400 & 264 & 563 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 557 & 79 & 40 & 1931 & 175 & 291 \\ 2997 & 278300 & 39474 & 19988 & 964950 & 87452 & 145425 \\ -5 & -500 & -72 & -37 & -1800 & -164 & -275 \end{bmatrix}$$

$L_{219.2}$: $1^{-2}_{2}8^{1}_{1}{,}1^{2}3^{1}{,}1^{-2}25^{-}\ \langle m\rangle$ $\quad 2^{l}_{2}200_{2}3^{r}_{2}4^{l}_{2}75_{2}8^{r}_{2}50^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -18996430200 & -94509600 & 964800 \\ -94509600 & -470197 & 4800 \\ 964800 & 4800 & -49 \end{bmatrix} \begin{bmatrix} 3894749 & 19377 & -198 \\ -780681000 & -3884013 & 39688 \\ 211182000 & 1050664 & -10737 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 407 & 28 & 55 & 653 & 117 & 191 \\ -1003 & -81600 & -5613 & -11024 & -130875 & -23448 & -38275 \\ 195 & 20200 & 1464 & 3026 & 36900 & 6736 & 11325 \end{bmatrix}$$

W_{220} 8 lattices, $\chi = 48$ — 10-gon: $2\infty2222\infty222 \rtimes C_2$

$L_{220.1} : 1^{-2}_{\mathrm{II}} 32^{1}_{1}, 1^{-}5^{-}25^{-}\ \langle 5\rangle$ — $32^{r}_{2}10^{40,9}_{\infty b}10^{b}_{2}32^{b}_{2}50^{l}_{2}\ (\times 2)$

shares genus with its 5-dual

$$\begin{bmatrix} -1055200 & 7200 & 1600 \\ 7200 & -10 & -15 \\ 1600 & -15 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1600 & 15 & 1 \end{bmatrix} \qquad \begin{bmatrix} -9 & -2 & -1 & -1 & 1 \\ -416 & -93 & -47 & -48 & 45 \\ -4128 & -910 & -450 & -448 & 450 \end{bmatrix}$$

W_{221} 8 lattices, $\chi = 12$ — 4-gon: $\infty 632$

$L_{221.1} : 1^{-2}_{\mathrm{II}} 8^{-}_{5}, 1^{-}3^{-}27^{1}\ \langle 2\rangle$ — $8^{6,5}_{\infty z}2_{6}6^{-}_{3}6^{b}_{2}$

$$\begin{bmatrix} -684504 & 16416 & -3024 \\ 16416 & -390 & 69 \\ -3024 & 69 & -10 \end{bmatrix} \qquad \begin{bmatrix} 1 & -3 & -1 & 3 \\ 52 & -155 & -52 & 155 \\ 56 & -163 & -57 & 162 \end{bmatrix}$$

W_{222} 8 lattices, $\chi = 24$ — 6-gon: $2\infty22\infty2 \rtimes C_2$

$L_{222.1} : 1^{-2}_{\mathrm{II}} 8^{-}_{5}, 1^{-}3^{1}27^{-}\ \langle 2\rangle$ — $54^{b}_{2}8^{6,1}_{\infty z}2^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -1367064 & -460944 & 12096 \\ -460944 & -155418 & 4077 \\ 12096 & 4077 & -106 \end{bmatrix} \begin{bmatrix} -106201 & -35813 & 944 \\ 327600 & 110473 & -2912 \\ 480600 & 162069 & -4273 \end{bmatrix} \qquad \begin{bmatrix} 73 & 35 & -36 \\ -225 & -108 & 111 \\ -324 & -160 & 161 \end{bmatrix}$$

W_{223} 24 lattices, $\chi = 36$ — 10-gon: $2222222222 \rtimes C_2$

$L_{223.1} : 1^{2}_{6}8^{1}_{7}, 1^{1}3^{-}27^{-}\ \langle 2\rangle$ — $216^{*}_{2}4^{*}_{2}24^{b}_{2}54^{s}_{2}6^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -401544 & 3240 & 3240 \\ 3240 & -21 & -27 \\ 3240 & -27 & -26 \end{bmatrix} \begin{bmatrix} 16523 & -126 & -135 \\ 290088 & -2213 & -2370 \\ 1751544 & -13356 & -14311 \end{bmatrix} \qquad \begin{bmatrix} 5 & -1 & -1 & 11 & 7 \\ 72 & -18 & -16 & 198 & 124 \\ 540 & -106 & -108 & 1161 & 741 \end{bmatrix}$$

$L_{223.2} : 1^{-2}_{2}16^{-}_{3}, 1^{-}3^{1}27^{1}\ \langle 2\rangle$ — $432^{r}_{2}2^{b}_{2}48^{*}_{2}108^{l}_{2}3_{2}\ (\times 2)$

shares genus with $L_{223.3}$

$$\begin{bmatrix} 432 & 0 & 432 \\ 0 & -6 & -51 \\ 432 & -51 & -1 \end{bmatrix} \begin{bmatrix} 6479 & -780 & -105 \\ 54864 & -6605 & -889 \\ -7776 & 936 & 125 \end{bmatrix} \qquad \begin{bmatrix} -17 & -2 & 1 & 49 & 13 \\ -144 & -17 & 8 & 414 & 110 \\ 0 & 2 & 0 & -54 & -15 \end{bmatrix}$$

$L_{223.3} : 1^{-2}_{2}16^{-}_{3}, 1^{-}3^{1}27^{1}\ \langle m\rangle$ — $432^{b}_{2}2^{l}_{2}48_{2}27^{r}_{2}12^{*}_{2}\ (\times 2)$

shares genus with $L_{223.2}$

$$\begin{bmatrix} -30672 & 432 & 1728 \\ 432 & -6 & -27 \\ 1728 & -27 & -13 \end{bmatrix} \begin{bmatrix} -10369 & 160 & 120 \\ -654480 & 10099 & 7575 \\ -23328 & 360 & 269 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 1 & -11 & -13 \\ -72 & 63 & 64 & -693 & -820 \\ 0 & 2 & 0 & -27 & -30 \end{bmatrix}$$

W_{224} 60 lattices, $\chi = 18$ — 7-gon: $\rlap{|}2222|222 \rtimes D_2$

$L_{224.1} : 1^{2}_{0}8^{-}_{5}, 1^{2}3^{1}, 1^{2}7^{1}$ — $1^{r}_{2}4^{*}_{2}28^{l}_{2}3^{r}_{2}56^{s}_{2}12^{l}_{2}7_{2}$

$$\begin{bmatrix} -13272 & 168 & 168 \\ 168 & -1 & -4 \\ 168 & -4 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & -5 & -1 & -1 & 1 & 1 \\ 0 & -50 & -252 & -51 & -56 & 48 & 49 \\ -1 & -32 & -154 & -30 & -28 & 30 & 28 \end{bmatrix}$$

$L_{224.2} : [1^{1}2^{-}]_{4}16^{1}_{1}, 1^{2}3^{1}, 1^{2}7^{1}\ \langle 2\rangle$ — $16_{2}1^{r}_{2}112^{*}_{2}12^{*}_{2}56^{s}_{2}48^{l}_{2}7_{2}$

$$\begin{bmatrix} 4368 & -336 & 336 \\ -336 & 10 & -6 \\ 336 & -6 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & -3 & -1 & -1 & 1 & 1 \\ 64 & 0 & -196 & -66 & -70 & 60 & 63 \\ 48 & -1 & -168 & -54 & -56 & 48 & 49 \end{bmatrix}$$

$L_{224.3} : [1^{-}2^{-}]_{0}16^{-}_{5}, 1^{2}3^{1}, 1^{2}7^{1}\ \langle m\rangle$ — $16^{s}_{2}4^{*}_{2}112^{l}_{2}3^{r}_{2}56^{*}_{2}48^{*}_{2}28^{s}_{2}$

$$\begin{bmatrix} 306768 & -1680 & -1680 \\ -1680 & 10 & 8 \\ -1680 & 8 & 11 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -9 & -1 & -3 & -1 & -1 \\ -112 & -110 & -980 & -108 & -322 & -108 & -112 \\ -72 & -74 & -672 & -75 & -224 & -72 & -70 \end{bmatrix}$$

$L_{224.4} : [1^{-}2^{1}]_{2}16^{1}_{7},1^{2}3^{1},1^{2}7^{1}\ \langle m\rangle$ $\qquad 16^{l}_{2}1_{2}112_{2}3_{2}14^{r}_{2}48^{s}_{2}28^{*}_{2}$

$$\begin{bmatrix} 38640 & 336 & -672 \\ 336 & -2 & -4 \\ -672 & -4 & 11 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 5 & 1 & 2 & 1 & -1 \\ -28 & -1 & 112 & 24 & 49 & 24 & -28 \\ -72 & -1 & 336 & 69 & 140 & 72 & -70 \end{bmatrix}$$

$L_{224.5} : [1^{1}2^{1}]_{6}16^{-}_{3},1^{2}3^{1},1^{2}7^{1}$ $\qquad 16^{*}_{2}4^{s}_{2}112^{s}_{2}12^{l}_{2}14_{2}48_{2}7^{r}_{2}$

$$\begin{bmatrix} -134736 & 672 & 1008 \\ 672 & -2 & -8 \\ 1008 & -8 & -1 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 & 1 & -1 & -1 & 1 & 2 \\ 356 & 118 & 112 & -120 & -119 & 120 & 238 \\ 160 & 54 & 56 & -54 & -56 & 48 & 105 \end{bmatrix}$$

W_{225} 120 lattices, $\chi = 72$ $\qquad$ 16-gon: $2222|2222|2222|2222| \rtimes D_4$

$L_{225.1} : 1^{-2}_{4}8^{1}_{1},1^{1}3^{-}9^{-},1^{2}7^{1}\ \langle 3\rangle$ $\qquad 24^{l}_{2}7_{2}72_{2}1^{r}_{2}504^{s}_{2}4^{*}_{2}72^{*}_{2}28^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -2306808 & 5544 & 7056 \\ 5544 & -3 & -21 \\ 7056 & -21 & -20 \end{bmatrix} \begin{bmatrix} 37925 & -84 & -119 \\ 3705912 & -8209 & -11628 \\ 9470664 & -20976 & -29717 \end{bmatrix} \quad \begin{bmatrix} 27 & 16 & 17 & 1 & 1 & -1 & -1 & 11 \\ 2636 & 1561 & 1656 & 97 & 84 & -98 & -96 & 1078 \\ 6744 & 3997 & 4248 & 250 & 252 & -250 & -252 & 2744 \end{bmatrix}$$

$L_{225.2} : [1^{1}2^{1}]_{2}16^{-}_{3},1^{1}3^{-}9^{-},1^{2}7^{1}\ \langle 3,2\rangle 6^{r}_{2}112^{l}_{2}18_{2}1^{r}_{2}504^{*}_{2}16^{s}_{2}72^{*}_{2}28^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} 996912 & -248976 & -1008 \\ -248976 & 62178 & 252 \\ -1008 & 252 & 1 \end{bmatrix} \begin{bmatrix} -22933 & 5649 & 28 \\ -85176 & 20981 & 104 \\ -1598688 & 393816 & 1951 \end{bmatrix} \quad \begin{bmatrix} -11 & -45 & -4 & 0 & 11 & 1 & -5 & -19 \\ -41 & -168 & -15 & 0 & 42 & 4 & -18 & -70 \\ -738 & -2968 & -252 & -1 & 504 & 8 & -468 & -1442 \end{bmatrix}$$

$L_{225.3} : [1^{1}2^{1}]_{0}16^{-}_{5},1^{1}3^{-}9^{-},1^{2}7^{1}\ \langle 32,3,m\rangle$

$24^{s}_{2}112^{*}_{2}72^{l}_{2}1_{2}126^{r}_{2}16^{l}_{2}18_{2}7^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -6762672 & 23184 & 11088 \\ 23184 & -66 & -42 \\ 11088 & -42 & -17 \end{bmatrix} \begin{bmatrix} 34271 & -132 & -52 \\ 3821328 & -14719 & -5798 \\ 12886272 & -49632 & -19553 \end{bmatrix} \begin{bmatrix} 17 & 43 & 13 & 1 & 4 & -1 & -1 & 2 \\ 1894 & 4788 & 1446 & 111 & 441 & -112 & -111 & 224 \\ 6396 & 16184 & 4896 & 377 & 1512 & -376 & -378 & 749 \end{bmatrix}$$

$L_{225.4} : [1^{-}2^{1}]_{6}16^{1}_{7},1^{-}3^{-}9^{1},1^{2}7^{1}\ \langle 3m,3,m\rangle$

$6_{2}63^{r}_{2}8^{*}_{2}144^{s}_{2}56^{*}_{2}36^{l}_{2}2_{2}1008_{2}\ (\times 2)$

$$\begin{bmatrix} -1402128 & -15120 & 6048 \\ -15120 & -138 & 60 \\ 6048 & 60 & -25 \end{bmatrix} \begin{bmatrix} -6581 & -55 & 25 \\ -655368 & -5479 & 2490 \\ -3174192 & -26532 & 12059 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -25 & -9 & -79 & -71 & -41 & -5 & -121 \\ -197 & -2478 & -894 & -7860 & -7070 & -4086 & -499 & -12096 \\ -960 & -12033 & -4336 & -38088 & -34244 & -19782 & -2414 & -58464 \end{bmatrix}$$

$L_{225.5} : [1^{-}2^{1}]_{4}16^{1}_{1},1^{1}3^{-}9^{-},1^{2}7^{1}\ \langle 3m,3\rangle$

$24^{*}_{2}112^{s}_{2}72^{*}_{2}4^{l}_{2}126_{2}16_{2}18^{r}_{2}28^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -249517296 & -3972528 & 294336 \\ -3972528 & -63246 & 4686 \\ 294336 & 4686 & -347 \end{bmatrix} \begin{bmatrix} -163297 & -2598 & 188 \\ 10287648 & 163673 & -11844 \\ 326592 & 5196 & -377 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 71 & 31 & 7 & 25 & -1 & -7 & -17 \\ -1346 & -4564 & -1998 & -452 & -1617 & 64 & 453 & 1106 \\ -360 & -1400 & -684 & -166 & -630 & 16 & 180 & 518 \end{bmatrix}$$

W_{226} 12 lattices, $\chi = 54$ $\qquad$ 12-gon: $422222422222 \rtimes C_2$

$L_{226.1} : 1^{-2}_{\mathrm{II}}4^{1}_{7},1^{2}9^{-},1^{2}19^{-}\ \langle 2\rangle$ $\qquad 4^{*}_{4}2^{s}_{2}38^{s}_{2}18^{b}_{2}2^{s}_{2}342^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -26853156 & -897408 & 53352 \\ -897408 & -29978 & 1783 \\ 53352 & 1783 & -106 \end{bmatrix} \begin{bmatrix} 186731 & 6265 & -371 \\ -186732 & -6266 & 371 \\ 90831780 & 3047475 & -180466 \end{bmatrix} \begin{bmatrix} 3 & -2 & -2 & 16 & 23 & 1001 \\ -2 & 2 & 0 & -18 & -24 & -1026 \\ 1476 & -973 & -1007 & 7749 & 11171 & 486495 \end{bmatrix}$$

W_{227} 6 lattices, $\chi = 8$ 5-gon: $\not{3}22|22 \rtimes D_2$

$L_{227.1} : 1_{\mathrm{II}}^{-2}4_1^1,1^13^-9^1,1^{-2}7^- \ \langle 2\rangle$ $6_3^+6_2^l36_2^r42_2^l4_2^r$

$$\begin{bmatrix} -203868 & -41580 & -24948 \\ -41580 & -8454 & -5115 \\ -24948 & -5115 & -3026 \end{bmatrix} \qquad \begin{bmatrix} -56 & 66 & 47 & -207 & -147 \\ 172 & -203 & -144 & 637 & 452 \\ 171 & -201 & -144 & 630 & 448 \end{bmatrix}$$

W_{228} 6 lattices, $\chi = 8$ 4-gon: $6|62|2 \rtimes D_2$

$L_{228.1} : 1_{\mathrm{II}}^{-2}4_1^1,1^-3^-9^-,1^{-2}7^- \ \langle 2\rangle$ $2_66_618_2^b42_2^b$

$$\begin{bmatrix} -115164 & -4536 & -33264 \\ -4536 & -174 & -1317 \\ -33264 & -1317 & -9598 \end{bmatrix} \qquad \begin{bmatrix} 10 & 26 & -28 & -51 \\ -41 & -110 & 117 & 217 \\ -29 & -75 & 81 & 147 \end{bmatrix}$$

W_{229} 26 lattices, $\chi = 36$ 10-gon: $\not{2}22|22\not{2}22|22 \rtimes D_4$

$L_{229.1} : 1_{\mathrm{II}}^{2}4_5^-,1^13^-9^1,1^27^1 \ \langle 2\rangle$ $36_2^*4_2^*252_2^b6_2^b28_2^* \ (\times 2)$

$$\begin{bmatrix} -1652364 & 4788 & 11088 \\ 4788 & -12 & -33 \\ 11088 & -33 & -74 \end{bmatrix} \begin{bmatrix} 4409 & -12 & -30 \\ 252840 & -689 & -1720 \\ 546840 & -1488 & -3721 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 5 & 2 & 31 \\ -54 & -58 & 252 & 112 & 1764 \\ -126 & -124 & 630 & 249 & 3850 \end{bmatrix}$$

$L_{229.2} : 1_2^28_3^-,1^-3^19^-,1^27^1 \ \langle 2\rangle$ $2_2^s18_2^b56_2^*12_2^*504_2^b \ (\times 2)$

$$\begin{bmatrix} -489384 & -8568 & -9072 \\ -8568 & -150 & -159 \\ -9072 & -159 & -163 \end{bmatrix} \begin{bmatrix} -4411 & -77 & -84 \\ 244440 & 4267 & 4656 \\ 7560 & 132 & 143 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 25 & 9 & 275 \\ 55 & 57 & -1372 & -496 & -15204 \\ 2 & 0 & -56 & -18 & -504 \end{bmatrix}$$

$L_{229.3} : 1_6^{-2}16_7^1,1^13^-9^1,1^27^1 \ \langle 2, m\rangle$ $36_2^l1_21008_2^r6_2^l112_29_2^r4_2^*1008_2^b6_2^b112_2^*$

$$\begin{bmatrix} -216720 & 4032 & 9072 \\ 4032 & -75 & -168 \\ 9072 & -168 & -335 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 19 & 4 & 129 & 20 & 15 & 527 & 9 & 53 \\ -54 & -56 & 1008 & 221 & 7168 & 1113 & 836 & 29400 & 503 & 2968 \\ 0 & 1 & 0 & -3 & -112 & -18 & -14 & -504 & -9 & -56 \end{bmatrix}$$

W_{230} 52 lattices, $\chi = 36$ 10-gon: $\not{2}22|22\not{2}22|22 \rtimes D_4$

$L_{230.1} : 1_2^{-2}8_7^1,1^13^19^1,1^27^1 \ \langle 2m, 2\rangle$ $4_2^*36_2^l7_23_263_2^r \ (\times 2)$

$$\begin{bmatrix} -6148296 & 15120 & 16128 \\ 15120 & -33 & -42 \\ 16128 & -42 & -41 \end{bmatrix} \begin{bmatrix} 44519 & -95 & -125 \\ 6206088 & -13244 & -17425 \\ 11138904 & -23769 & -31276 \end{bmatrix} \qquad \begin{bmatrix} 31 & 91 & 32 & 2 & 2 \\ 4322 & 12684 & 4459 & 278 & 273 \\ 7756 & 22770 & 8008 & 501 & 504 \end{bmatrix}$$

$L_{230.2} : 1_0^28_5^-,1^13^19^1,1^27^1 \ \langle m\rangle$ $4_2^l9_27_2^r12_2^l63_21_2^r36_2^*28_2^l3_2^r252_2^*$

$$\begin{bmatrix} -54728856 & 46872 & 95760 \\ 46872 & -33 & -84 \\ 95760 & -84 & -167 \end{bmatrix} \qquad \begin{bmatrix} 59 & 86 & 60 & 7 & 2 & -1 & -1 & 13 & 6 & 395 \\ 8242 & 12012 & 8379 & 976 & 273 & -140 & -138 & 1820 & 839 & 55188 \\ 29680 & 43263 & 30184 & 3522 & 1008 & -503 & -504 & 6538 & 3018 & 198702 \end{bmatrix}$$

$L_{230.3} : 1_2^28_3^-,1^13^19^1,1^27^1 \ \langle m\rangle$ $1_29_2^r28_2^*12_2^*252_2^l \ (\times 2)$

$$\begin{bmatrix} -2211048 & 7056 & 5040 \\ 7056 & -15 & -18 \\ 5040 & -18 & -11 \end{bmatrix} \begin{bmatrix} 2519 & -11 & -5 \\ 209160 & -914 & -415 \\ 808920 & -3531 & -1606 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & -1 & -1 & 17 \\ 82 & 81 & -84 & -82 & 1428 \\ 323 & 324 & -322 & -324 & 5418 \end{bmatrix}$$

$L_{230.4} : [1^-2^-]_016_5^-,1^13^19^1,1^27^1 \ \langle 2\rangle$

$16_2^s36_2^*112_2^l3_2^r1008_2^*4_2^s144_2^s28_2^*48_2^*252_2^s$

$$\begin{bmatrix} -60073776 & 1097712 & -42336 \\ 1097712 & -20058 & 774 \\ -42336 & 774 & -29 \end{bmatrix} \qquad \begin{bmatrix} 7 & -1 & -37 & -7 & -743 & -53 & -295 & -97 & -5 & 25 \\ 376 & -54 & -1988 & -376 & -39900 & -2846 & -15840 & -5208 & -268 & 1344 \\ -184 & 18 & 952 & 183 & 19656 & 1406 & 7848 & 2590 & 144 & -630 \end{bmatrix}$$

$L_{230.5}$: $[1^{-}2^{1}]_{2}16^{1}_{7},1^{1}3^{1}9^{1},1^{2}7^{1}$ $\langle m \rangle$

$16^{l}_{2}9_{2}112_{2}3_{2}1008_{2}1^{r}_{2}144^{*}_{2}28^{s}_{2}48^{s}_{2}252^{*}_{2}$

$$\begin{bmatrix} -209138832 & -3664080 & -1008 \\ -3664080 & -64194 & -18 \\ -1008 & -18 & 1 \end{bmatrix} \quad \begin{bmatrix} 213 & 170 & 523 & 14 & 209 & 0 & -41 & -13 & 15 & 641 \\ -12156 & -9702 & -29848 & -799 & -11928 & 0 & 2340 & 742 & -856 & -36582 \\ -3992 & -3195 & -9856 & -267 & -4032 & -1 & 792 & 266 & -264 & -11970 \end{bmatrix}$$

$L_{230.6}$: $[1^{1}2^{1}]_{6}16^{-}_{3},1^{1}3^{1}9^{1},1^{2}7^{1}$ $16^{*}_{2}36^{s}_{2}112^{s}_{2}12^{s}_{2}1008^{s}_{2}4^{*}_{2}144^{l}_{2}7_{2}48_{2}63^{r}_{2}$

$$\begin{bmatrix} -1269072 & -35280 & 14112 \\ -35280 & -978 & 390 \\ 14112 & 390 & -155 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 5 & 3 & 223 & 17 & 101 & 18 & 5 & 2 \\ 68 & 72 & -336 & -206 & -15456 & -1180 & -7020 & -1253 & -352 & -147 \\ 80 & 90 & -392 & -246 & -18648 & -1426 & -8496 & -1519 & -432 & -189 \end{bmatrix}$$

$L_{230.7}$: $[1^{1}2^{-}]_{4}16^{1}_{1},1^{1}3^{1}9^{1},1^{2}7^{1}$ $\langle m \rangle$

$16_{2}9^{r}_{2}112^{*}_{2}12^{*}_{2}1008^{l}_{2}1_{2}144_{2}7^{r}_{2}48^{l}_{2}63_{2}$

$$\begin{bmatrix} 248976 & 5040 & -2016 \\ 5040 & 102 & -42 \\ -2016 & -42 & -41 \end{bmatrix} \quad \begin{bmatrix} -97 & -70 & -193 & -5 & 5 & 1 & 1 & -6 & -21 & -164 \\ 4744 & 3423 & 9436 & 244 & -252 & -49 & -48 & 294 & 1028 & 8022 \\ -112 & -81 & -224 & -6 & 0 & 1 & 0 & -7 & -24 & -189 \end{bmatrix}$$

W_{231} 8 lattices, $\chi = 96$ 16-gon: $2\infty2|2\infty22|22\infty2|2\infty22|2 \rtimes D_4$

$L_{231.1}$: $1^{-2}_{2}64^{1}_{1},1^{-}3^{1}9^{1}$ $\langle 3 \rangle$ $576^{*}_{2}12^{48,25}_{\infty z}3^{r}_{2}576^{s}_{2}12^{48,1}_{\infty z}3_{2}576^{r}_{2}2^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 14400 & 0 & -576 \\ 0 & 3 & -3 \\ -576 & -3 & 26 \end{bmatrix} \begin{bmatrix} 383 & 10 & -24 \\ 8832 & 229 & -552 \\ 9792 & 255 & -613 \end{bmatrix} \quad \begin{bmatrix} -349 & -27 & -10 & -181 & -17 & -9 & -205 & -1 \\ -8064 & -622 & -229 & -4128 & -386 & -203 & -4608 & -22 \\ -8928 & -690 & -255 & -4608 & -432 & -228 & -5184 & -25 \end{bmatrix}$$

W_{232} 16 lattices, $\chi = 12$ 6-gon: $2|222|22 \rtimes D_2$

$L_{232.1}$: $[1^{1}2^{-}]_{4}32^{1}_{7},1^{2}3^{-}$ $96^{s}_{2}8^{*}_{2}96^{l}_{2}1_{2}6^{r}_{2}4^{*}_{2}$

$$\begin{bmatrix} 8160 & 3936 & -96 \\ 3936 & 1898 & -46 \\ -96 & -46 & 1 \end{bmatrix} \quad \begin{bmatrix} 31 & -1 & -11 & 0 & 4 & 7 \\ -72 & 2 & 24 & 0 & -9 & -16 \\ -288 & 0 & 48 & -1 & -30 & -58 \end{bmatrix}$$

$L_{232.2}$: $[1^{1}2^{1}]_{0}32^{-}_{3},1^{2}3^{-}$ $96_{2}2^{r}_{2}96^{s}_{2}4^{*}_{2}24^{l}_{2}1_{2}$

$$\begin{bmatrix} -20640 & 384 & 384 \\ 384 & -2 & -8 \\ 384 & -8 & -7 \end{bmatrix} \quad \begin{bmatrix} 19 & 1 & 1 & -1 & -1 & 1 \\ 144 & 7 & 0 & -8 & -6 & 8 \\ 864 & 46 & 48 & -46 & -48 & 45 \end{bmatrix}$$

$L_{232.3}$: $1^{-}_{3}4^{1}_{1}32^{1}_{7},1^{2}3^{1}$ $12^{l}_{2}4_{2}3^{r}_{2}32^{*}_{2}48^{s}_{2}32^{*}_{2}$

$$\begin{bmatrix} -11040 & -288 & 384 \\ -288 & 4 & 8 \\ 384 & 8 & -13 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 1 & -1 & -5 & -5 \\ -12 & 5 & 6 & -4 & -30 & -36 \\ -42 & 32 & 33 & -32 & -168 & -176 \end{bmatrix}$$

$L_{232.4}$: $1^{-}_{3}4^{1}_{7}32^{1}_{1},1^{2}3^{1}$ $12^{*}_{2}16^{l}_{2}3_{2}32_{2}12^{r}_{2}32^{s}_{2}$

$$\begin{bmatrix} 3360 & 384 & 0 \\ 384 & 44 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 1 & 1 & -1 & -3 \\ 6 & -10 & -9 & -8 & 9 & 24 \\ -18 & -8 & -3 & 0 & 0 & -16 \end{bmatrix}$$

W_{233} 32 lattices, $\chi = 24$ 7-gon: $2\infty\!\!\!\!|\;\,222|22 \rtimes D_2$

$L_{233.1}$: $[1^{1}2^{1}]_{2}32^{-}_{5},1^{-}3^{1}9^{1}$ $\langle 3 \rangle$ $32^{l}_{2}3^{24,5}_{\infty}12^{*}_{2}32^{l}_{2}9_{2}2^{r}_{2}36^{*}_{2}$

$$\begin{bmatrix} -3115872 & -67968 & 25344 \\ -67968 & -1482 & 552 \\ 25344 & 552 & -205 \end{bmatrix} \quad \begin{bmatrix} 7 & 3 & 7 & 11 & 1 & -1 & -1 \\ -440 & -190 & -448 & -712 & -69 & 63 & 66 \\ -320 & -141 & -342 & -560 & -63 & 46 & 54 \end{bmatrix}$$

$L_{233.2}$: $[1^{-}2^{1}]_{2}32^{1}_{1},1^{-}3^{1}9^{1}$ $\langle 3 \rangle$ $32_{2}3^{24,17}_{\infty}12^{s}_{2}32^{s}_{2}36^{*}_{2}8^{l}_{2}9_{2}$

$$\begin{bmatrix} -4736736 & 310176 & -9216 \\ 310176 & -20310 & 606 \\ -9216 & 606 & -13 \end{bmatrix} \quad \begin{bmatrix} 67 & 23 & 25 & -17 & -65 & -23 & -1 \\ 1008 & 346 & 376 & -256 & -978 & -346 & -15 \\ -512 & -177 & -198 & 112 & 486 & 176 & 9 \end{bmatrix}$$

$L_{233.3}$: $1^1_1 4^1_7 32^-_3, 1^1 3^- 9^-$ $\langle 3 \rangle$ $\qquad$ $1_2 96^{12,11}_{\infty} 96^s_2 4^*_2 288^s_2 16^*_2 288^l_2$

$$\begin{bmatrix} -277920 & -7776 & 2592 \\ -7776 & -84 & 60 \\ 2592 & 60 & -23 \end{bmatrix} \qquad \begin{bmatrix} 1 & 11 & 5 & -1 & -19 & -3 & -1 \\ 14 & 152 & 64 & -16 & -276 & -42 & -12 \\ 149 & 1632 & 720 & -158 & -2880 & -448 & -144 \end{bmatrix}$$

$L_{233.4}$: $1^-_5 4^1_1 32^1_1, 1^1 3^- 9^-$ $\langle 3 \rangle$ $\qquad$ $1^r_2 96^{24,23}_{\infty z} 96^*_2 4^s_2 288^l_2 4_2 288_2$

$$\begin{bmatrix} 288 & 0 & 0 \\ 0 & -156 & -60 \\ 0 & -60 & -23 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & -3 & -1 & -5 & 0 & 1 \\ -2 & -20 & -4 & 4 & 48 & 3 & 0 \\ 5 & 48 & 0 & -14 & -144 & -8 & 0 \end{bmatrix}$$

W_{234} 6 lattices, $\chi = 12$ $\qquad$ 6-gon: $\not{2}|\not{2}|\not{2}|\not{2}|\not{2}|\not{2}| \rtimes D_{12}$

$L_{234.1}$: $1^2_2 16^-_5, 1^1 3^- 9^1$ $\qquad$ $36^s_2 16^l_2 9_2 1^r_2 144^s_2 4^*_2$

$$\begin{bmatrix} -22320 & 432 & 144 \\ 432 & -3 & -6 \\ 144 & -6 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 1 & 0 & -5 & -1 \\ -36 & 32 & 33 & 0 & -168 & -34 \\ -54 & 56 & 54 & -1 & -288 & -56 \end{bmatrix}$$

$L_{234.2}$: $1^-_5 4^1_1 16^1_1, 1^1 3^- 9^1$ $\langle 3 \rangle$ $\qquad$ $36^r_2 16^s_2 36^l_2 4_2 144_2 1_2$

shares genus with its 2-dual $\cong$ 3-dual; isometric to its own 2.3-dual

$$\begin{bmatrix} -3312 & 720 & 0 \\ 720 & -12 & -12 \\ 0 & -12 & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & -1 & -1 & -5 & 0 \\ 9 & 4 & -6 & -5 & -24 & 0 \\ 108 & 56 & -54 & -56 & -288 & -1 \end{bmatrix}$$

W_{235} 34 lattices, $\chi = 24$ $\qquad$ 8-gon: $2|22|22|22|2 \rtimes D_4$

$L_{235.1}$: $[1^1 2^1]_6 32^-_5, 1^- 3^- 9^-$ $\langle 3 \rangle$ $\qquad$ $288^*_2 8^s_2 288^*_2 24^s_2 32^*_2 72^s_2 32^*_2 24^s_2$

$$\begin{bmatrix} -3168 & -1728 & -864 \\ -1728 & -930 & -456 \\ -864 & -456 & -217 \end{bmatrix} \qquad \begin{bmatrix} 119 & 11 & 53 & -1 & -13 & -13 & 9 & 21 \\ -360 & -34 & -168 & 2 & 40 & 42 & -24 & -62 \\ 288 & 28 & 144 & 0 & -32 & -36 & 16 & 48 \end{bmatrix}$$

$L_{235.2}$: $[1^- 2^1]_2 64^1_1, 1^1 3^1 9^1$ $\langle 3m, 3, m \rangle$ $\qquad$ $576_2 1^r_2 576^*_2 12^s_2 64^s_2 36^*_2 64^l_2 3_2$

shares genus with its 3-dual

$$\begin{bmatrix} 318528 & 159552 & -576 \\ 159552 & 79914 & -288 \\ -576 & -288 & 1 \end{bmatrix} \qquad \begin{bmatrix} 91 & 0 & -23 & -1 & 15 & 17 & 53 & 9 \\ -192 & 0 & 48 & 2 & -32 & -36 & -112 & -19 \\ -2880 & -1 & 576 & 6 & -544 & -558 & -1696 & -285 \end{bmatrix}$$

$L_{235.3}$: $1^1_1 4^1_1 32^-_5, 1^1 3^1 9^1$ $\langle 3 \rangle$ $\qquad$ $36^l_2 4_2 9^r_2 48^*_2 4^l_2 36_2 1^r_2 48^*_2$

$$\begin{bmatrix} -1685088 & 10944 & 10944 \\ 10944 & -60 & -72 \\ 10944 & -72 & -71 \end{bmatrix} \qquad \begin{bmatrix} 29 & 4 & 4 & -1 & -1 & 1 & 3 & 27 \\ 336 & 47 & 48 & -10 & -12 & 9 & 34 & 310 \\ 4122 & 568 & 567 & -144 & -142 & 144 & 427 & 3840 \end{bmatrix}$$

$L_{235.4}$: $1^-_5 4^1_7 32^1_7, 1^1 3^1 9^1$ $\langle 3 \rangle$ $\qquad$ $36^*_2 16^l_2 9_2 12^r_2 4^*_2 144^l_2 1_2 12^r_2$

$$\begin{bmatrix} -2571552 & 27072 & 13536 \\ 27072 & -276 & -144 \\ 13536 & -144 & -71 \end{bmatrix} \qquad \begin{bmatrix} 35 & 9 & 4 & -1 & -1 & 5 & 4 & 17 \\ 828 & 214 & 96 & -23 & -24 & 114 & 94 & 401 \\ 4986 & 1280 & 567 & -144 & -142 & 720 & 571 & 2424 \end{bmatrix}$$

$L_{235.5}$: $1^-_3 8^1_7 64^1_1, 1^1 3^1 9^1$ $\qquad$ $576^r_2 4^b_2 576^s_2 12^s_2 64^b_2 36^l_2 64_2 3_2$

$$\begin{bmatrix} -400320 & -241920 & 4608 \\ -241920 & -146184 & 2784 \\ 4608 & 2784 & -53 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -19 & -1 & 3 & 4 & 9 & 1 \\ 24 & 3 & 48 & 2 & -8 & -9 & -16 & -1 \\ 1152 & 70 & 864 & 18 & -160 & -126 & -64 & 33 \end{bmatrix}$$

W_{236} 8 lattices, $\chi = 48$ $\qquad$ 12-gon: $2|222|222|222|22 \rtimes D_4$

$L_{236.1}$: $1^{-2}_6 64^1_1, 1^1 3^- 9^1$ $\langle 3 \rangle$ $\qquad$ $64^b_2 6^l_2 64_2 9^r_2 64^s_2 36^*_2$ $(\times 2)$

shares genus with its 3-dual

$$\begin{bmatrix} -18800064 & -286272 & 6912 \\ -286272 & -4359 & 105 \\ 6912 & 105 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -6912 & -105 & 1 \end{bmatrix} \qquad \begin{bmatrix} -27 & -5 & -53 & -7 & -13 & 1 \\ 1792 & 332 & 3520 & 465 & 864 & -66 \\ 736 & 147 & 1600 & 216 & 416 & -18 \end{bmatrix}$$

W_{237} 32 lattices, $\chi = 90$ 18-gon: $242222222242222222 \rtimes C_2$

$L_{237.1} : 1_2^2 8_5^-,1^2 9^-,1^2 11^1 \ \langle 2 \rangle$ $396_2^l 1_4 2_2^b 8_2^* 44_2^* 72_2^b 2_2^s 18_2^b 8_2^* \ (\times 2)$

$$\begin{bmatrix} -437976 & 159984 & 0 \\ 159984 & -58438 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -5044843 & 1837617 & 4917 \\ -13814064 & 5031863 & 13464 \\ -13317480 & 4850980 & 12979 \end{bmatrix}$$

$$\begin{bmatrix} 217 & 0 & -42 & -317 & -1165 & -2143 & -340 & -4000 & -20581 \\ 594 & 0 & -115 & -868 & -3190 & -5868 & -931 & -10953 & -56356 \\ 594 & -1 & -114 & -848 & -3102 & -5688 & -900 & -10566 & -54336 \end{bmatrix}$$

$L_{237.2} : 1_{-2}^{-2} 8_1^1,1^2 9^-,1^2 11^1 \ \langle m \rangle$ $99_2^r 4_4^* 2_2^l 8_2 11_2 72_2^r 2_2^b 18_2^l 8_2 \ (\times 2)$

$$\begin{bmatrix} 9379656 & -31680 & 0 \\ -31680 & 107 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1117315 & 3773 & -374 \\ -329506056 & 1112691 & -110296 \\ 13814064 & -46648 & 4623 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 11 & 18 & 61 & 9 & 100 & 507 \\ 297 & 296 & 591 & 3248 & 5313 & 18000 & 2655 & 29493 & 149520 \\ 0 & -2 & -13 & -96 & -176 & -648 & -103 & -1215 & -6256 \end{bmatrix}$$

W_{238} 23 lattices, $\chi = 24$ 8-gon: $2|2|2|2|2|2|2|2| \rtimes D_8$

$L_{238.1} : [1^- 2^1]_2 16_1^1,1^- 3^1 9^- \ \langle 2 \rangle$ $3_2^r 8_2^* 48_2^* 72_2^l \ (\times 2)$

$$\begin{bmatrix} 30096 & 3168 & -576 \\ 3168 & 318 & -60 \\ -576 & -60 & 11 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 96 & 13 & -2 \\ 576 & 84 & -13 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 5 \\ -4 & -6 & -4 & 6 \\ -75 & -88 & 24 & 288 \end{bmatrix}$$

$L_{238.2} : [1^1 2^1]_6 16_5^-,1^- 3^1 9^- \ \langle m \rangle$ $12_2^* 8_2^s 48_2^s 72_2^* \ (\times 2)$

$$\begin{bmatrix} 100944 & 24480 & -720 \\ 24480 & 5934 & -174 \\ -720 & -174 & 5 \end{bmatrix} \begin{bmatrix} -49 & -10 & 0 \\ 240 & 49 & 0 \\ 1440 & 300 & -1 \end{bmatrix} \quad \begin{bmatrix} 7 & 7 & 9 & 1 \\ -34 & -34 & -44 & -6 \\ -174 & -176 & -240 & -72 \end{bmatrix}$$

$L_{238.3} : [1^1 2^-]_4 32_7^1,1^1 3^- 9^1 \ \langle 2, m \rangle$ $96_2^* 36_2^l 6_2 1_2^r 96_2^l 9_2 6_2^r 4_2^*$

$$\begin{bmatrix} 13536 & 864 & 1440 \\ 864 & -282 & 222 \\ 1440 & 222 & 103 \end{bmatrix} \quad \begin{bmatrix} -19 & -59 & -22 & -3 & 63 & 46 & 33 & 17 \\ 56 & 174 & 65 & 9 & -184 & -135 & -97 & -50 \\ 144 & 450 & 168 & 23 & -480 & -351 & -252 & -130 \end{bmatrix}$$

$L_{238.4} : [1^1 2^1]_0 32_3^-,1^1 3^- 9^1 \ \langle m \rangle$ $96_2^s 36_2^* 24_2^l 1_2 96_2 9_2^r 24_2^* 4_2^s$

$$\begin{bmatrix} 37728 & 0 & -288 \\ 0 & -30 & -6 \\ -288 & -6 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -1 & 0 & 3 & 2 & 3 & 1 \\ -32 & 24 & 26 & 0 & -80 & -54 & -82 & -28 \\ 144 & -126 & -132 & -1 & 384 & 261 & 396 & 134 \end{bmatrix}$$

$L_{238.5} : 1_3^- 4_1^1 32_7^1,1^- 3^1 9^- \ \langle m \rangle$ $48_2^* 32_2^l 3_2^r 288_2^* 48_2^s 32_2^* 12_2^* 288_2^s$

$$\begin{bmatrix} -11808 & -1728 & 576 \\ -1728 & -60 & 12 \\ 576 & 12 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & -5 & -3 & -3 & -1 & -1 \\ -62 & -60 & 1 & 324 & 190 & 188 & 62 & 60 \\ -168 & -160 & 3 & 864 & 504 & 496 & 162 & 144 \end{bmatrix}$$

$L_{238.6} : 1_3^- 4_7^1 32_1^1,1^- 3^1 9^-$ $12_2 32_2 3_2 288_2 12_2^r 32_2^s 12_2^s 288_2^l$

$$\begin{bmatrix} 90144 & 0 & -1440 \\ 0 & 12 & 0 \\ -1440 & 0 & 23 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 23 & 5 & 9 & 3 & 7 \\ 1 & 0 & -1 & -24 & -5 & -8 & -2 & 0 \\ 0 & 64 & 63 & 1440 & 312 & 560 & 186 & 432 \end{bmatrix}$$

W_{239} 4 lattices, $\chi = 36$ 9-gon: $22\dagger 2242|24 \rtimes D_2$

$L_{239.1} : 1_2^2 32_1^1,1^2 7^-$ $2_2^b 32_2^* 4_2^l 1_2 32_2^r 2_4 1_2^r 32_2^s 4_4^*$

$$\begin{bmatrix} -231392 & 56672 & 3584 \\ 56672 & -13879 & -879 \\ 3584 & -879 & -54 \end{bmatrix} \quad \begin{bmatrix} 18 & 313 & 155 & 94 & 577 & 84 & 41 & -37 & -17 \\ 70 & 1216 & 602 & 365 & 2240 & 326 & 159 & -144 & -66 \\ 55 & 976 & 486 & 296 & 1824 & 267 & 132 & -112 & -54 \end{bmatrix}$$

W_{240} 8 lattices, $\chi = 18$ 6-gon: $24|422|2 \rtimes D_2$

$L_{240.1} : 1_2^2 16_1^1, 1^2 3^1, 1^2 5^-$ $16_2^* 4_4^* 2_4 1_2 16_2^r 10_2^b$

$$\begin{bmatrix} -1422960 & 4080 & 4800 \\ 4080 & -11 & -15 \\ 4800 & -15 & -14 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & 1 & 3 & 13 & 2 \\ -208 & -210 & 208 & 627 & 2720 & 420 \\ -120 & -118 & 119 & 355 & 1536 & 235 \end{bmatrix}$$

W_{241} 12 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{241.1} : 1_{\mathrm{II}}^{-2} 16_3^-, 1^2 3^1, 1^- 5^- 25^-$ $\langle 5 \rangle$ $48_2^r 10_2^b 1200_2^b 2_2^l 1200_2^r 10_2^b 48_2^b 50_2^l$

$$\begin{bmatrix} -6478800 & 1299600 & 25200 \\ 1299600 & -260690 & -5055 \\ 25200 & -5055 & -98 \end{bmatrix} \qquad \begin{bmatrix} -65 & -9 & -31 & 1 & 101 & 2 & -5 & -12 \\ -288 & -39 & -120 & 5 & 480 & 9 & -24 & -55 \\ -1872 & -305 & -1800 & -1 & 1200 & 50 & -48 & -250 \end{bmatrix}$$

W_{242} 8 lattices, $\chi = 12$ 6-gon: $222|222| \rtimes D_2$

$L_{242.1} : 1_2^{-2} 16_5^-, 1^2 3^1, 1^{-2} 5^1$ $16_2^l 3_2 80_2^r 2_2^b 80_2^* 12_2^s$

$$\begin{bmatrix} 106320 & 17520 & -480 \\ 17520 & 2887 & -79 \\ -480 & -79 & 2 \end{bmatrix} \qquad \begin{bmatrix} 9 & 14 & 103 & 0 & -13 & -1 \\ -56 & -87 & -640 & 0 & 80 & 6 \\ -48 & -75 & -560 & -1 & 40 & 0 \end{bmatrix}$$

W_{243} 8 lattices, $\chi = 12$ 6-gon: $22|222|2 \rtimes D_2$

$L_{243.1} : 1_6^2 16_1^1, 1^2 3^1, 1^{-2} 5^1$ $16_2 3_2^r 80_2^s 12_2^* 16_2^b 30_2^l$

$$\begin{bmatrix} 1595280 & -21360 & 3360 \\ -21360 & 286 & -45 \\ 3360 & -45 & 7 \end{bmatrix} \qquad \begin{bmatrix} -3 & -1 & -1 & 1 & 1 & -1 \\ -224 & -75 & -80 & 72 & 72 & -75 \\ 0 & -3 & -40 & -18 & -16 & 0 \end{bmatrix}$$

W_{244} 16 lattices, $\chi = 24$ 8-gon: $2222|2222| \rtimes D_2$

$L_{244.1} : 1_2^2 16_1^1, 1^- 3^- 9^1, 1^{-2} 5^1$ $\langle 3 \rangle$ $80_2^l 9_2^r 20_2^* 144_2^b 2_2^l 144_2 5_2^r 36_2^s$

$$\begin{bmatrix} -1568880 & 7920 & 4320 \\ 7920 & -39 & -24 \\ 4320 & -24 & -7 \end{bmatrix} \qquad \begin{bmatrix} 3 & -1 & -1 & 5 & 1 & 41 & 7 & 7 \\ 480 & -159 & -160 & 792 & 159 & 6528 & 1115 & 1116 \\ 200 & -72 & -70 & 360 & 71 & 2880 & 490 & 486 \end{bmatrix}$$

W_{245} 16 lattices, $\chi = 24$ 8-gon: $222|2222|2 \rtimes D_2$

$L_{245.1} : 1_6^{-2} 16_5^-, 1^- 3^- 9^1, 1^{-2} 5^1$ $\langle 3 \rangle$ $80_2 9_2 5_2^r 144_2^s 20_2^* 36_2^* 80_2^b 6_2^l$

$$\begin{bmatrix} -488880 & 1440 & 4320 \\ 1440 & -3 & -15 \\ 4320 & -15 & -34 \end{bmatrix} \qquad \begin{bmatrix} 23 & 4 & 2 & 1 & -1 & -1 & 3 & 1 \\ 2960 & 513 & 255 & 120 & -130 & -126 & 400 & 130 \\ 1600 & 279 & 140 & 72 & -70 & -72 & 200 & 69 \end{bmatrix}$$

W_{246} 6 lattices, $\chi = 36$ 8-gon: $26322632 \rtimes C_2$

$L_{246.1} : 1_{\mathrm{II}}^{-2} 4_7^1, 1^- 3^- 81^-$ $\langle 2 \rangle$ $162_2^b 2_6 6_3^- 6_2^s$ $(\times 2)$

$$\begin{bmatrix} -1920996 & 17172 & 602316 \\ 17172 & -138 & -5427 \\ 602316 & -5427 & -188734 \end{bmatrix} \begin{bmatrix} 2411801 & -23467 & -750944 \\ 19702224 & -191705 & -6134528 \\ 7130268 & -69378 & -2220097 \end{bmatrix} \qquad \begin{bmatrix} -274 & 115 & 1377 & 3923 \\ -2241 & 939 & 11248 & 32047 \\ -810 & 340 & 4071 & 11598 \end{bmatrix}$$

W_{247} 4 lattices, $\chi = 24$ 4-gon: $\infty|\infty|\infty|\infty| \rtimes D_4$

$L_{247.1} : 1_{\mathrm{II}}^{-2} 8_1^1, 1^1 5^- 25^1$ $\langle 2 \rangle$ $40_{\infty z}^{10,3} 10_{\infty b}^{20,13}$ $(\times 2)$

$$\begin{bmatrix} -124600 & 2400 & 1800 \\ 2400 & -10 & -35 \\ 1800 & -35 & -26 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -480 & 1 & 7 \\ 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 7 & 3 \\ -4 & 2 \\ 480 & 205 \end{bmatrix}$$

W_{248} 10 lattices, $\chi = 48$ 10-gon: $2\infty 22|22\infty 22|2 \rtimes D_4$

$L_{248.1} : [1^{1}2^{1}]_{0}128^{1}_{1}$ $128^{l}_{2}1^{16,9}_{\infty}4^{*}_{2}128^{s}_{2}8^{*}_{2}\ (\times 2)$

shares genus with main($L_{248.2}$)

$$\begin{bmatrix} 3200 & -384 & -768 \\ -384 & 46 & 92 \\ -768 & 92 & 185 \end{bmatrix} \begin{bmatrix} 255 & -28 & -64 \\ 1152 & -127 & -288 \\ 512 & -56 & -129 \end{bmatrix} \quad \begin{bmatrix} -117 & -8 & -15 & -101 & -3 \\ -480 & -34 & -68 & -480 & -18 \\ -256 & -17 & -30 & -192 & -4 \end{bmatrix}$$

$L_{248.2} : [1^{1}2^{1}]_{0}256^{1}_{1}\ \langle m* \rangle$ $256^{s}_{2}8^{32,1}_{\infty z}2_{2}256_{2}1^{r}_{2}256^{*}_{2}8^{32,17}_{\infty z}2^{r}_{2}256^{s}_{2}4^{*}_{2}$

$$\begin{bmatrix} 4352 & 0 & -256 \\ 0 & 2 & -2 \\ -256 & -2 & 17 \end{bmatrix} \quad \begin{bmatrix} -73 & -11 & -6 & -89 & -1 & -23 & -1 & 0 & -7 & -1 \\ -1088 & -162 & -87 & -1280 & -14 & -320 & -14 & -1 & -128 & -16 \\ -1280 & -192 & -104 & -1536 & -17 & -384 & -16 & 0 & -128 & -18 \end{bmatrix}$$

$L_{248.3} : 1^{1}_{1}8^{1}_{7}128^{1}_{1}$ $128_{2}1^{8,1}_{\infty}4^{s}_{2}128^{b}_{2}8^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} 4224 & -1024 & -896 \\ -1024 & 248 & 216 \\ -896 & 216 & 185 \end{bmatrix} \begin{bmatrix} -289 & 66 & 48 \\ -1824 & 417 & 304 \\ 768 & -176 & -129 \end{bmatrix} \quad \begin{bmatrix} 111 & 7 & 11 & 63 & 0 \\ 672 & 43 & 70 & 416 & 3 \\ -256 & -17 & -30 & -192 & -4 \end{bmatrix}$$

$L_{248.4} : 1^{1}_{7}8^{1}_{1}128^{1}_{1}$ $128^{r}_{2}4^{8,1}_{\infty a}4^{b}_{2}128^{l}_{2}8_{2}\ (\times 2)$

$$\begin{bmatrix} 254080 & 5888 & -3456 \\ 5888 & 136 & -80 \\ -3456 & -80 & 47 \end{bmatrix} \begin{bmatrix} 671 & 14 & -9 \\ 672 & 13 & -9 \\ 51072 & 1064 & -685 \end{bmatrix} \quad \begin{bmatrix} -33 & -4 & -3 & -17 & 0 \\ -64 & -5 & 1 & 32 & 5 \\ -2560 & -306 & -222 & -1216 & 8 \end{bmatrix}$$

W_{249} 60 lattices, $\chi = 30$ 9-gon: $222|2222|2 \rtimes D_2$

$L_{249.1} : 1^{2}_{0}8^{1}_{1},1^{2}3^{-},1^{2}11^{1}$ $132^{*}_{2}8^{*}_{2}44^{s}_{2}24^{l}_{2}11_{2}8_{2}33^{r}_{2}4^{l}_{2}1^{r}_{2}$

$$\begin{bmatrix} -217272 & 792 & 528 \\ 792 & -1 & -4 \\ 528 & -4 & 1 \end{bmatrix} \quad \begin{bmatrix} -5 & -1 & -1 & 1 & 5 & 5 & 14 & 1 & 0 \\ -858 & -172 & -176 & 168 & 847 & 848 & 2376 & 170 & 0 \\ -792 & -156 & -154 & 156 & 770 & 768 & 2145 & 152 & -1 \end{bmatrix}$$

$L_{249.2} : [1^{1}2^{1}]_{2}16^{1}_{7},1^{2}3^{-},1^{2}11^{1}\ \langle 2 \rangle$ $33_{2}2^{r}_{2}176^{l}_{2}6_{2}11^{r}_{2}8^{*}_{2}528^{l}_{2}1^{r}_{2}16^{l}_{2}$

$$\begin{bmatrix} -1833744 & 4752 & 8448 \\ 4752 & -10 & -24 \\ 8448 & -24 & -37 \end{bmatrix} \quad \begin{bmatrix} 61 & 10 & 73 & 1 & -1 & -1 & 13 & 1 & 11 \\ 7953 & 1303 & 9504 & 129 & -132 & -130 & 1716 & 131 & 1436 \\ 8745 & 1434 & 10472 & 144 & -143 & -144 & 1848 & 143 & 1576 \end{bmatrix}$$

$L_{249.3} : [1^{1}2^{1}]_{0}16^{1}_{1},1^{2}3^{-},1^{2}11^{1}\ \langle m \rangle$ $33^{r}_{2}8^{*}_{2}176^{s}_{2}24^{*}_{2}44^{l}_{2}2_{2}528_{2}1_{2}16_{2}$

$$\begin{bmatrix} 528 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} -8 & -3 & -13 & -1 & -1 & 0 & 1 & 0 & -1 \\ -99 & -34 & -132 & -6 & 0 & 1 & 0 & -1 & -16 \\ -231 & -84 & -352 & -24 & -22 & 0 & 0 & -1 & -32 \end{bmatrix}$$

$L_{249.4} : [1^{-}2^{1}]_{6}16^{-}_{3},1^{2}3^{-},1^{2}11^{1}\ \langle m \rangle$ $132^{l}_{2}2_{2}176_{2}6^{r}_{2}44^{*}_{2}8^{s}_{2}528^{*}_{2}4^{*}_{2}16^{*}_{2}$

$$\begin{bmatrix} -243408 & 1584 & 1584 \\ 1584 & -10 & -12 \\ 1584 & -12 & -1 \end{bmatrix} \quad \begin{bmatrix} 83 & 7 & 53 & 1 & -1 & -1 & 1 & 1 & 7 \\ 10758 & 907 & 6864 & 129 & -132 & -130 & 132 & 130 & 908 \\ 1914 & 162 & 1232 & 24 & -22 & -24 & 0 & 22 & 160 \end{bmatrix}$$

$L_{249.5} : [1^{-}2^{1}]_{4}16^{-}_{5},1^{2}3^{-},1^{2}11^{1}$ $132^{*}_{2}8^{s}_{2}176^{*}_{2}24^{l}_{2}11_{2}2^{r}_{2}528^{s}_{2}4^{s}_{2}16^{s}_{2}$

$$\begin{bmatrix} 19536 & 4752 & 1056 \\ 4752 & 1154 & 238 \\ 1056 & 238 & -131 \end{bmatrix} \quad \begin{bmatrix} 157 & 9 & -639 & -347 & -1191 & -558 & -12061 & -197 & -19 \\ -660 & -38 & 2684 & 1458 & 5005 & 2345 & 50688 & 828 & 80 \\ 66 & 4 & -264 & -144 & -495 & -232 & -5016 & -82 & -8 \end{bmatrix}$$

W_{250} 8 lattices, $\chi = 8$ 5-gon: $2|22|2 \rtimes D_2$

$L_{250.1} : 1^{-2}_{\mathrm{II}}8^{-}_{3},1^{1}3^{-}9^{1},1^{-2}5^{-}\ \langle 2 \rangle$ $90^{b}_{2}6^{+}_{3}6^{b}_{2}10^{l}_{2}24^{r}_{2}$

$$\begin{bmatrix} -377640 & 5040 & -56160 \\ 5040 & -66 & 765 \\ -56160 & 765 & -8162 \end{bmatrix} \quad \begin{bmatrix} 28 & 15 & -14 & -39 & -45 \\ 1095 & 589 & -548 & -1530 & -1768 \\ -90 & -48 & 45 & 125 & 144 \end{bmatrix}$$

W_{251} 8 lattices, $\chi = 8$ 4-gon: $26|62| \rtimes D_2$

$L_{251.1}$: $1_{\mathrm{II}}^{-2}8_{3}^{-},1^{-}3^{-}9^{-},1^{-2}5^{-}$ $\langle 2\rangle$ $24_2^r 18_6 6_6 2_2^l$

$$\begin{bmatrix} -1572840 & 10440 & 213120 \\ 10440 & -66 & -1515 \\ 213120 & -1515 & -25822 \end{bmatrix} \quad \begin{bmatrix} -81 & -79 & 80 & 26 \\ -7304 & -7125 & 7214 & 2345 \\ -240 & -234 & 237 & 77 \end{bmatrix}$$

W_{252} 16 lattices, $\chi = 18$ 7-gon: $22{\nmid}222|2 \rtimes D_2$

$L_{252.1}$: $1_{2}^{-2}8_{5}^{-},1^{-}3^{-}9^{-},1^{2}5^{1}$ $\langle 2\rangle$ $20_2^* 72_2^b 2_2^b 18_2^b 8_2^* 180_2^s 24_2^s$

$$\begin{bmatrix} 49320 & -4320 & -360 \\ -4320 & 375 & 33 \\ -360 & 33 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 & 0 & -7 & -17 & -67 & -3 \\ 10 & 48 & 0 & -66 & -160 & -630 & -28 \\ 20 & 108 & -1 & -171 & -412 & -1620 & -72 \end{bmatrix}$$

$L_{252.2}$: $1_{2}^{2}8_{1}^{1},1^{-}3^{-}9^{-},1^{2}5^{1}$ $\langle m\rangle$ $5_2 72_2^r 2_2^s 18_2^l 8_2 45_2^r 24_2^l$

$$\begin{bmatrix} 105480 & 4320 & 360 \\ 4320 & 177 & 15 \\ 360 & 15 & 2 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 0 & -8 & -21 & -43 & -5 \\ 25 & -24 & 0 & 198 & 520 & 1065 & 124 \\ -5 & 0 & -1 & -45 & -112 & -225 & -24 \end{bmatrix}$$

W_{253} 4 lattices, $\chi = 36$ 8-gon: $24{\nmid}\infty 422{\nmid}2 \rtimes D_2$

$L_{253.1}$: $1_{2}^{2}32_{7}^{1},1^{2}9^{1}$ $36_2^l 1_4 2_{\infty b}^{24,23} 2_4^* 4_2^l 9_2 1_2^r 4_2^*$

$$\begin{bmatrix} -525600 & 3744 & 2016 \\ 3744 & -23 & -16 \\ 2016 & -16 & -7 \end{bmatrix} \quad \begin{bmatrix} 43 & 12 & 6 & 1 & -1 & -1 & 1 & 7 \\ 2736 & 763 & 381 & 63 & -64 & -63 & 64 & 446 \\ 6102 & 1704 & 853 & 143 & -142 & -144 & 141 & 992 \end{bmatrix}$$

W_{254} 22 lattices, $\chi = 30$ 9-gon: $22{\nmid}2222|22 \rtimes D_2$

$L_{254.1}$: $1_{\mathrm{II}}^{2}4_{1}^{1},1^{1}3^{1}9^{1},1^{2}11^{-}$ $\langle 2\rangle$ $22_2^b 12_2^* 4_2^* 36_2^* 12_2^b 198_2^l 4_2^r 66_2^l 36_2^r$

$$\begin{bmatrix} -489852 & 1980 & 1188 \\ 1980 & 12 & -9 \\ 1188 & -9 & -2 \end{bmatrix} \quad \begin{bmatrix} 15 & 7 & 1 & -1 & -1 & 1 & 1 & 9 & 11 \\ 946 & 440 & 62 & -66 & -64 & 66 & 64 & 572 & 696 \\ 4565 & 2130 & 304 & -306 & -306 & 297 & 304 & 2739 & 3348 \end{bmatrix}$$

$L_{254.2}$: $1_{2}^{2}8_{7}^{1},1^{-}3^{-}9^{-},1^{2}11^{1}$ $\langle 2\rangle$ $396_2^* 24_2^b 18_2^s 2_2^b 24_2^* 44_2^s 72_2^l 33_2^r 8_2^s$

$$\begin{bmatrix} -490248 & 1584 & 1584 \\ 1584 & -3 & -9 \\ 1584 & -9 & 2 \end{bmatrix} \quad \begin{bmatrix} -41 & -7 & -2 & 0 & 1 & 1 & -1 & -3 & -3 \\ -8250 & -1408 & -402 & 0 & 200 & 198 & -204 & -605 & -604 \\ -4554 & -780 & -225 & -1 & 108 & 110 & -108 & -330 & -332 \end{bmatrix}$$

$L_{254.3}$: $1_{2}^{-2}8_{3}^{-},1^{-}3^{-}9^{-},1^{2}11^{1}$ $\langle m\rangle$ $99_2 24_2^r 18_2^b 2_2^l 24_2 11_2^r 72_2^s 132_2^s 8_2^l$

$$\begin{bmatrix} 127512 & -3960 & 0 \\ -3960 & 123 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -1 & 0 & 3 & 4 & 7 & 7 & 1 \\ 33 & -32 & -33 & -1 & 88 & 121 & 216 & 220 & 32 \\ 0 & 0 & -9 & -11 & -96 & -88 & -108 & -66 & -4 \end{bmatrix}$$

W_{255} 24 lattices, $\chi = 27$ 8-gon: 42222222

$L_{255.1}$: $1_{\mathrm{II}}^{-2}4_{7}^{1},1^{2}9^{1},1^{2}5^{-},1^{2}7^{1}$ $\langle 2\rangle$ $2_4^* 4_2^* 36_2^b 14_2^s 90_2^l 28_2^r 10_2^l 252_2^r$

$$\begin{bmatrix} -102537540 & 52920 & 63000 \\ 52920 & -26 & -35 \\ 63000 & -35 & -34 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -1 & 3 & 23 & 23 & 6 & 85 \\ 1188 & -1190 & -1188 & 3570 & 27360 & 27356 & 7135 & 101052 \\ 629 & -628 & -630 & 1883 & 14445 & 14448 & 3770 & 53424 \end{bmatrix}$$

W_{256} 24 lattices, $\chi = 48$ 12-gon: $222222222222 \rtimes C_2$

$L_{256.1}$: $1_{\mathrm{II}}^{-2}4_{7}^{1},1^{2}9^{1},1^{-2}5^{1},1^{-2}7^{-}$ $\langle 2\rangle$ $2_2^s 630_2^b 4_2^b 70_2^b 36_2^* 20_2^b$ $(\times 2)$

$$\begin{bmatrix} -658980 & 1260 & 2520 \\ 1260 & -2 & -7 \\ 2520 & -7 & 2 \end{bmatrix} \begin{bmatrix} -3961 & 10 & 2 \\ -1510740 & 3814 & 763 \\ -291060 & 735 & 146 \end{bmatrix} \quad \begin{bmatrix} -2 & -52 & -1 & -1 & 1 & 1 \\ -763 & -19845 & -382 & -385 & 378 & 380 \\ -146 & -3780 & -72 & -70 & 72 & 70 \end{bmatrix}$$

W_{257} 16 lattices, $\chi = 24$ 8-gon: $222|2222|2 \rtimes D_2$

$L_{257.1} : [1^1 2^1]_0 32^-_5, 1^{-2} 5^-$ $160_2 1^r_2 32^*_2 40^s_2 32^*_2 4^s_2 160^l_2 2_2$

$$\begin{bmatrix} -70240 & 800 & 800 \\ 800 & -2 & -10 \\ 800 & -10 & -9 \end{bmatrix} \quad \begin{bmatrix} 101 & 7 & 19 & 7 & -1 & -1 & 1 & 1 \\ 960 & 67 & 184 & 70 & -8 & -10 & 0 & 9 \\ 7840 & 543 & 1472 & 540 & -80 & -78 & 80 & 78 \end{bmatrix}$$

$L_{257.2} : [1^1 2^-]_4 32^1_1, 1^{-2} 5^-$ $160^*_2 4^s_2 32^l_2 10_2 32_2 1^r_2 160^*_2 8^s_2$

$$\begin{bmatrix} 67360 & -1600 & -160 \\ -1600 & 38 & 4 \\ -160 & 4 & -7 \end{bmatrix} \quad \begin{bmatrix} -101 & -13 & -15 & -1 & 5 & 1 & -1 & -3 \\ -4280 & -552 & -640 & -45 & 208 & 42 & -40 & -126 \\ -240 & -34 & -48 & -10 & 0 & 1 & 0 & -4 \end{bmatrix}$$

$L_{257.3} : 1^-_5 4^1_1 32^1_7, 1^{-2} 5^1$ $20^*_2 32^l_2 1_2 20^r_2 4^*_2 32^l_2 5_2 4^r_2$

$$\begin{bmatrix} -219680 & 1920 & 2080 \\ 1920 & -12 & -20 \\ 2080 & -20 & -19 \end{bmatrix} \quad \begin{bmatrix} 29 & 29 & 2 & 1 & -1 & -1 & 2 & 2 \\ 840 & 836 & 57 & 25 & -30 & -28 & 60 & 59 \\ 2270 & 2272 & 157 & 80 & -78 & -80 & 155 & 156 \end{bmatrix}$$

$L_{257.4} : 1^-_5 4^1_7 32^1_1, 1^{-2} 5^1$ $5_2 32_2 1^r_2 80^*_2 4^s_2 32^s_2 20^*_2 16^l_2$

$$\begin{bmatrix} -593120 & 39840 & -2560 \\ 39840 & -2676 & 172 \\ -2560 & 172 & -11 \end{bmatrix} \quad \begin{bmatrix} -9 & -17 & -1 & 1 & 1 & 1 & -3 & -3 \\ -130 & -248 & -15 & 10 & 14 & 16 & -40 & -42 \\ 55 & 64 & -3 & -80 & -14 & 16 & 70 & 40 \end{bmatrix}$$

W_{258} 8 lattices, $\chi = 36$ 8-gon: $24|42|24|42| \rtimes D_4$

$L_{258.1} : 1^2_2 16^-_3, 1^2 3^-, 1^2 7^1$ $112^l_2 1_4 2^*_4 4^s_2$ $(\times 2)$

$$\begin{bmatrix} -2258256 & 557424 & 22176 \\ 557424 & -137591 & -5477 \\ 22176 & -5477 & -214 \end{bmatrix} \begin{bmatrix} 742937 & -183749 & -6859 \\ 2914128 & -720745 & -26904 \\ 2403744 & -594512 & -22193 \end{bmatrix} \quad \begin{bmatrix} 5211 & 297 & 78 & -77 \\ 20440 & 1165 & 306 & -302 \\ 16856 & 960 & 251 & -250 \end{bmatrix}$$

W_{259} 8 lattices, $\chi = 16$ 6-gon: 222226

$L_{259.1} : 1^{-2}_{\mathrm{II}} 16^1_1, 1^2 3^-, 1^{-2} 7^-$ $6^b_2 16^b_2 42^l_2 16^r_2 6^b_2 2_6$

$$\begin{bmatrix} 2549904 & -6384 & -3696 \\ -6384 & -2 & 33 \\ -3696 & 33 & -26 \end{bmatrix} \quad \begin{bmatrix} 2 & 7 & -1 & -21 & -10 & -3 \\ 555 & 1944 & -273 & -5824 & -2775 & -833 \\ 420 & 1472 & -210 & -4416 & -2103 & -631 \end{bmatrix}$$

W_{260} 8 lattices, $\chi = 18$ 7-gon: $22|222\rlap{|}22 \rtimes D_2$

$L_{260.1} : 1^{-2}_6 16^1_7, 1^2 3^-, 1^2 7^1$ $112^b_2 6^s_2 14^b_2 6^l_2 112_2 1^r_2 4^*_2$

$$\begin{bmatrix} 76272 & 25200 & -336 \\ 25200 & 8326 & -111 \\ -336 & -111 & 1 \end{bmatrix} \quad \begin{bmatrix} 831 & 92 & 30 & -1 & -37 & 0 & 31 \\ -2520 & -279 & -91 & 3 & 112 & 0 & -94 \\ -448 & -48 & -14 & 0 & 0 & -1 & -18 \end{bmatrix}$$

W_{261} 16 lattices, $\chi = 72$ 16-gon: $2222|2222|2222|2222| \rtimes D_4$

$L_{261.1} : 1^2_6 16^-_3, 1^1 3^1 9^-, 1^2 7^1$ $\langle 3 \rangle$ $112^s_2 12^l_2 7_2 48^r_2 126^b_2 48^*_2 28^l_2 3^r_2$ $(\times 2)$

$$\begin{bmatrix} -3865680 & 8064 & 3024 \\ 8064 & 3 & -9 \\ 3024 & -9 & -2 \end{bmatrix} \begin{bmatrix} -15553 & 4 & 16 \\ -2013984 & 517 & 2072 \\ -14614992 & 3759 & 15035 \end{bmatrix}$$

$$\begin{bmatrix} -9 & -5 & -17 & -59 & -71 & -65 & -41 & -4 \\ -1176 & -650 & -2205 & -7648 & -9198 & -8416 & -5306 & -517 \\ -8456 & -4698 & -15974 & -55440 & -66717 & -61080 & -38528 & -3759 \end{bmatrix}$$

W_{262} 16 lattices, $\chi = 72$ 16-gon: $222|2222|2222|2222|2 \rtimes D_4$

$L_{262.1} : 1^{-2}_{2} 16^{1}_{7}, 1^{1} 3^{1} 9^{-}, 1^{2} 7^{1}\ \langle 3 \rangle$ $112_{2} 3_{2} 7^{r}_{2} 48^{s}_{2} 28^{*}_{2} 12^{*}_{2} 112^{b}_{2} 18^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} 2339568 & 5040 & -7056 \\ 5040 & -6 & -9 \\ -7056 & -9 & 19 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 28560 & 139 & -115 \\ 34272 & 168 & -139 \end{bmatrix} \quad \begin{bmatrix} 13 & 1 & 2 & 3 & 3 & 1 & 1 & -1 \\ 2128 & 163 & 322 & 472 & 434 & 116 & -392 & -255 \\ 5824 & 447 & 889 & 1320 & 1274 & 390 & -224 & -558 \end{bmatrix}$$

W_{263} 22 lattices, $\chi = 36$ 7-gon: $2\infty\infty\infty222$

$L_{263.1} : 1^{2}_{\mathrm{II}} 4^{1}_{7}, 1^{-} 7^{1} 49^{1}\ \langle 2 \rangle$ $98^{l}_{2} 28^{7,1}_{\infty} 28^{7,4}_{\infty b} 28^{7,4}_{\infty} 28^{*}_{2} 196^{b}_{2} 14^{s}_{2}$

$$\begin{bmatrix} 2940 & 1568 & -588 \\ 1568 & 840 & -315 \\ -588 & -315 & 118 \end{bmatrix} \quad \begin{bmatrix} -5 & -3 & 1 & 3 & 3 & 1 & -1 \\ 28 & 16 & -2 & 16 & 54 & 112 & 10 \\ 49 & 28 & 0 & 56 & 154 & 294 & 21 \end{bmatrix}$$

$L_{263.2} : 1^{2}_{6} 8^{1}_{1}, 1^{-} 7^{1} 49^{1}\ \langle 2 \rangle$ $196^{s}_{2} 56^{28,1}_{\infty z} 14^{28,25}_{\infty a} 56^{28,25}_{\infty z} 14^{l}_{2} 392_{2} 7^{r}_{2}$

$$\begin{bmatrix} -526456 & -258328 & -68992 \\ -258328 & -126749 & -33845 \\ -68992 & -33845 & -9034 \end{bmatrix} \quad \begin{bmatrix} -135 & -71 & 68 & 903 & 907 & 2869 & 36 \\ 406 & 212 & -204 & -2700 & -2710 & -8568 & -107 \\ -490 & -252 & 245 & 3220 & 3227 & 10192 & 126 \end{bmatrix}$$

$L_{263.3} : 1^{-2}_{6} 8^{-}_{5}, 1^{-} 7^{1} 49^{1}\ \langle m \rangle$ $49^{r}_{2} 56^{28,15}_{\infty z} 14^{28,25}_{\infty b} 56^{28,11}_{\infty z} 14^{b}_{2} 392^{*}_{2} 28^{l}_{2}$

$$\begin{bmatrix} -756952 & -80752 & -25872 \\ -80752 & -8610 & -2765 \\ -25872 & -2765 & -879 \end{bmatrix} \quad \begin{bmatrix} -36 & -21 & 29 & 281 & 259 & 769 & 7 \\ 259 & 152 & -209 & -2032 & -1875 & -5572 & -52 \\ 245 & 140 & -196 & -1876 & -1722 & -5096 & -42 \end{bmatrix}$$

W_{264} 3 lattices, $\chi = 24$ 8-gon: $\not2 2 \not2 2 \not2 2 \not2 2 \rtimes D_4$

$L_{264.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{3}, 1^{1} 7^{-} 49^{1}\ \langle 2 \rangle$ $196^{*}_{2} 4^{b}_{2} 98^{b}_{2} 2^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -140532 & 2352 & 980 \\ 2352 & -14 & -21 \\ 980 & -21 & -6 \end{bmatrix} \begin{bmatrix} -1177 & 15 & 9 \\ -21560 & 274 & 165 \\ -117992 & 1505 & 902 \end{bmatrix} \quad \begin{bmatrix} -45 & -7 & -20 & -1 \\ -826 & -128 & -364 & -18 \\ -4508 & -702 & -2009 & -101 \end{bmatrix}$$

W_{265} 6 lattices, $\chi = 28$ 6-gon: $\not2 6|6 \not2 6|6 \rtimes D_4$

$L_{265.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{3}, 1^{-} 3^{-} 9^{-}, 1^{2} 13^{1}\ \langle 2 \rangle$ $18^{b}_{2} 2_{6} 6_{6}\ (\times 2)$

$$\begin{bmatrix} -2291796 & 17784 & 8892 \\ 17784 & -138 & -69 \\ 8892 & -69 & -34 \end{bmatrix} \begin{bmatrix} 2573 & -20 & -9 \\ 344916 & -2681 & -1206 \\ -30888 & 240 & 107 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -129 & -1 & 130 \\ 0 & 2 & -3 \end{bmatrix}$$

W_{266} 6 lattices, $\chi = 14$ 6-gon: $\not2 22 \not2 22 \rtimes D_2$

$L_{266.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{3}, 1^{1} 3^{-} 9^{1}, 1^{2} 13^{1}\ \langle 2 \rangle$ $36^{*}_{2} 4^{*}_{2} 468^{b}_{2} 6^{-}_{3} 6^{b}_{2} 52^{*}_{2}$

$$\begin{bmatrix} -2535156 & 9828 & 9828 \\ 9828 & -30 & -39 \\ 9828 & -39 & -38 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 7 & 2 & 3 & 11 \\ -24 & -26 & 156 & 50 & 77 & 286 \\ -234 & -232 & 1638 & 465 & 696 & 2548 \end{bmatrix}$$

W_{267} 44 lattices, $\chi = 72$ 12-gon: $2\infty|\infty22|22\infty|\infty22|2 \rtimes D_4$

$L_{267.1} : 1^{2}_{\mathrm{II}} 4^{1}_{7}, 1^{1} 3^{-} 9^{-}, 1^{-} 5^{-} 25^{-}\ \langle 23, 3, 2 \rangle$ $450^{l}_{2} 60^{15,1}_{\infty} 60^{15,8}_{\infty b} 60^{r}_{2} 18^{b}_{2} 10^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} 13974300 & -264600 & -19800 \\ -264600 & 5010 & 375 \\ -19800 & 375 & 28 \end{bmatrix} \begin{bmatrix} -2021 & 38 & 3 \\ -78780 & 1481 & 117 \\ -363600 & 6840 & 539 \end{bmatrix} \quad \begin{bmatrix} 7 & 5 & 3 & 11 & 5 & 2 \\ 195 & 176 & 124 & 476 & 219 & 89 \\ 2250 & 1140 & 450 & 1380 & 594 & 220 \end{bmatrix}$$

$L_{267.2}$: $1^{2}_{6}8^{1}_{1},1^{-}3^{1}9^{1},1^{1}5^{1}25^{1}$ $\langle 3m,3,2\rangle$

$$36^{s}_{2}120^{60,49}_{\infty z}30^{60,17}_{\infty a}120^{s}_{2}900^{*}_{2}20^{*}_{2}\ (\times 2)$$

$$\begin{bmatrix} -11417400 & -5657400 & -703800 \\ -5657400 & -2803245 & -348705 \\ -703800 & -348705 & -43354 \end{bmatrix} \begin{bmatrix} 7762399 & 3846770 & 478890 \\ -18108960 & -8974159 & -1117206 \\ 19641600 & 9733680 & 1211759 \end{bmatrix}$$

$$\begin{bmatrix} 2513 & 8517 & 2686 & 25491 & 63487 & 5959 \\ -5862 & -19868 & -6266 & -59468 & -148110 & -13902 \\ 6354 & 21540 & 6795 & 64500 & 160650 & 15080 \end{bmatrix}$$

$L_{267.3}$: $1^{-2}_{6}8^{-}_{5},1^{-}3^{1}9^{1},1^{1}5^{1}25^{1}$ $\langle 32,3,m\rangle$

$$9^{r}_{2}120^{60,19}_{\infty z}30^{60,17}_{\infty b}120^{l}_{2}225_{2}5_{2}\ (\times 2)$$

$$\begin{bmatrix} -16061400 & -797400 & -266400 \\ -797400 & -39570 & -13245 \\ -266400 & -13245 & -4399 \end{bmatrix} \begin{bmatrix} 1343899 & 67195 & 21805 \\ -20439360 & -1021969 & -331632 \\ -19841400 & -992070 & -321931 \end{bmatrix}$$

$$\begin{bmatrix} 169 & 1167 & 376 & 3621 & 4526 & 427 \\ -2571 & -17752 & -5719 & -55072 & -68835 & -6494 \\ -2493 & -17220 & -5550 & -53460 & -66825 & -6305 \end{bmatrix}$$

W_{268} 44 lattices, $\chi = 36$ 8-gon: $\infty 222|222\infty| \rtimes D_2$

$L_{268.1}$: $1^{2}_{\mathrm{II}}4^{1}_{7},1^{-}3^{-}9^{1},1^{1}5^{-}25^{1}$ $\langle 23,3,2\rangle$

$$60^{15,8}_{\infty}60^{*}_{2}900^{b}_{2}6^{s}_{2}90^{s}_{2}150^{b}_{2}36^{*}_{2}60^{15,4}_{\infty b}$$

$$\begin{bmatrix} -188100 & 6300 & 0 \\ 6300 & -210 & -15 \\ 0 & -15 & 224 \end{bmatrix} \quad \begin{bmatrix} 1 & -15 & -1 & 6 & 46 & 156 & 151 & 111 \\ 28 & -448 & -30 & 179 & 1371 & 4645 & 4494 & 3302 \\ 0 & -30 & 0 & 12 & 90 & 300 & 288 & 210 \end{bmatrix}$$

$L_{268.2}$: $1^{2}_{6}8^{1}_{1},1^{1}3^{1}9^{-},1^{-}5^{1}25^{-}$ $\langle 3m,3,2\rangle$

$$120^{60,53}_{\infty z}30^{l}_{2}1800_{2}3^{r}_{2}180^{l}_{2}75_{2}72^{r}_{2}30^{60,49}_{\infty a}$$

$$\begin{bmatrix} 28529065800 & -37254600 & -18932400 \\ -37254600 & 48630 & 24705 \\ -18932400 & 24705 & 12547 \end{bmatrix}$$

$$\begin{bmatrix} -29 & 14 & 1 & -7 & -127 & -248 & -511 & -199 \\ -47912 & 23131 & 1680 & -11564 & -209814 & -409720 & -844224 & -328769 \\ 50580 & -24420 & -1800 & 12207 & 221490 & 432525 & 891216 & 347070 \end{bmatrix}$$

$L_{268.3}$: $1^{-2}_{6}8^{-}_{5},1^{1}3^{1}9^{-},1^{-}5^{1}25^{-}$ $\langle 32,3,m\rangle$

$$120^{60,23}_{\infty z}30^{b}_{2}1800^{*}_{2}12^{l}_{2}45^{r}_{2}300^{*}_{2}72^{b}_{2}30^{60,49}_{\infty b}$$

$$\begin{bmatrix} 719389800 & -5914800 & -3114000 \\ -5914800 & 48630 & 25605 \\ -3114000 & 25605 & 13477 \end{bmatrix} \quad \begin{bmatrix} 31 & -16 & -29 & 13 & 64 & 509 & 527 & 206 \\ 2728 & -1409 & -2580 & 1142 & 5628 & 44770 & 46356 & 18121 \\ 1980 & -1020 & -1800 & 834 & 4095 & 32550 & 33696 & 13170 \end{bmatrix}$$

W_{269} 6 lattices, $\chi = 6$ 5-gon: $2|22\!|\!2 2 \rtimes D_2$

$L_{269.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{3},1^{2}3^{1},1^{-}5^{-}25^{-}$ $\langle 2\rangle$ $12^{r}_{2}10^{l}_{2}300^{r}_{2}2^{b}_{2}50^{l}_{2}$

$$\begin{bmatrix} -107700 & -40200 & 5700 \\ -40200 & -14990 & 2135 \\ 5700 & 2135 & -298 \end{bmatrix} \quad \begin{bmatrix} 23 & 14 & -431 & -24 & -29 \\ -48 & -29 & 900 & 50 & 60 \\ 96 & 60 & -1800 & -101 & -125 \end{bmatrix}$$

W_{270} 24 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{270.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{3},1^{-}3^{-}9^{1},1^{-}5^{1}25^{1}$ $\langle 23,3,2\rangle$ $2^{b}_{2}900^{*}_{2}20^{b}_{2}150^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} 81900 & 5400 & -900 \\ 5400 & 330 & -45 \\ -900 & -45 & 2 \end{bmatrix} \begin{bmatrix} 1919 & 92 & -2 \\ -41280 & -1979 & 43 \\ -57600 & -2760 & 59 \end{bmatrix} \quad \begin{bmatrix} 2 & 71 & 5 & 6 \\ -43 & -1530 & -108 & -130 \\ -59 & -2250 & -170 & -225 \end{bmatrix}$$

W_{271} 9 lattices, $\chi = 12$ 6-gon: $\not 2 2|2 \not 2 2|2 \rtimes D_4$

$L_{271.1}: 1_{\mathrm{II}}^{-2} 4_3^-, 1^1 3^1 9^1, 1^1 5^- 25^1 \ \langle 23, 3, 2\rangle$ $4_2^* 900_2^b 10_2^b 36_2^* 100_2^b 90_2^b$

$$\begin{bmatrix} -522900 & 2249100 & -449100 \\ 2249100 & -9335490 & 1863315 \\ -449100 & 1863315 & -371906 \end{bmatrix} \quad \begin{bmatrix} 253 & 3287 & 0 & -251 & -249 & 254 \\ 5060 & 65730 & -1 & -5022 & -4980 & 5082 \\ 25046 & 325350 & -5 & -24858 & -24650 & 25155 \end{bmatrix}$$

W_{272} 44 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{272.1}: 1_{\mathrm{II}}^2 4_7^1, 1^2 3^1, 1^1 5^1 25^- \ \langle 2\rangle$ $300_2^* 4_2^b 30_2^s 50_2^b 20_2^* \ (\times 2)$

$$\begin{bmatrix} 270300 & 13800 & -4500 \\ 13800 & 680 & -225 \\ -4500 & -225 & 74 \end{bmatrix} \begin{bmatrix} 2069 & 129 & -39 \\ 64860 & 4041 & -1222 \\ 324300 & 20210 & -6111 \end{bmatrix} \quad \begin{bmatrix} -89 & -13 & -11 & -11 & -3 \\ -2790 & -404 & -336 & -330 & -88 \\ -13950 & -2026 & -1695 & -1675 & -450 \end{bmatrix}$$

$L_{272.2}: 1_6^2 8_1^1, 1^2 3^-, 1^- 5^- 25^1 \ \langle 2\rangle$ $150_2^l 8_2 15_2^r 100_2^* 40_2^b \ (\times 2)$

$$\begin{bmatrix} -348600 & 1200 & 0 \\ 1200 & 35 & 30 \\ 0 & 30 & 23 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -2160 & 26 & 15 \\ 3600 & -45 & -26 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 & -3 & -1 \\ -285 & -288 & -288 & -850 & -268 \\ 375 & 376 & 375 & 1100 & 340 \end{bmatrix}$$

$L_{272.3}: 1_6^{-2} 8_5^-, 1^2 3^-, 1^- 5^- 25^1 \ \langle m\rangle$ $150_2^b 8_2^* 60_2^l 25_2 40_2^r \ (\times 2)$

$$\begin{bmatrix} -701400 & -43800 & 1200 \\ -43800 & -2735 & 75 \\ 1200 & 75 & -2 \end{bmatrix} \begin{bmatrix} -2761 & -173 & 4 \\ 38640 & 2421 & -56 \\ -234600 & -14705 & 339 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 1 & -2 & -7 \\ -30 & -16 & -18 & 25 & 96 \\ 75 & -4 & -90 & -300 & -680 \end{bmatrix}$$

W_{273} 8 lattices, $\chi = 24$ 8-gon: $2|22|22|22|2 \rtimes D_4$

$L_{273.1}: [1^1 2^1]_0 64_3^-, 1^2 3^1$ $192_2^* 4_2^s 192_2^l 2_2 192_2 1_2^r 192_2^* 8_2^s$

$$\begin{bmatrix} -27456 & 960 & 192 \\ 960 & -2 & -10 \\ 192 & -10 & -1 \end{bmatrix} \quad \begin{bmatrix} -5 & -1 & 1 & 1 & 41 & 2 & 35 & 1 \\ -48 & -10 & 0 & 9 & 384 & 19 & 336 & 10 \\ -480 & -94 & 96 & 94 & 3840 & 187 & 3264 & 92 \end{bmatrix}$$

$L_{273.2}: 1_3^- 8_1^1 64_7^1, 1^2 3^1$ $12_2^* 64_2^l 3_2 8_2^r \ (\times 2)$

$$\begin{bmatrix} -6720 & 192 & 192 \\ 192 & 8 & -8 \\ 192 & -8 & -5 \end{bmatrix} \begin{bmatrix} 95 & -2 & -3 \\ 480 & -11 & -15 \\ 2688 & -56 & -85 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 2 \\ -6 & -4 & 6 & 11 \\ -30 & -32 & 27 & 56 \end{bmatrix}$$

W_{274} 16 lattices, $\chi = 48$ 10-gon: $2\not\infty 22|22\not\infty 22|2 \rtimes D_4$

$L_{274.1}: [1^- 2^1]_6 64_1^1, 1^- 3^- 9^1 \ \langle 3\rangle$ $576_2 6_\infty^{48,1} 24_2^* 576_2^s 8_2^* 576_2^s 24_{\infty z}^{48,1} 6_2^r 576_2^l 2_2$

$$\begin{bmatrix} 14400 & 2880 & -576 \\ 2880 & 546 & -114 \\ -576 & -114 & 23 \end{bmatrix} \quad \begin{bmatrix} 29 & 4 & 11 & 65 & 3 & 25 & 1 & -1 & -11 & 0 \\ 96 & 11 & 26 & 144 & 6 & 48 & 2 & -1 & 0 & 1 \\ 1152 & 150 & 396 & 2304 & 104 & 864 & 36 & -30 & -288 & 4 \end{bmatrix}$$

$L_{274.2}: 1_7^1 8_3^- 64_1^1, 1^- 3^- 9^1 \ \langle 3\rangle$ $576_2 24_\infty^{48,1} 24_2^r 576_2^b 8_2^l \ (\times 2)$

$$\begin{bmatrix} 14400 & -576 & -576 \\ -576 & 24 & 24 \\ -576 & 24 & 23 \end{bmatrix} \begin{bmatrix} 53 & -3 & -2 \\ -216 & 11 & 8 \\ 1728 & -96 & -65 \end{bmatrix} \quad \begin{bmatrix} -19 & -3 & -2 & -7 & 0 \\ 48 & 11 & 13 & 72 & 3 \\ -576 & -96 & -72 & -288 & -4 \end{bmatrix}$$

W_{275} 12 lattices, $\chi = 12$ 5-gon: 62223

$L_{275.1}: 1_{\mathrm{II}}^{-2} 4_1^1, 1^- 3^- 27^1, 1^{-2} 5^- \ \langle 2\rangle$ $6_6 2_2^b 60_2^* 108_2^b 6_3^-$

$$\begin{bmatrix} -194940 & -76680 & 62640 \\ -76680 & -29982 & 24027 \\ 62640 & 24027 & -18046 \end{bmatrix} \quad \begin{bmatrix} 384 & 909 & 1249 & -823 & -326 \\ -1285 & -3042 & -4180 & 2754 & 1091 \\ -378 & -895 & -1230 & 810 & 321 \end{bmatrix}$$

W_{276} 12 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{276.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{1}3^{1}27^{1},1^{2}5^{1}$ $\langle 2\rangle$ $270^{l}_{2}4^{r}_{2}30^{b}_{2}108^{*}_{2}12^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 401220 & 5400 & -4860 \\ 5400 & 66 & -63 \\ -4860 & -63 & 58 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 13860 & 244 & -189 \\ 17820 & 315 & -244 \end{bmatrix} \begin{bmatrix} -1 & -1 & -2 & -7 & -1 \\ -675 & -132 & -130 & -396 & -50 \\ -945 & -244 & -315 & -1026 & -138 \end{bmatrix}$$

W_{277} 12 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{277.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{-}3^{1}27^{-},1^{-2}5^{-}$ $\langle 2\rangle$ $540^{b}_{2}2^{s}_{2}54^{b}_{2}12^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -307260 & 7560 & -3780 \\ 7560 & -186 & 93 \\ -3780 & 93 & -46 \end{bmatrix} \begin{bmatrix} -2161 & 53 & -24 \\ -73440 & 1801 & -816 \\ 32400 & -795 & 359 \end{bmatrix} \begin{bmatrix} 11 & 1 & 7 & 3 \\ 450 & 39 & 261 & 106 \\ 0 & -4 & -54 & -36 \end{bmatrix}$$

W_{278} 28 lattices, $\chi = 96$ 18-gon: $22\infty\!\!\!|\,2222|2222\infty\!\!\!|\,2222|22 \rtimes D_4$

$L_{278.1}$: $[1^{1}2^{-}]_{4}32^{-}_{3},1^{-2}7^{1}$ $224^{s}_{2}8^{*}_{2}28^{8,3}_{\infty z}7^{r}_{2}8^{*}_{2}224^{l}_{2}1^{r}_{2}56^{*}_{2}4^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -230048 & 1344 & 2464 \\ 1344 & -6 & -16 \\ 2464 & -16 & -25 \end{bmatrix} \begin{bmatrix} 14503 & -63 & -175 \\ 783216 & -3403 & -9450 \\ 919968 & -3996 & -11101 \end{bmatrix} \begin{bmatrix} 555 & 77 & 89 & 50 & 99 & 775 & 13 & 71 & 13 \\ 30072 & 4170 & 4816 & 2702 & 5346 & 41832 & 701 & 3822 & 698 \\ 35168 & 4880 & 5642 & 3171 & 6280 & 49168 & 825 & 4508 & 826 \end{bmatrix}$$

$L_{278.2}$: $[1^{1}2^{1}]_{0}64^{1}_{7},1^{-2}7^{1}$ $\langle m\rangle$ $448^{s}_{2}4^{*}_{2}56^{16,7}_{\infty z}14_{2}1^{r}_{2}448^{*}_{2}8^{*}_{2}28^{l}_{2}2^{r}_{2}$ $(\times 2)$

shares genus with $L_{278.3}$

$$\begin{bmatrix} 1430464 & -12544 & -448 \\ -12544 & 110 & 4 \\ -448 & 4 & -7 \end{bmatrix} \begin{bmatrix} 59135 & -520 & 116 \\ 6771072 & -59541 & 13282 \\ 206976 & -1820 & 405 \end{bmatrix}$$

$$\begin{bmatrix} -181 & -7 & -1 & 4 & 1 & -1 & -3 & -19 & -10 \\ -20832 & -808 & -126 & 455 & 114 & -112 & -342 & -2170 & -1143 \\ -1120 & -54 & -56 & 0 & 1 & 0 & -4 & -42 & -26 \end{bmatrix}$$

$L_{278.3}$: $[1^{1}2^{1}]_{0}64^{1}_{7},1^{-2}7^{1}$ $448_{2}1^{r}_{2}56^{16,15}_{\infty z}14^{r}_{2}4^{*}_{2}448^{s}_{2}8^{l}_{2}7_{2}2_{2}$ $(\times 2)$

shares genus with $L_{278.2}$

$$\begin{bmatrix} 1179584 & 11648 & 11200 \\ 11648 & 110 & 108 \\ 11200 & 108 & 105 \end{bmatrix} \begin{bmatrix} -7841 & -94 & -83 \\ -799680 & -9589 & -8466 \\ 1646400 & 19740 & 17429 \end{bmatrix}$$

$$\begin{bmatrix} -181 & -5 & -15 & -3 & -1 & -1 & 1 & 1 & 0 \\ -19712 & -548 & -1666 & -343 & -116 & -112 & 118 & 133 & 23 \\ 39424 & 1093 & 3304 & 672 & 226 & 224 & -228 & -245 & -26 \end{bmatrix}$$

$L_{278.4}$: $1^{1}_{7}4^{1}_{7}32^{1}_{1},1^{-2}7^{1}$ $28^{*}_{2}16^{s}_{2}224^{8,1}_{\infty z}224^{*}_{2}16^{l}_{2}7_{2}32_{2}28^{r}_{2}32^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} -509152 & -2464 & 3584 \\ -2464 & -4 & 16 \\ 3584 & 16 & -25 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -39984 & -351 & 308 \\ -45696 & -400 & 351 \end{bmatrix} \begin{bmatrix} 23 & 11 & 19 & 9 & 1 & -1 & -3 & -5 & -7 \\ 392 & 166 & 196 & -140 & -170 & -224 & -192 & -217 & -256 \\ 3430 & 1616 & 2688 & 1008 & -64 & -385 & -608 & -896 & -1200 \end{bmatrix}$$

$L_{278.5}$: $1^{1}_{7}4^{1}_{1}32^{1}_{7},1^{-2}7^{1}$ $28^{l}_{2}4_{2}224^{4,1}_{\infty}224^{l}_{2}4_{2}7^{r}_{2}32^{s}_{2}112^{*}_{2}32^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -161056 & -3584 & 2240 \\ -3584 & -28 & 32 \\ 2240 & 32 & -25 \end{bmatrix} \begin{bmatrix} 8035 & 301 & -154 \\ 433944 & 16253 & -8316 \\ 1267392 & 47472 & -24289 \end{bmatrix}$$

$$\begin{bmatrix} 149 & 41 & 187 & 205 & 50 & 97 & 51 & 67 & 23 \\ 8092 & 2225 & 10136 & 11088 & 2701 & 5236 & 2748 & 3598 & 1228 \\ 23590 & 6488 & 29568 & 32368 & 7888 & 15295 & 8032 & 10528 & 3600 \end{bmatrix}$$

$L_{278.6}$: $1^{1}_{7}8^{1}_{7}64^{1}_{1},1^{-2}7^{1}$ $28^{s}_{2}64^{l}_{2}56^{16,9}_{\infty}56_{2}64_{2}7^{r}_{2}32^{s}_{2}448^{*}_{2}32^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} 6720 & -896 & 0 \\ -896 & 120 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 461 & -63 & 6 \\ 4312 & -589 & 56 \\ 9856 & -1344 & 127 \end{bmatrix} \begin{bmatrix} -19 & -23 & -15 & -20 & -43 & -22 & -13 & -41 & -9 \\ -196 & -232 & -147 & -189 & -400 & -203 & -118 & -364 & -78 \\ -602 & -672 & -392 & -448 & -896 & -441 & -240 & -672 & -128 \end{bmatrix}$$

W_{279} 8 lattices, $\chi = 36$ 9-gon: $222|2224\!\!\!|\,4 \rtimes D_2$

$L_{279.1}$: $1_2^2 32_1^1, 1^2 3^-, 1^2 5^1$ $2_2^b 32_2^* 20_2^s 32_2^l 5_2 32_2^r 2_4^* 4_2^l 1_4$

$$\begin{bmatrix} -5553120 & -2770080 & 18240 \\ -2770080 & -1381807 & 9098 \\ 18240 & 9098 & -59 \end{bmatrix} \quad \begin{bmatrix} 66 & 135 & -263 & -1723 & -2124 & -18993 & -3919 & -931 & -67 \\ -133 & -272 & 530 & 3472 & 4280 & 38272 & 7897 & 1876 & 135 \\ -105 & -208 & 420 & 2720 & 3345 & 29888 & 6165 & 1462 & 104 \end{bmatrix}$$

W_{280} 8 lattices, $\chi = 16$ 6-gon: 222622

$L_{280.1}$: $1_{\mathrm{II}}^{-2} 32_3^-, 1^2 3^-, 1^{-2} 5^-$ $96_2^r 10_2^b 96_2^b 2_6 6_2^b 2_2^l$

$$\begin{bmatrix} -24512160 & 61440 & 30720 \\ 61440 & -154 & -77 \\ 30720 & -77 & -38 \end{bmatrix} \quad \begin{bmatrix} 13 & 2 & 5 & 0 & -1 & 0 \\ 5184 & 800 & 2016 & 4 & -399 & -1 \\ 0 & -5 & -48 & -9 & 0 & 2 \end{bmatrix}$$

W_{281} 12 lattices, $\chi = 48$ 12-gon: $2|222|222|222|22 \rtimes D_4$

$L_{281.1}$: $1_6^2 32_1^1, 1^- 3^- 9^-, 1^{-2} 5^-$ $\langle 3 \rangle$
$288_2^r 6_2^b 32_2^* 60_2^s 288_2^l 15_2 32_2^r 6_2^b 288_2^* 60_2^s 32_2^l 15_2$

$$\begin{bmatrix} -1859040 & -8640 & -10080 \\ -8640 & 6 & -3 \\ -10080 & -3 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 0 & -1 & -1 & 1 & 2 & 7 & 2 & 35 & 11 & 5 & 3 \\ 10368 & -1 & -944 & -940 & 960 & 1895 & 6624 & 1891 & 33072 & 10390 & 4720 & 2830 \\ -10944 & 0 & 992 & 990 & -1008 & -1995 & -6976 & -1992 & -34848 & -10950 & -4976 & -2985 \end{bmatrix}$$

W_{282} 12 lattices, $\chi = 48$ 12-gon: $2|222|222|222|22 \rtimes D_4$

$L_{282.1}$: $1_6^{-2} 32_5^-, 1^1 3^- 9^1, 1^{-2} 5^-$ $\langle 3 \rangle$
$160_2^b 6_2^l 1440_2 1_2^r 1440_2^s 4_2^* 1440_2^b 6_2^l 160_2 9_2^r 160_2^s 36_2^*$

$$\begin{bmatrix} 32142240 & -10710720 & -34560 \\ -10710720 & 3569118 & 11517 \\ -34560 & 11517 & 37 \end{bmatrix}$$

$$\begin{bmatrix} 27 & 55 & 5831 & 82 & 8099 & 463 & 41711 & 1251 & 11987 & 407 & 783 & -83 \\ 80 & 163 & 17280 & 243 & 24000 & 1372 & 123600 & 3707 & 35520 & 1206 & 2320 & -246 \\ 320 & 636 & 67680 & 953 & 94320 & 5398 & 486720 & 14604 & 140000 & 4761 & 9200 & -954 \end{bmatrix}$$

W_{283} 8 lattices, $\chi = 36$ 6-gon: $\infty\infty 3 \infty 26$

$L_{283.1}$: $1_{\mathrm{II}}^{-2} 8_7^1, 1^- 3^- 81^1$ $\langle 2 \rangle$ $6_{\infty b}^{36,35} 24_{\infty z}^{18,5} 6_3^- 6_{\infty a}^{36,23} 24_2^b 2_6$

$$\begin{bmatrix} -669384 & 6480 & -648 \\ 6480 & -30 & -3 \\ -648 & -3 & 2 \end{bmatrix} \quad \begin{bmatrix} 4 & 3 & 1 & 0 & -1 & 0 \\ 635 & 476 & 158 & -1 & -160 & 0 \\ 2250 & 1692 & 567 & 3 & -564 & -1 \end{bmatrix}$$

W_{284} 8 lattices, $\chi = 72$ 10-gon: $2\infty\infty\infty 22\infty\infty\infty 2 \rtimes C_2$

$L_{284.1}$: $1_{\mathrm{II}}^{-2} 8_7^1, 1^1 3^- 81^-$ $\langle 2 \rangle$ $162_2^s 6_{\infty b}^{36,19} 24_{\infty z}^{18,13} 6_{\infty a}^{36,31} 24_2^b$ $(\times 2)$

$$\begin{bmatrix} -679752 & 361584 & -213192 \\ 361584 & -180210 & 101043 \\ -213192 & 101043 & -54266 \end{bmatrix} \begin{bmatrix} -285698017 & 133304292 & -70578468 \\ -1274348376 & 594600236 & -314813373 \\ -1250420328 & 583435611 & -308902220 \end{bmatrix}$$

$$\begin{bmatrix} 3868 & -1266 & -14663 & -33826 & -1431831 \\ 17253 & -5647 & -65404 & -150880 & -6386644 \\ 16929 & -5541 & -64176 & -148047 & -6266724 \end{bmatrix}$$

W_{285} 4 lattices, $\chi = 72$ 8-gon: $\infty|\infty|\infty|\infty|\infty|\infty|\infty|\infty| \rtimes D_8$

$L_{285.1} : 1^{-2}_{4} 4^{1}_{1}, 1^{-} 5^{1} 25^{-}$ $\qquad 20^{5,2}_{\infty a} 20^{10,3}_{\infty} 20^{5,2}_{\infty z} 5^{20,17}_{\infty}$ $(\times 2)$

$$\begin{bmatrix} -8700 & -700 & 2300 \\ -700 & -55 & 185 \\ 2300 & 185 & -608 \end{bmatrix} \begin{bmatrix} -641 & -48 & 168 \\ 160 & 11 & -42 \\ -2400 & -180 & 629 \end{bmatrix} \qquad \begin{bmatrix} 39 & 37 & 51 & 11 \\ 4 & -4 & -16 & -7 \\ 150 & 140 & 190 & 40 \end{bmatrix}$$

$L_{285.2} : 1^{2}_{\mathrm{II}} 8^{-}_{5}, 1^{1} 5^{-} 25^{1}$ $\qquad 40^{5,1}_{\infty b} 40^{5,2}_{\infty} 40^{5,4}_{\infty z} 10^{20,13}_{\infty a}$ $(\times 2)$

$$\begin{bmatrix} -1821400 & -14400 & 10800 \\ -14400 & -110 & 85 \\ 10800 & 85 & -64 \end{bmatrix} \begin{bmatrix} -3041 & -24 & 18 \\ -59280 & -469 & 351 \\ -592800 & -4680 & 3509 \end{bmatrix} \qquad \begin{bmatrix} 15 & 17 & 27 & 7 \\ 304 & 336 & 524 & 133 \\ 2940 & 3320 & 5260 & 1360 \end{bmatrix}$$

W_{286} 1 lattice, $\chi = 24$ 4-gon: $\not\infty|\not\infty|\not\infty|\not\infty| \rtimes D_8$

$L_{286.1} : 1^{-}_{5} 8^{1}_{7} 64^{-}_{5}$ $\qquad 32^{16,3}_{\infty z} 8^{8,5}_{\infty b}$ $(\times 2)$

$$\begin{bmatrix} 6976 & 768 & -192 \\ 768 & -8 & -16 \\ -192 & -16 & 5 \end{bmatrix} \begin{bmatrix} -17 & 6 & 0 \\ -48 & 17 & 0 \\ -768 & 288 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -4 \\ 2 & -11 \\ 48 & -188 \end{bmatrix}$$

W_{287} 8 lattices, $\chi = 60$ 12-gon: $242|242|242|242| \rtimes D_4$

$L_{287.1} : 1^{2}_{2} 16^{1}_{7}, 1^{2} 3^{1}, 1^{2} 11^{-}$ $\qquad 66^{b}_{2} 2^{*}_{4} 4^{s}_{2} 48^{l}_{2} 1_{4} 2^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} -2319504 & -584496 & 14256 \\ -584496 & -147287 & 3591 \\ 14256 & 3591 & -86 \end{bmatrix} \begin{bmatrix} -599105 & -151108 & 3848 \\ 2428800 & 612599 & -15600 \\ 2100912 & 529899 & -13495 \end{bmatrix} \qquad \begin{bmatrix} -749 & -38 & 37 & 77 & -128 & -331 \\ 3036 & 154 & -150 & -312 & 519 & 1342 \\ 2607 & 131 & -130 & -264 & 452 & 1165 \end{bmatrix}$$

W_{288} 12 lattices, $\chi = 48$ 10-gon: $3222632226 \rtimes C_2$

$L_{288.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{3}, 1^{-} 3^{-} 27^{-}, 1^{-2} 7^{1}$ $\langle 2 \rangle$ $\qquad 6^{-}_{3} 6^{b}_{2} 14^{b}_{2} 54^{s}_{2} 2_{6}$ $(\times 2)$

$$\begin{bmatrix} -52346196 & 1488564 & -35532 \\ 1488564 & -42330 & 1011 \\ -35532 & 1011 & -22 \end{bmatrix} \begin{bmatrix} 1581929 & -45036 & 891 \\ 55269900 & -1573481 & 31130 \\ -14999040 & 427008 & -8449 \end{bmatrix} \begin{bmatrix} -323 & -583 & -568 & -619 & -33 \\ -11285 & -20369 & -19845 & -21627 & -1153 \\ 3066 & 5529 & 5383 & 5859 & 311 \end{bmatrix}$$

W_{289} 12 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{289.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{3}, 1^{1} 3^{1} 27^{-}, 1^{2} 7^{-}$ $\langle 2 \rangle$ $\qquad 378^{b}_{2} 4^{*}_{2} 84^{b}_{2} 54^{l}_{2} 12^{r}_{2}$ $(\times 2)$

$$\begin{bmatrix} -72721908 & -1136268 & 12096 \\ -1136268 & -17754 & 189 \\ 12096 & 189 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -32508 & -509 & 5 \\ -3315816 & -51816 & 509 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 5 & 4 & 1 \\ -126 & -64 & -322 & -261 & -68 \\ 189 & -2 & -210 & -513 & -408 \end{bmatrix}$$

W_{290} 12 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{290.1} : 1^{-2}_{\mathrm{II}} 4^{-}_{3}, 1^{1} 3^{-} 27^{1}, 1^{2} 7^{-}$ $\langle 2 \rangle$ $\qquad 756^{*}_{2} 4^{b}_{2} 42^{l}_{2} 108^{r}_{2} 6^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} -285012 & 1512 & 756 \\ 1512 & 186 & -3 \\ 756 & -3 & -2 \end{bmatrix} \begin{bmatrix} -5167 & -107 & 13 \\ -10332 & -215 & 26 \\ -2138724 & -44298 & 5381 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 3 & 7 & 0 \\ 0 & -2 & -7 & -36 & -4 \\ 378 & 380 & 1134 & 2484 & -33 \end{bmatrix}$$

W_{291} 28 lattices, $\chi = 36$ 9-gon: $222|2222\not\infty 2 \rtimes D_2$

$L_{291.1} : [1^{1} 2^{-}]_{4} 32^{-}_{5}, 1^{2} 9^{-}$ $\qquad 288^{*}_{2} 8^{s}_{2} 32^{s}_{2} 72^{*}_{2} 32^{*}_{2} 8^{s}_{2} 288^{*}_{2} 4^{24,13}_{\infty z} 1^{r}_{2}$

$$\begin{bmatrix} 72864 & 36000 & -288 \\ 36000 & 17786 & -142 \\ -288 & -142 & 1 \end{bmatrix} \qquad \begin{bmatrix} -35 & -1 & 27 & 113 & 143 & 115 & 1009 & 29 & 0 \\ 72 & 2 & -56 & -234 & -296 & -238 & -2088 & -60 & 0 \\ 144 & 0 & -160 & -648 & -816 & -656 & -5760 & -166 & -1 \end{bmatrix}$$

$L_{291.2}: [1^1 2^1]_0 64^1_1, 1^2 9^1 \ \langle m \rangle$ $\qquad$ $576^l_2 1_2 64_2 9^r_2 64^l_2 1_2 576_2 2^{48,1}_{\infty} 8^*_2$

shares genus with $L_{291.4}$

$$\begin{bmatrix} 28224 & 0 & -1152 \\ 0 & 2 & -2 \\ -1152 & -2 & 49 \end{bmatrix} \qquad \begin{bmatrix} -59 & -2 & -13 & -4 & -9 & -1 & -23 & 0 & -1 \\ -1296 & -44 & -288 & -90 & -208 & -24 & -576 & -1 & -22 \\ -1440 & -49 & -320 & -99 & -224 & -25 & -576 & 0 & -24 \end{bmatrix}$$

$L_{291.3}: 1^1_1 4^1_1 32^1_7, 1^2 9^1$ $\qquad$ $36^l_2 4_2 1_2 36^r_2 4^l_2 4_2 9^r_2 32^{24,7}_{\infty z} 32^*_2$

$$\begin{bmatrix} 6624 & -576 & 288 \\ -576 & 68 & -40 \\ 288 & -40 & 25 \end{bmatrix} \qquad \begin{bmatrix} -13 & -3 & -1 & -4 & -1 & -1 & -2 & -1 & -3 \\ -396 & -91 & -30 & -117 & -28 & -27 & -54 & -28 & -92 \\ -486 & -112 & -37 & -144 & -34 & -32 & -63 & -32 & -112 \end{bmatrix}$$

$L_{291.4}: [1^1 2^1]_0 64^1_1, 1^2 9^1$ $\qquad$ $576^s_2 4^*_2 64^*_2 36^s_2 64^s_2 4^*_2 576^s_2 8^{48,1}_{\infty z} 2^r_2$

shares genus with $L_{291.2}$

$$\begin{bmatrix} -1575360 & 5184 & 5184 \\ 5184 & -2 & -18 \\ 5184 & -18 & -17 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 & -1 & 7 & 27 & 13 & 253 & 9 & 1 \\ 0 & -18 & -16 & 126 & 480 & 230 & 4464 & 158 & 17 \\ 288 & -286 & -288 & 1998 & 7712 & 3714 & 72288 & 2572 & 286 \end{bmatrix}$$

$L_{291.5}: 1^1_1 4^1_7 32^1_1, 1^2 9^1$ $\qquad$ $36^*_2 16^l_2 1^r_2 144^*_2 4^*_2 16^l_2 9_2 32^{12,7}_{\infty} 32^s_2$

$$\begin{bmatrix} 1435680 & 44640 & 6048 \\ 44640 & 1388 & 188 \\ 6048 & 188 & 25 \end{bmatrix} \qquad \begin{bmatrix} -35 & -15 & -2 & -7 & 1 & 5 & 5 & 1 & -9 \\ 1152 & 494 & 66 & 234 & -32 & -162 & -162 & -32 & 296 \\ -198 & -88 & -13 & -72 & -2 & 8 & 9 & 0 & -48 \end{bmatrix}$$

$L_{291.6}: 1^1_1 8^1_1 64^1_7, 1^2 9^1$ $\qquad$ $36^*_2 64^l_2 1^r_2 576^*_2 4^*_2 64^l_2 9_2 8^{48,31}_{\infty} 8^r_2$

$$\begin{bmatrix} -8193600 & 23616 & 24192 \\ 23616 & -56 & -72 \\ 24192 & -72 & -71 \end{bmatrix} \qquad \begin{bmatrix} 53 & 39 & 2 & 5 & -1 & -1 & 4 & 4 & 9 \\ 2826 & 2076 & 106 & 252 & -54 & -52 & 216 & 215 & 481 \\ 15174 & 11168 & 573 & 1440 & -286 & -288 & 1143 & 1144 & 2576 \end{bmatrix}$$

W_{292} 6 lattices, $\chi = 48$ $\qquad$ 12-gon: $22|222|222|222|2 \rtimes D_4$

$L_{292.1}: 1^{-2}_{\mathrm{II}} 4^1_1, 1^- 3^1 9^-, 1^{-2} 23^1 \ \langle 2 \rangle$ $\qquad$ $92^b_2 18^b_2 138^b_2 2^b_2 828^*_2 12^*_2 \ (\times 2)$

$$\begin{bmatrix} 105156 & -35604 & 0 \\ -35604 & 12054 & 3 \\ 0 & 3 & -10 \end{bmatrix} \begin{bmatrix} 6071 & -2090 & 110 \\ 17940 & -6176 & 325 \\ 5796 & -1995 & 104 \end{bmatrix} \qquad \begin{bmatrix} -1121 & -199 & -358 & -23 & -373 & 21 \\ -3312 & -588 & -1058 & -68 & -1104 & 62 \\ -1058 & -189 & -345 & -23 & -414 & 18 \end{bmatrix}$$

W_{293} 6 lattices, $\chi = 48$ $\qquad$ 12-gon: $222|222|222|222| \rtimes D_4$

$L_{293.1}: 1^{-2}_{\mathrm{II}} 4^1_1, 1^1 3^1 9^1, 1^{-2} 23^1 \ \langle 2 \rangle$ $\qquad$ $12^b_2 46^l_2 36^r_2 138^l_2 4^r_2 414^b_2 \ (\times 2)$

$$\begin{bmatrix} -378396 & 15732 & 4968 \\ 15732 & -654 & -207 \\ 4968 & -207 & -62 \end{bmatrix} \begin{bmatrix} 94391 & -3952 & -1083 \\ 2180952 & -91313 & -25023 \\ 268272 & -11232 & -3079 \end{bmatrix} \qquad \begin{bmatrix} -3 & 7 & 1 & -43 & -21 & -347 \\ -70 & 161 & 24 & -989 & -484 & -8004 \\ -6 & 23 & 0 & -138 & -64 & -1035 \end{bmatrix}$$

W_{294} 8 lattices, $\chi = 42$ $\qquad$ 10-gon: $42222|22224| \rtimes D_2$

$L_{294.1}: 1^2_2 16^1_1, 1^2 3^-, 1^2 13^1$ $\qquad$ $2^*_4 4^l_2 13_2 16_2 1^r_2 208^s_2 4^*_2 16^*_2 52^l_2 1_4$

$$\begin{bmatrix} -70657392 & 58656 & 83616 \\ 58656 & -47 & -72 \\ 83616 & -72 & -95 \end{bmatrix} \qquad \begin{bmatrix} 2 & 67 & 148 & 99 & 18 & 241 & 13 & 7 & -3 & -1 \\ 1247 & 41756 & 92235 & 61696 & 11217 & 150176 & 8100 & 4360 & -1872 & -623 \\ 815 & 27318 & 60346 & 40368 & 7340 & 98280 & 5302 & 2856 & -1222 & -408 \end{bmatrix}$$

W_{295} 32 lattices, $\chi = 42$ 11-gon: $2222\!\!|22222|2 \rtimes D_2$

$L_{295.1}$: $1_2^2 8_1^1,1^-3^1 9^-,1^2 13^-$ $\langle 2, m\rangle$

$104_2^b 18_2^l 8_2^r 234_2^b 2_2^s 18_2^b 26_2^l 72_2^r 2_2^b 936_2^* 12_2^*$

$$\begin{bmatrix} -8501688 & 12168 & -2959632 \\ 12168 & -15 & 4485 \\ -2959632 & 4485 & -1004638 \end{bmatrix}$$

$$\begin{bmatrix} -1263 & 58 & 115 & -406 & -173 & -1325 & -3628 & -6103 & -1324 & -45701 & -345 \\ -262704 & 12066 & 23920 & -84474 & -35988 & -275622 & -754676 & -1269504 & -275408 & -9506328 & -71762 \\ 2548 & -117 & -232 & 819 & 349 & 2673 & 7319 & 12312 & 2671 & 92196 & 696 \end{bmatrix}$$

$L_{295.2}$: $1_6^{-2} 16_5^-,1^1 3^- 9^1,1^2 13^1$ $\langle 3, m\rangle$

$1872_2 1_2^r 144_2^l 13_2 9_2^r 4_2^* 468_2^s 16_2^s 36_2^* 208_2^b 6_2^l$

shares genus with its 3-dual

$$\begin{bmatrix} -65008944 & 10829520 & 41184 \\ 10829520 & -1804035 & -6861 \\ 41184 & -6861 & -26 \end{bmatrix} \quad \begin{bmatrix} 4117 & 56 & 467 & 121 & 32 & -1 & -145 & -23 & 1 & 211 & 23 \\ 24336 & 331 & 2760 & 715 & 189 & -6 & -858 & -136 & 6 & 1248 & 136 \\ 99216 & 1355 & 11376 & 2977 & 810 & -2 & -3276 & -544 & 0 & 4888 & 543 \end{bmatrix}$$

W_{296} 32 lattices, $\chi = 72$ 16-gon: $2222222222222222 \rtimes C_2$

$L_{296.1}$: $1_{\text{II}}^{-2} 8_7^1,1^2 9^-,1^{-2} 5^-,1^2 7^1$ $\langle 2\rangle$ $504_2^r 10_2^s 126_2^b 2_2^l 56_2^r 18_2^b 14_2^b 2_2^l$ $(\times 2)$

$$\begin{bmatrix} 2552760 & -511560 & -2520 \\ -511560 & 102514 & 505 \\ -2520 & 505 & 2 \end{bmatrix} \begin{bmatrix} 2615759 & -524536 & 2249 \\ 13048560 & -2616617 & 11219 \\ 997920 & -200112 & 857 \end{bmatrix}$$

$$\begin{bmatrix} -174485 & -6921 & -21457 & -1298 & -8049 & -866 & -87 & 0 \\ -870408 & -34525 & -107037 & -6475 & -40152 & -4320 & -434 & 0 \\ -66528 & -2640 & -8190 & -496 & -3080 & -333 & -35 & -1 \end{bmatrix}$$

W_{297} 8 lattices, $\chi = 36$ 10-gon: $22222|22222| \rtimes D_2$

$L_{297.1}$: $1_2^{-2} 32_7^1,1^2 3^1,1^2 7^1$ $224_2^s 12_2^* 28_2^l 3_2 224_2^r 2_2^b 224_2^* 12_2^l 7_2 3_2^r$

$$\begin{bmatrix} -8766240 & 7392 & 16128 \\ 7392 & -5 & -16 \\ 16128 & -16 & -25 \end{bmatrix} \quad \begin{bmatrix} 37 & 11 & 19 & 13 & 177 & 1 & 9 & -1 & -1 & 1 \\ 20608 & 6132 & 10598 & 7254 & 98784 & 559 & 5040 & -558 & -560 & 555 \\ 10640 & 3162 & 5460 & 3735 & 50848 & 287 & 2576 & -288 & -287 & 288 \end{bmatrix}$$

W_{298} 6 lattices, $\chi = 40$ 8-gon: $26|62|26|62| \rtimes D_4$

$L_{298.1}$: $1_{\text{II}}^{-2} 4_7^1,1^-3^-9^-,1^{-2} 25^1$ $\langle 2\rangle$ $150_2^s 18_6 6_6 2_2^s$ $(\times 2)$

$$\begin{bmatrix} -30392100 & 1091700 & -9061200 \\ 1091700 & -34842 & 329757 \\ -9061200 & 329757 & -2697358 \end{bmatrix} \begin{bmatrix} -362227951 & 13177346 & -107833621 \\ -1062468900 & 38651131 & -316292182 \\ 1086939000 & -39541320 & 323576819 \end{bmatrix}$$

$$\begin{bmatrix} 17021 & 8527 & -8511 & -83754 \\ 49925 & 25011 & -24964 & -245663 \\ -51075 & -25587 & 25539 & 251321 \end{bmatrix}$$

W_{299} 6 lattices, $\chi = 20$ 7-gon: $222|222\!\!| \rtimes D_2$

$L_{299.1}$: $1_{\text{II}}^{-2} 4_7^1,1^1 3^- 9^1,1^{-2} 25^1$ $\langle 2\rangle$ $6_2^b 100_2^* 36_2^b 150_2^b 4_2^* 900_2^b 6_3^+$

$$\begin{bmatrix} -3536100 & 4500 & 3600 \\ 4500 & 6 & -9 \\ 3600 & -9 & -2 \end{bmatrix} \quad \begin{bmatrix} 0 & 3 & 1 & -1 & -1 & -41 & -1 \\ -1 & 750 & 252 & -250 & -252 & -10350 & -253 \\ 0 & 2000 & 666 & -675 & -670 & -27450 & -669 \end{bmatrix}$$

W_{300} 8 lattices, $\chi = 24$ 5-gon: $\infty 22\infty\infty$

$L_{300.1} : 1^{-2}_{\mathrm{II}} 8^{-}_{3}, 1^{-} 7^{1} 49^{1}$ $\langle 2\rangle$ $56^{14,9}_{\infty z} 14^{b}_{2} 98^{b}_{2} 56^{14,1}_{\infty z} 14^{28,23}_{\infty b}$

$$\begin{bmatrix} -171304 & 2744 & -8624 \\ 2744 & -42 & 147 \\ -8624 & 147 & -394 \end{bmatrix} \quad \begin{bmatrix} -45 & -7 & 12 & 7 & -6 \\ -1672 & -263 & 441 & 260 & -221 \\ 364 & 56 & -98 & -56 & 49 \end{bmatrix}$$

W_{301} 8 lattices, $\chi = 24$ 8-gon: 22222222

$L_{301.1} : 1^{-2}_{\mathrm{II}} 16^{1}_{1}, 1^{2} 9^{1}, 1^{-2} 5^{-}$ $144^{b}_{2} 10^{l}_{2} 16^{r}_{2} 90^{b}_{2} 16^{b}_{2} 10^{l}_{2} 144^{r}_{2} 2^{b}_{2}$

$$\begin{bmatrix} -633878640 & -7824240 & 146880 \\ -7824240 & -96578 & 1813 \\ 146880 & 1813 & -34 \end{bmatrix} \quad \begin{bmatrix} 109 & 61 & 131 & 128 & 7 & -4 & -55 & -1 \\ -8856 & -4955 & -10640 & -10395 & -568 & 325 & 4464 & 81 \\ -1368 & -705 & -1456 & -1350 & -48 & 50 & 432 & -1 \end{bmatrix}$$

W_{302} 22 lattices, $\chi = 108$ 14-gon: $\infty\infty 2\infty 22\infty\infty\infty 2\infty 22\infty \rtimes C_2$

$L_{302.1} : 1^{2}_{\mathrm{II}} 4^{1}_{1}, 1^{1} 9^{1} 81^{-}$ $\langle 2\rangle$ $36^{9,5}_{\infty a} 36^{9,4}_{\infty} 36^{*}_{2} 4^{3,2}_{\infty b} 4^{r}_{2} 162^{l}_{2} 36^{9,1}_{\infty}$ $(\times 2)$

$$\begin{bmatrix} 1895076 & -40176 & -5508 \\ -40176 & 846 & 117 \\ -5508 & 117 & 16 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 55440 & -1079 & -165 \\ -362880 & 7056 & 1079 \end{bmatrix} \quad \begin{bmatrix} -1 & -3 & -5 & -1 & -1 & -7 & -3 \\ -10 & -44 & -100 & -26 & -52 & -657 & -512 \\ -270 & -720 & -1026 & -166 & 4 & 1944 & 2340 \end{bmatrix}$$

$L_{302.2} : 1^{-2}_{2} 8^{-}_{3}, 1^{-} 9^{-} 81^{1}$ $\langle 2\rangle$ $18^{36,23}_{\infty b} 72^{36,13}_{\infty z} 18^{b}_{2} 2^{12,11}_{\infty a} 8^{s}_{2} 324^{s}_{2} 72^{36,1}_{\infty z}$ $(\times 2)$

$$\begin{bmatrix} -1470312 & -725112 & -90720 \\ -725112 & -357597 & -44703 \\ -90720 & -44703 & -5350 \end{bmatrix} \begin{bmatrix} 44039519 & 21740144 & 2857326 \\ -91012320 & -44928305 & -5904966 \\ 13698720 & 6762384 & 888785 \end{bmatrix}$$

$$\begin{bmatrix} 42910 & 54863 & 20691 & 1772 & 451 & -1559 & -811 \\ -88678 & -113380 & -42760 & -3662 & -932 & 3222 & 1676 \\ 13347 & 17064 & 6435 & 551 & 140 & -486 & -252 \end{bmatrix}$$

$L_{302.3} : 1^{2}_{2} 8^{1}_{7}, 1^{-} 9^{-} 81^{1}$ $\langle m\rangle$ $18^{36,23}_{\infty a} 72^{36,31}_{\infty z} 18^{s}_{2} 2^{12,11}_{\infty b} 8^{l}_{2} 81^{r}_{2} 72^{36,19}_{\infty z}$ $(\times 2)$

$$\begin{bmatrix} -2240136 & -133488 & -112752 \\ -133488 & -7758 & -6921 \\ -112752 & -6921 & -5467 \end{bmatrix} \begin{bmatrix} 14785091 & 949601 & 673630 \\ -136284120 & -8753111 & -6209300 \\ -132392232 & -8503146 & -6031981 \end{bmatrix}$$

$$\begin{bmatrix} 10662 & 13609 & 5120 & 436 & 105 & -208 & -197 \\ -98279 & -125444 & -47195 & -4019 & -968 & 1917 & 1816 \\ -95472 & -121860 & -45846 & -3904 & -940 & 1863 & 1764 \end{bmatrix}$$

W_{303} 16 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{303.1} : 1^{-2}_{\mathrm{II}} 8^{-}_{3}, 1^{2} 3^{-}, 1^{1} 5^{-} 25^{-}$ $\langle 2\rangle$ $24^{r}_{2} 50^{s}_{2} 6^{b}_{2} 10^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -877800 & -352200 & 4800 \\ -352200 & -141310 & 1925 \\ 4800 & 1925 & -26 \end{bmatrix} \begin{bmatrix} -13201 & -5313 & 77 \\ 34800 & 14006 & -203 \\ 138000 & 55545 & -806 \end{bmatrix} \quad \begin{bmatrix} -55 & -2 & 8 & 5 \\ 144 & 5 & -21 & -13 \\ 504 & 0 & -78 & -40 \end{bmatrix}$$

W_{304} 36 lattices, $\chi = 36$ 10-gon: $2|22\not{2}22|22\not{2}2 \rtimes D_4$

$L_{304.1} : 1^{-2}_{2} 8^{-}_{5}, 1^{-} 3^{-} 9^{-}, 1^{-} 5^{1} 25^{-}$ $\langle 23, 3m, 3, 2, m\rangle$

$200^{*}_{2} 180^{*}_{2} 8^{b}_{2} 450^{b}_{2} 2^{b}_{2} 1800^{*}_{2} 20^{*}_{2} 72^{b}_{2} 50^{b}_{2} 18^{b}_{2}$

$$\begin{bmatrix} -12202200 & 2309400 & 12600 \\ 2309400 & -437070 & -2385 \\ 12600 & -2385 & -13 \end{bmatrix} \quad \begin{bmatrix} -13 & -5 & 1 & 17 & 2 & 119 & 7 & 13 & 1 & -2 \\ -60 & -24 & 4 & 75 & 9 & 540 & 32 & 60 & 5 & -9 \\ -1600 & -450 & 232 & 2700 & 286 & 16200 & 910 & 1584 & 50 & -288 \end{bmatrix}$$

$L_{304.2}\,:\; 1_2^2 16_1^1{,}1^1 3^1 9^1{,}1^1 5^- 25^1\;\langle 5,3m,3,m\rangle$

$400_2^b 90_2^l 16_2 225_2 1_2 3600_2^r 10_2^b 144_2^* 100_2^* 36_2^*$

shares genus with its 3-dual ≅ 5-dual; isometric to its own 3.5-dual

$$\begin{bmatrix} -433292400 & -2851200 & 298800 \\ -2851200 & -18735 & 1965 \\ 298800 & 1965 & -206 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 & 14 & 1 & 91 & 2 & 5 & -1 & -1 \\ 320 & 444 & 560 & 1575 & 113 & 10320 & 228 & 576 & -110 & -114 \\ 7400 & 10035 & 12592 & 35325 & 2528 & 230400 & 5075 & 12744 & -2500 & -2538 \end{bmatrix}$$

W_{305} 46 lattices, $\chi = 36$ 10-gon: $2\mkern-6mu|\mkern-3mu22|22\mkern-6mu|\mkern-3mu22|2 \rtimes D_4$

$L_{305.1}\,:\; 1_4^{-2} 8_3^-{,}1^2 3^-{,}1^1 5^1 25^1\;\langle 2\rangle$ $24_2 25_2^r 4_2^* 600_2^* 20_2^* 24_2^* 100_2^l 1_2 600_2 5_2$

$$\begin{bmatrix} -258600 & 2400 & 2400 \\ 2400 & -20 & -25 \\ 2400 & -25 & -19 \end{bmatrix} \begin{bmatrix} 11 & 11 & 3 & 13 & -1 & -1 & 3 & 1 & 37 & 3 \\ 648 & 650 & 178 & 780 & -58 & -60 & 170 & 58 & 2160 & 176 \\ 528 & 525 & 142 & 600 & -50 & -48 & 150 & 49 & 1800 & 145 \end{bmatrix}$$

$L_{305.2}\,:\; 1_2^2 8_1^1{,}1^2 3^-{,}1^1 5^1 25^1\;\langle m\rangle$ $24_2^l 25_2 1_2^r 600_2^l 5_2^r\;(\times 2)$

$$\begin{bmatrix} -2292600 & 17400 & 6600 \\ 17400 & -95 & -50 \\ 6600 & -50 & -19 \end{bmatrix} \begin{bmatrix} 4159 & -36 & -12 \\ -3120 & 26 & 9 \\ 1450800 & -12555 & -4186 \end{bmatrix} \begin{bmatrix} 17 & 16 & 2 & 13 & -1 \\ -12 & -10 & -1 & 0 & 1 \\ 5928 & 5575 & 696 & 4500 & -350 \end{bmatrix}$$

$L_{305.3}\,:\; 1_2^{-2} 8_5^-{,}1^2 3^-{,}1^1 5^1 25^1$ $24_2^s 100_2^* 4_2^s 600_2^s 20_2^s\;(\times 2)$

$$\begin{bmatrix} -2789400 & 9000 & 9000 \\ 9000 & -5 & -30 \\ 9000 & -30 & -29 \end{bmatrix} \begin{bmatrix} 14839 & -21 & -49 \\ 171720 & -244 & -567 \\ 4420200 & -6255 & -14596 \end{bmatrix} \begin{bmatrix} 23 & 71 & 15 & 139 & 9 \\ 264 & 820 & 174 & 1620 & 106 \\ 6852 & 21150 & 4468 & 41400 & 2680 \end{bmatrix}$$

$L_{305.4}\,:\; [1^- 2^-]_0 16_7^1{,}1^2 3^-{,}1^1 5^1 25^1\;\langle 2\rangle$

$600_2^s 16_2^* 100_2^l 6_2 5_2 150_2^r 4_2^* 400_2^s 24_2^* 80_2^*$

$$\begin{bmatrix} -1232400 & -49200 & 10800 \\ -49200 & -1930 & 420 \\ 10800 & 420 & -91 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 & 2 & 32 & 7 & 67 & 11 & 5 \\ -90 & 76 & 80 & -75 & -152 & -2445 & -536 & -5140 & -846 & -388 \\ -300 & 232 & 250 & -228 & -465 & -7500 & -1646 & -15800 & -2604 & -1200 \end{bmatrix}$$

$L_{305.5}\,:\; [1^- 2^1]_6 16_5^-{,}1^2 3^-{,}1^1 5^1 25^1\;\langle m\rangle$

$150_2^r 16_2^s 100_2^* 24_2^l 5_2^r 600_2^* 4_2^s 400_2^l 6_2 80_2$

$$\begin{bmatrix} -53223600 & 17756400 & 69600 \\ 17756400 & -5923870 & -23220 \\ 69600 & -23220 & -91 \end{bmatrix} \begin{bmatrix} -26 & -11 & 7 & 31 & 22 & 587 & 59 & 521 & 38 & 19 \\ -75 & -32 & 20 & 90 & 64 & 1710 & 172 & 1520 & 111 & 56 \\ -750 & -248 & 250 & 744 & 495 & 12600 & 1234 & 10600 & 738 & 240 \end{bmatrix}$$

$L_{305.6}\,:\; [1^1 2^-]_4 16_3^-{,}1^2 3^-{,}1^1 5^1 25^1\;\langle m\rangle$

$600_2^* 16_2^l 25_2 6_2^r 20_2^l 150_2 1_2^r 400_2^* 24_2^s 80_2^s$

$$\begin{bmatrix} 66404400 & 1581600 & -8400 \\ 1581600 & 37670 & -200 \\ -8400 & -200 & 1 \end{bmatrix} \begin{bmatrix} 73 & 19 & 26 & 10 & 7 & 11 & 0 & -9 & -1 & 5 \\ -3090 & -804 & -1100 & -423 & -296 & -465 & 0 & 380 & 42 & -212 \\ -4500 & -1136 & -1525 & -576 & -390 & -600 & -1 & 400 & 12 & -360 \end{bmatrix}$$

$L_{305.7}\,:\; [1^1 2^1]_2 16_1^1{,}1^2 3^-{,}1^1 5^1 25^1$ $150_2 16_2 25_2^r 24_2^* 20_2^* 600_2^l 1_2 400_2 6_2^r 80_2^l$

$$\begin{bmatrix} 560400 & -14400 & 1200 \\ -14400 & 370 & -30 \\ 1200 & -30 & -29 \end{bmatrix} \begin{bmatrix} -121 & -51 & -59 & -37 & -7 & 7 & 1 & 1 & -5 & -33 \\ -4725 & -1992 & -2305 & -1446 & -274 & 270 & 39 & 40 & -195 & -1288 \\ -150 & -64 & -75 & -48 & -10 & 0 & 1 & 0 & -6 & -40 \end{bmatrix}$$

W_{306} 32 lattices, $\chi = 96$ 14-gon: $2\infty|\infty 22\mkern-6mu|\mkern-3mu\infty 22\infty|\infty 22\mkern-6mu|\mkern-3mu\infty 2 \rtimes D_4$

$L_{306.1}\,:\; 1_7^1 4_7^1 64_5^-{,}1^- 3^1 9^1\;\langle 3\rangle$ $576_2^* 48_{\infty z}^{48,25} 12_{\infty}^{48,13} 48_2^s 576_2^* 12_{\infty z}^{24,13} 3_2^r\;(\times 2)$

$$\begin{bmatrix} -1222848 & -37440 & -38592 \\ -37440 & -1140 & -1176 \\ -38592 & -1176 & -1213 \end{bmatrix} \begin{bmatrix} -6913 & -198 & -206 \\ -1168128 & -33463 & -34814 \\ 1354752 & 38808 & 40375 \end{bmatrix} \begin{bmatrix} 71 & 13 & 0 & -1 & 1 & 1 & 2 \\ 11304 & 2018 & -25 & -190 & 264 & 196 & 364 \\ -13248 & -2376 & 24 & 216 & -288 & -222 & -417 \end{bmatrix}$$

$L_{306.2}$: $1^{1}_{1}4^{1}_{7}64^{-}_{3},1^{-}3^{1}9^{1}$ $\langle 3\rangle$ $36^{*}_{2}48^{48,7}_{\infty z}12^{48,43}_{\infty}48^{l}_{2}9_{2}192^{12,7}_{\infty}192^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} -403776 & 7488 & 77184 \\ 7488 & -132 & -1488 \\ 77184 & -1488 & -14287 \end{bmatrix} \begin{bmatrix} 373751 & -6981 & -71063 \\ 8957520 & -167311 & -1703130 \\ 1085760 & -20280 & -206441 \end{bmatrix}$$

$$\begin{bmatrix} -31 & -33 & 62 & 1603 & 1007 & 3371 & 4197 \\ -744 & -790 & 1487 & 38426 & 24138 & 80800 & 100592 \\ -90 & -96 & 180 & 4656 & 2925 & 9792 & 12192 \end{bmatrix}$$

$L_{306.3}$: $1^{-}_{3}4^{1}_{7}64^{1}_{1},1^{-}3^{1}9^{1}$ $\langle 3\rangle$ $576^{l}_{2}12^{48,25}_{\infty}48^{48,37}_{\infty z}12_{2}576_{2}3^{24,1}_{\infty}12^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} 112997952 & 55301760 & 2316096 \\ 55301760 & 27064956 & 1133508 \\ 2316096 & 1133508 & 47471 \end{bmatrix} \begin{bmatrix} -24431401 & -11953935 & -495830 \\ 51191280 & 25047161 & 1038916 \\ -30340800 & -14845320 & -615761 \end{bmatrix}$$

$$\begin{bmatrix} 56425 & 5429 & 505 & -264 & -505 & 198 & 2221 \\ -118224 & -11375 & -1058 & 553 & 1056 & -415 & -4654 \\ 69984 & 6732 & 624 & -324 & -576 & 249 & 2766 \end{bmatrix}$$

$L_{306.4}$: $1^{-}_{5}4^{1}_{7}64^{1}_{7},1^{-}3^{1}9^{1}$ $\langle 3\rangle$ $36^{l}_{2}12^{48,7}_{\infty}48^{48,19}_{\infty z}12_{2}9^{r}_{2}192^{24,7}_{\infty z}192^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} -1155502656 & 8633088 & 549064512 \\ 8633088 & -64500 & -4102212 \\ 549064512 & -4102212 & -260900947 \end{bmatrix} \begin{bmatrix} -204281569 & 1526502 & 97074020 \\ 11446812000 & -85536751 & -5439492500 \\ -609890688 & 4557432 & 289818319 \end{bmatrix}$$

$$\begin{bmatrix} -223 & -201 & 1069 & 7733 & 9806 & 33119 & 41769 \\ 12510 & 11257 & -59930 & -433367 & -549525 & -1855928 & -2340568 \\ -666 & -600 & 3192 & 23088 & 29277 & 98880 & 124704 \end{bmatrix}$$

W_{307} 16 lattices, $\chi = 48$ 12-gon: $\not{2}2|2\not{2}2|2\not{2}2|2\not{2}2|2 \rtimes D_8$

$L_{307.1}$: $1^{1}_{1}4^{1}_{1}64^{-}_{5},1^{1}3^{-}9^{1}$ $\langle 3\rangle$ $64^{*}_{2}36^{l}_{2}4_{2}9^{r}_{2}64^{*}_{2}144^{s}_{2}$ $(\times 2)$

shares genus with its 3-dual

$$\begin{bmatrix} -61632 & -1152 & -576 \\ -1152 & -12 & -24 \\ -576 & -24 & 13 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 192 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -9 & -7 & -1 & -1 & 1 & 5 \\ 360 & 282 & 41 & 45 & -24 & -186 \\ 256 & 198 & 28 & 27 & -32 & -144 \end{bmatrix}$$

$L_{307.2}$: $1^{-}_{5}4^{1}_{1}64^{1}_{1},1^{1}3^{-}9^{1}$ $\langle 3\rangle$ $64^{s}_{2}36^{*}_{2}16^{l}_{2}9_{2}64_{2}36^{r}_{2}$ $(\times 2)$

shares genus with its 3-dual

$$\begin{bmatrix} 54398016 & -3626496 & 57600 \\ -3626496 & 241764 & -3840 \\ 57600 & -3840 & 61 \end{bmatrix} \begin{bmatrix} 83879 & -5595 & 90 \\ 1330896 & -88775 & 1428 \\ 4563072 & -304368 & 4895 \end{bmatrix} \begin{bmatrix} 25 & 35 & 21 & 41 & 119 & 46 \\ 400 & 558 & 334 & 651 & 1888 & 729 \\ 1568 & 2070 & 1192 & 2259 & 6464 & 2448 \end{bmatrix}$$

W_{308} 16 lattices, $\chi = 48$ 10-gon: $6222362223 \rtimes C_2$

$L_{308.1}$: $1^{-2}_{\mathrm{II}}8^{1}_{1},1^{-}3^{-}27^{-},1^{-2}5^{1}$ $\langle 2\rangle$ $6_{6}2^{s}_{2}54^{l}_{2}8^{r}_{2}6^{-}_{3}$ $(\times 2)$

$$\begin{bmatrix} -233462520 & 127440 & 247320 \\ 127440 & -66 & -135 \\ 247320 & -135 & -262 \end{bmatrix} \begin{bmatrix} 955151 & -550 & -1012 \\ -868320 & 499 & 920 \\ 901967400 & -519375 & -955651 \end{bmatrix} \begin{bmatrix} 19 & 4 & 4 & -1 & -1 \\ -19 & -5 & -9 & 0 & 2 \\ 17943 & 3778 & 3780 & -944 & -945 \end{bmatrix}$$

W_{309} 120 lattices, $\chi = 48$ 12-gon: $222|222222|222 \rtimes D_2$

$L_{309.1}$: $1^{2}_{0}8^{1}_{1},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$ $105_{2}8_{2}5^{r}_{2}56^{s}_{2}20^{*}_{2}8^{*}_{2}420^{l}_{2}1_{2}7^{r}_{2}24^{s}_{2}28^{*}_{2}4^{l}_{2}$

$$\begin{bmatrix} -4037880 & 8400 & 9240 \\ 8400 & -16 & -21 \\ 9240 & -21 & -19 \end{bmatrix} \begin{bmatrix} 163 & 17 & 7 & 5 & -1 & -1 & 11 & 1 & 5 & 11 & 31 & 17 \\ 40950 & 4272 & 1760 & 1260 & -250 & -252 & 2730 & 250 & 1253 & 2760 & 7784 & 4270 \\ 33915 & 3536 & 1455 & 1036 & -210 & -208 & 2310 & 209 & 1043 & 2292 & 6454 & 3538 \end{bmatrix}$$

$L_{309.2}$: $[1^{1}2^{1}]_{2}16^{1}_{7},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$ $\langle 2\rangle$

$$105_{2}2^{r}_{2}20^{*}_{2}56^{*}_{2}80^{s}_{2}8^{*}_{2}1680^{l}_{2}1_{2}112_{2}6^{r}_{2}28^{*}_{2}16^{l}_{2}$$

$$\begin{bmatrix} 2471280 & 1233120 & -1680 \\ 1233120 & 615302 & -838 \\ -1680 & -838 & 1 \end{bmatrix} \quad \begin{bmatrix} 5014 & 289 & 577 & 383 & 189 & -1 & -209 & 0 & 195 & 97 & 773 & 965 \\ -10080 & -581 & -1160 & -770 & -380 & 2 & 420 & 0 & -392 & -195 & -1554 & -1940 \\ -22995 & -1326 & -2650 & -1764 & -880 & 0 & 840 & -1 & -896 & -444 & -3542 & -4424 \end{bmatrix}$$

$L_{309.3}$: $[1^{-}2^{1}]_{6}16^{-}_{3},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$ $\langle m\rangle$

$$420^{l}_{2}2_{2}5^{r}_{2}56^{s}_{2}80^{*}_{2}8^{s}_{2}1680^{*}_{2}4^{s}_{2}112^{l}_{2}6_{2}7^{r}_{2}16^{*}_{2}$$

$$\begin{bmatrix} -186730320 & 73920 & 146160 \\ 73920 & -26 & -60 \\ 146160 & -60 & -113 \end{bmatrix}$$

$$\begin{bmatrix} 23 & 2 & 6 & 27 & 59 & 25 & 781 & 17 & 45 & 1 & -1 & -1 \\ 14490 & 1259 & 3775 & 16982 & 37100 & 15718 & 490980 & 10686 & 28280 & 627 & -630 & -628 \\ 22050 & 1918 & 5755 & 25900 & 56600 & 23984 & 749280 & 16310 & 43176 & 960 & -959 & -960 \end{bmatrix}$$

$L_{309.4}$: $[1^{-}2^{1}]_{4}16^{-}_{5},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$ $\langle m\rangle$

$$420^{*}_{2}8^{l}_{2}5_{2}14_{2}80_{2}2^{r}_{2}1680^{s}_{2}4^{*}_{2}112^{s}_{2}24^{*}_{2}28^{s}_{2}16^{s}_{2}$$

$$\begin{bmatrix} -21871920 & 25200 & 25200 \\ 25200 & -2 & -30 \\ 25200 & -30 & -29 \end{bmatrix}$$

$$\begin{bmatrix} 473 & 29 & 16 & 13 & 19 & 1 & 1 & -1 & -1 & 5 & 31 & 43 \\ 14070 & 862 & 475 & 385 & 560 & 29 & 0 & -30 & -28 & 150 & 924 & 1280 \\ 396270 & 24296 & 13405 & 10892 & 15920 & 838 & 840 & -838 & -840 & 4188 & 25970 & 36024 \end{bmatrix}$$

$L_{309.5}$: $[1^{1}2^{1}]_{0}16^{1}_{1},1^{2}3^{-},1^{-2}5^{1},1^{-2}7^{1}$

$$105^{r}_{2}8^{*}_{2}20^{l}_{2}14^{r}_{2}80^{l}_{2}2_{2}1680_{2}1^{r}_{2}112^{*}_{2}24^{l}_{2}7_{2}16_{2}$$

$$\begin{bmatrix} 1680 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} -26 & -3 & -3 & -1 & -1 & 0 & 1 & 0 & -1 & -1 & -2 & -5 \\ -315 & -34 & -30 & -7 & 0 & 1 & 0 & -1 & -28 & -18 & -28 & -64 \\ -1155 & -132 & -130 & -42 & -40 & 0 & 0 & -1 & -56 & -48 & -91 & -224 \end{bmatrix}$$

W_{310} 120 lattices, $\chi = 72$ 16-gon: $2222|2222|2222|2222| \rtimes D_4$

$L_{310.1}$: $1^{-2}_{4}8^{-}_{5},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}$ $280^{l}_{2}1_{2}168_{2}5^{r}_{2}24^{s}_{2}20^{*}_{2}168^{*}_{2}4^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} 19418280 & -57960 & 0 \\ -57960 & 173 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -47251 & 141 & -12 \\ -15813000 & 47187 & -4016 \\ 252000 & -752 & 63 \end{bmatrix} \begin{bmatrix} 159 & 8 & 125 & 16 & 19 & 23 & 71 & 7 \\ 53200 & 2677 & 41832 & 5355 & 6360 & 7700 & 23772 & 2344 \\ -980 & -46 & -672 & -80 & -84 & -90 & -252 & -22 \end{bmatrix}$$

$L_{310.2}$: $[1^{-}2^{1}]_{2}16^{-}_{3},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}$ $\langle 2\rangle$

$$280^{*}_{2}16^{s}_{2}168^{*}_{2}80^{s}_{2}24^{*}_{2}20^{l}_{2}42_{2}1^{r}_{2}\ (\times 2)$$

$$\begin{bmatrix} -64681680 & 45360 & 38640 \\ 45360 & -22 & -28 \\ 38640 & -28 & -23 \end{bmatrix} \begin{bmatrix} 30239 & -22 & -18 \\ 4203360 & -3059 & -2502 \\ 45662400 & -33220 & -27181 \end{bmatrix}$$

$$\begin{bmatrix} 43 & 11 & 53 & 33 & 13 & 21 & 40 & 5 \\ 5950 & 1524 & 7350 & 4580 & 1806 & 2920 & 5565 & 696 \\ 64960 & 16616 & 80052 & 49840 & 19632 & 31710 & 60396 & 7549 \end{bmatrix}$$

$L_{310.3}$: $[1^{1}2^{-}]_{4}16^{-}_{5},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}$ $\langle m\rangle$

$$70^{r}_{2}16^{l}_{2}42_{2}80_{2}6^{r}_{2}20^{*}_{2}168^{l}_{2}1_{2}\ (\times 2)$$

$$\begin{bmatrix} -108076080 & 114240 & 50400 \\ 114240 & -106 & -56 \\ 50400 & -56 & -23 \end{bmatrix} \begin{bmatrix} 61739 & -75 & -27 \\ 17246040 & -20951 & -7542 \\ 93268560 & -113300 & -40789 \end{bmatrix}$$

$$\begin{bmatrix} 39 & 19 & 44 & 53 & 10 & 31 & 115 & 7 \\ 10885 & 5304 & 12285 & 14800 & 2793 & 8660 & 32130 & 1956 \\ 58940 & 28712 & 66486 & 80080 & 15108 & 46830 & 173712 & 10573 \end{bmatrix}$$

$L_{310.4}$: $[1^{-}2^{-}]_{0}16^{1}_{1},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}$ $\quad 70_{2}16_{2}42^{r}_{2}80^{l}_{2}6_{2}5^{r}_{2}168^{*}_{2}4^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -22900080 & 8400 & 23520 \\ 8400 & 22 & -20 \\ 23520 & -20 & -19 \end{bmatrix} \begin{bmatrix} 69299 & 5 & -85 \\ 26306280 & 1897 & -32266 \\ 58045680 & 4188 & -71197 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 & 17 & 33 & 10 & 23 & 211 & 31 \\ -385 & 1136 & 6447 & 12520 & 3795 & 8730 & 80094 & 11768 \\ -840 & 2512 & 14238 & 27640 & 8376 & 19265 & 176736 & 25966 \end{bmatrix}$$

$L_{310.5}$: $[1^{1}2^{1}]_{6}16^{1}_{7},1^{2}3^{-},1^{-2}5^{1},1^{2}7^{-}$ $\langle m \rangle$

$280^{s}_{2}16^{*}_{2}168^{s}_{2}80^{*}_{2}24^{l}_{2}5_{2}42^{r}_{2}4^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} 1745520 & -588000 & 6720 \\ -588000 & 198074 & -2260 \\ 6720 & -2260 & 13 \end{bmatrix} \begin{bmatrix} 309119 & -103868 & 276 \\ 920640 & -309347 & 822 \\ 255360 & -85804 & 227 \end{bmatrix} \begin{bmatrix} 2797 & 689 & 3173 & 1887 & 691 & 507 & 1798 & 415 \\ 8330 & 2052 & 9450 & 5620 & 2058 & 1510 & 5355 & 1236 \\ 2240 & 560 & 2604 & 1560 & 576 & 425 & 1512 & 350 \end{bmatrix}$$

W_{311} 6 lattices, $\chi = 36$ $\quad$ 10-gon: $\not2 22|22\not2 22|22 \rtimes D_4$

$L_{311.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{7},1^{1}5^{-}25^{1},1^{2}7^{1}$ $\langle 2 \rangle$ $\quad 100^{*}_{2}4^{b}_{2}350^{s}_{2}10^{s}_{2}14^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} -1332100 & -67200 & 11200 \\ -67200 & -3390 & 565 \\ 11200 & 565 & -94 \end{bmatrix} \begin{bmatrix} 3919 & 198 & -32 \\ -82320 & -4159 & 672 \\ -29400 & -1485 & 239 \end{bmatrix} \begin{bmatrix} 93 & 17 & 47 & -1 & -1 \\ -1970 & -362 & -1015 & 19 & 21 \\ -800 & -158 & -525 & -5 & 7 \end{bmatrix}$$

W_{312} 6 lattices, $\chi = 36$ $\quad$ 10-gon: $\not2 22|22\not2 22|22 \rtimes D_4$

$L_{312.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{7},1^{-}5^{-}25^{-},1^{2}7^{1}$ $\langle 2 \rangle$ $\quad 50^{b}_{2}2^{l}_{2}700^{r}_{2}10^{l}_{2}28^{r}_{2}$ $(\times 2)$

$$\begin{bmatrix} 169820700 & -3086300 & 55300 \\ -3086300 & 56090 & -1005 \\ 55300 & -1005 & 18 \end{bmatrix} \begin{bmatrix} 174299 & -3165 & 55 \\ 9795660 & -177874 & 3091 \\ 11329500 & -205725 & 3574 \end{bmatrix} \begin{bmatrix} 28 & 3 & 149 & 2 & 1 \\ 1575 & 169 & 8400 & 113 & 56 \\ 1900 & 218 & 11200 & 165 & 56 \end{bmatrix}$$

W_{313} 12 lattices, $\chi = 12$ $\quad$ 6-gon: $22|222|2 \rtimes D_2$

$L_{313.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{5},1^{-}3^{1}9^{-},1^{-2}5^{-},1^{2}7^{-}$ $\langle 2 \rangle$ $\quad 140^{b}_{2}18^{l}_{2}84^{r}_{2}2^{b}_{2}1260^{*}_{2}12^{*}_{2}$

$$\begin{bmatrix} -7006860 & 12600 & 3780 \\ 12600 & -6 & -15 \\ 3780 & -15 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 3 & 0 & -17 & -1 \\ -350 & 345 & 1036 & 0 & -5880 & -346 \\ -700 & 702 & 2100 & -1 & -11970 & -702 \end{bmatrix}$$

W_{314} 12 lattices, $\chi = 24$ $\quad$ 8-gon: $22|2222|22 \rtimes D_2$

$L_{314.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{5},1^{-}3^{1}9^{-},1^{2}5^{1},1^{-2}7^{1}$ $\langle 2 \rangle$ $\quad 180^{r}_{2}14^{b}_{2}30^{b}_{2}126^{l}_{2}20^{r}_{2}18^{b}_{2}12^{b}_{2}2^{l}_{2}$

$$\begin{bmatrix} 764820 & 220500 & -1260 \\ 220500 & 63570 & -363 \\ -1260 & -363 & 2 \end{bmatrix} \begin{bmatrix} -17 & -2 & 7 & 230 & 191 & 59 & 9 & 0 \\ 60 & 7 & -25 & -819 & -680 & -210 & -32 & 0 \\ 180 & 14 & -120 & -3654 & -3020 & -927 & -138 & -1 \end{bmatrix}$$

W_{315} 12 lattices, $\chi = 24$ $\quad$ 8-gon: $22|22|22|22| \rtimes D_4$

$L_{315.1}$: $1^{-2}_{\mathrm{II}}4^{-}_{5},1^{1}3^{1}9^{1},1^{-2}5^{-},1^{2}7^{-}$ $\langle 2 \rangle$ $\quad 84^{r}_{2}10^{b}_{2}12^{b}_{2}90^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -817740 & 311220 & -76860 \\ 311220 & -117618 & 28407 \\ -76860 & 28407 & -6362 \end{bmatrix} \begin{bmatrix} 12599 & -4605 & 990 \\ 43680 & -15965 & 3432 \\ 42840 & -15657 & 3365 \end{bmatrix} \begin{bmatrix} -2819 & -789 & -371 & 662 \\ -9772 & -2735 & -1286 & 2295 \\ -9576 & -2680 & -1260 & 2250 \end{bmatrix}$$

W_{316} 44 lattices, $\chi = 96$ 20-gon: $22222|22222|22222|22222| \rtimes D_4$

$L_{316.1} : 1^{2}_{\mathrm{II}}4^{1}_{1},1^{1}3^{1}9^{1},1^{-2}5^{-},1^{-2}7^{1}\ \langle 2\rangle$

$210^{l}_{2}4^{r}_{2}90^{b}_{2}28^{*}_{2}36^{*}_{2}12^{*}_{2}4^{*}_{2}252^{b}_{2}10^{l}_{2}36^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} -15499260 & -187740 & 12600 \\ -187740 & -2274 & 153 \\ 12600 & 153 & -8 \end{bmatrix} \begin{bmatrix} 306641 & 3724 & -196 \\ -25032000 & -304001 & 16000 \\ 4130280 & 50160 & -2641 \end{bmatrix}$$

$$\begin{bmatrix} 653 & 81 & 611 & 419 & 139 & 21 & 5 & 17 & -3 & -5 \\ -53305 & -6612 & -49875 & -34202 & -11346 & -1714 & -408 & -1386 & 245 & 408 \\ 8820 & 1096 & 8280 & 5684 & 1890 & 288 & 70 & 252 & -40 & -72 \end{bmatrix}$$

$L_{316.2} : 1^{-2}_{2}8^{-}_{3},1^{-}3^{-}9^{-},1^{-2}5^{1},1^{-2}7^{1}\ \langle 2\rangle$

$420^{s}_{2}8^{s}_{2}180^{*}_{2}56^{b}_{2}18^{l}_{2}24^{r}_{2}2^{b}_{2}504^{*}_{2}20^{s}_{2}72^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -199080 & 20160 & 0 \\ 20160 & -2037 & -3 \\ 0 & -3 & 2 \end{bmatrix} \begin{bmatrix} 44981 & -4662 & 63 \\ 442680 & -45881 & 620 \\ 642600 & -66600 & 899 \end{bmatrix} \begin{bmatrix} -761 & -89 & -637 & -421 & -64 & -13 & 0 & 17 & 1 & -11 \\ -7490 & -876 & -6270 & -4144 & -630 & -128 & 0 & 168 & 10 & -108 \\ -10920 & -1280 & -9180 & -6076 & -927 & -192 & -1 & 252 & 20 & -144 \end{bmatrix}$$

$L_{316.3} : 1^{2}_{2}8^{1}_{7},1^{-}3^{-}9^{-},1^{-2}5^{1},1^{-2}7^{1}\ \langle m\rangle$

$105^{r}_{2}8^{l}_{2}45_{2}56^{r}_{2}18^{b}_{2}24^{b}_{2}2^{l}_{2}504_{2}5^{r}_{2}72^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} 68415480 & -204120 & -5040 \\ -204120 & 609 & 15 \\ -5040 & 15 & 2 \end{bmatrix} \begin{bmatrix} 656501 & -1960 & 49 \\ 219995160 & -656801 & 16420 \\ 4019400 & -12000 & 299 \end{bmatrix}$$

$$\begin{bmatrix} -289 & -67 & -238 & -313 & -47 & -9 & 0 & 1 & -1 & -13 \\ -96845 & -22452 & -79755 & -104888 & -15750 & -3016 & 0 & 336 & -335 & -4356 \\ -1785 & -416 & -1485 & -1960 & -297 & -60 & -1 & 0 & -5 & -72 \end{bmatrix}$$

W_{317} 32 lattices, $\chi = 36$ 10-gon: $2222|22222|2 \rtimes D_2$

$L_{317.1} : [1^{1}2^{1}]_{2}32^{1}_{1},1^{2}3^{1},1^{2}5^{-}$ $32_{2}3^{r}_{2}160^{*}_{2}4^{*}_{2}40^{l}_{2}1^{r}_{2}160^{*}_{2}12^{s}_{2}32^{l}_{2}2_{2}$

$$\begin{bmatrix} 222240 & -36960 & -480 \\ -36960 & 6146 & 80 \\ -480 & 80 & 1 \end{bmatrix} \begin{bmatrix} -55 & -32 & -253 & -13 & -9 & 0 & 7 & 1 & -3 & -2 \\ -304 & -177 & -1400 & -72 & -50 & 0 & 40 & 6 & -16 & -11 \\ -2016 & -1167 & -9200 & -470 & -320 & -1 & 160 & 6 & -144 & -76 \end{bmatrix}$$

$L_{317.2} : [1^{-}2^{1}]_{2}32^{-}_{5},1^{2}3^{1},1^{2}5^{-}$ $32^{*}_{2}12^{s}_{2}160^{s}_{2}4^{l}_{2}10_{2}1_{2}160_{2}3^{r}_{2}32^{*}_{2}8^{s}_{2}$

$$\begin{bmatrix} 1011360 & 12480 & 0 \\ 12480 & 154 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -85 & -97 & -379 & -19 & -6 & 0 & 1 & -1 & -9 & -7 \\ 6872 & 7842 & 30640 & 1536 & 485 & 0 & -80 & 81 & 728 & 566 \\ 64 & 66 & 240 & 10 & 0 & -1 & 0 & 3 & 16 & 8 \end{bmatrix}$$

$L_{317.3} : 1^{1}_{1}4^{1}_{1}32^{1}_{1},1^{2}3^{-},1^{2}5^{1}$ $1^{r}_{2}96^{*}_{2}20^{s}_{2}32^{l}_{2}20_{2}32_{2}5^{r}_{2}96^{*}_{2}4^{l}_{2}4_{2}$

$$\begin{bmatrix} 206880 & -61920 & -480 \\ -61920 & 18532 & 144 \\ -480 & 144 & 1 \end{bmatrix} \begin{bmatrix} 0 & 11 & 3 & -5 & -17 & -79 & -91 & -359 & -37 & -4 \\ 0 & 36 & 10 & -16 & -55 & -256 & -295 & -1164 & -120 & -13 \\ -1 & 96 & 10 & -80 & -220 & -992 & -1135 & -4464 & -458 & -48 \end{bmatrix}$$

$L_{317.4} : 1^{-}_{5}4^{1}_{7}32^{-}_{3},1^{2}3^{-},1^{2}5^{1}$ $4^{s}_{2}96^{s}_{2}20^{*}_{2}32^{*}_{2}80^{s}_{2}32^{l}_{2}5_{2}96_{2}1^{r}_{2}16^{*}_{2}$

$$\begin{bmatrix} 5520480 & 221760 & -5280 \\ 221760 & 8908 & -212 \\ -5280 & -212 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 & -3 & -5 & -11 & -15 & -14 & -49 & -2 & 1 \\ -26 & -24 & 80 & 132 & 290 & 396 & 370 & 1296 & 53 & -26 \\ -46 & 48 & 230 & 320 & 680 & 944 & 895 & 3168 & 133 & -48 \end{bmatrix}$$

W_{318} 48 lattices, $\chi = 48$ 12-gon: $22|222|222|222|2 \rtimes D_4$

$L_{318.1} : [1^{-}2^{1}]_{6}32^{-}_{5},1^{-}3^{1}9^{-},1^{-2}5^{1}\ \langle 3\rangle$

$32^{l}_{2}45_{2}2^{r}_{2}180^{*}_{2}32^{s}_{2}120^{*}_{2}288^{l}_{2}5_{2}18^{r}_{2}20^{*}_{2}288^{s}_{2}120^{*}_{2}$

$$\begin{bmatrix} -1703520 & 18720 & 1440 \\ 18720 & 102 & -24 \\ 1440 & -24 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 & 61 & 37 & 73 & 115 & 11 & 4 & 3 & 1 & -3 \\ -24 & 45 & 23 & 1410 & 856 & 1690 & 2664 & 255 & 93 & 70 & 24 & -70 \\ -880 & 1755 & 878 & 53550 & 32480 & 64080 & 100944 & 9655 & 3510 & 2630 & 864 & -2640 \end{bmatrix}$$

$L_{318.2} : [1^{1}2^{1}]_{6}32^{1}_{1},1^{-}3^{1}9^{-},1^{-2}5^{1}\ \langle 3\rangle$

$32^{s}_{2}180^{*}_{2}8^{l}_{2}45_{2}32_{2}30^{r}_{2}288^{s}_{2}20^{*}_{2}72^{l}_{2}5_{2}288_{2}30^{r}_{2}$

$$\begin{bmatrix} -5418720 & 21600 & 2880 \\ 21600 & -6 & -18 \\ 2880 & -18 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 1 & 17 & 21 & 21 & 67 & 13 & 5 & 1 & 1 & -1 \\ -96 & 90 & 94 & 1605 & 1984 & 1985 & 6336 & 1230 & 474 & 95 & 96 & -95 \\ -1168 & 1170 & 1168 & 19845 & 24512 & 24510 & 78192 & 15170 & 5832 & 1165 & 1152 & -1170 \end{bmatrix}$$

$L_{318.3} : 1^{1}_{1}4^{1}_{7}32^{1}_{7},1^{1}3^{-}9^{1},1^{-2}5^{-}\ \langle 3\rangle$

$4^{*}_{2}1440^{s}_{2}16^{*}_{2}1440^{l}_{2}1_{2}60^{r}_{2}36^{*}_{2}160^{s}_{2}144^{*}_{2}160^{l}_{2}9_{2}60^{r}_{2}$

$$\begin{bmatrix} -5916960 & -56160 & 12960 \\ -56160 & 204 & 60 \\ 12960 & 60 & -23 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -109 & -3 & -1 & 1 & 13 & 13 & 23 & 5 & -13 & -7 & -23 \\ -436 & -6780 & -186 & -60 & 62 & 805 & 804 & 1420 & 306 & -820 & -438 & -1435 \\ -5090 & -79200 & -2176 & -720 & 725 & 9420 & 9414 & 16640 & 3600 & -9520 & -5103 & -16740 \end{bmatrix}$$

$L_{318.4} : 1^{-}_{5}4^{1}_{1}32^{-}_{5},1^{1}3^{-}9^{1},1^{-2}5^{-}\ \langle 3\rangle$

$4^{s}_{2}1440^{l}_{2}4_{2}1440_{2}1^{r}_{2}240^{*}_{2}36^{s}_{2}160^{l}_{2}36_{2}160_{2}9^{r}_{2}240^{*}_{2}$

$$\begin{bmatrix} 1440 & 0 & 0 \\ 0 & -156 & -60 \\ 0 & -60 & -23 \end{bmatrix} \quad \begin{bmatrix} -1 & -11 & 0 & 1 & 0 & -1 & -1 & -3 & -1 & -7 & -2 & -9 \\ 16 & 240 & 3 & 0 & -2 & -50 & -24 & -40 & -3 & 40 & 18 & 110 \\ -50 & -720 & -8 & 0 & 5 & 120 & 54 & 80 & 0 & -160 & -63 & -360 \end{bmatrix}$$

W_{319} 40 lattices, $\chi = 48$ 12-gon: $22|222|222|222|2 \rtimes D_4$

$L_{319.1} : [1^{1}2^{-}]_{4}32^{-}_{3},1^{2}3^{1},1^{-2}5^{1}$ $480^{l}_{2}1^{r}_{2}120^{*}_{2}4^{*}_{2}480^{s}_{2}8^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -988559520 & -12201600 & 88800 \\ -12201600 & -150602 & 1096 \\ 88800 & 1096 & -7 \end{bmatrix} \begin{bmatrix} -4330241 & -53448 & 408 \\ 350940480 & 4331645 & -33066 \\ 14901120 & 183924 & -1405 \end{bmatrix}$$

$$\begin{bmatrix} -3179 & -113 & -1427 & -295 & -5249 & -205 \\ 257640 & 9158 & 115650 & 23908 & 425400 & 16614 \\ 11040 & 391 & 4920 & 1014 & 18000 & 700 \end{bmatrix}$$

$L_{319.2} : 1^{1}_{7}4^{1}_{1}32^{1}_{7},1^{2}3^{-},1^{-2}5^{-}$ $15^{r}_{2}32^{s}_{2}240^{*}_{2}32^{*}_{2}60^{l}_{2}4_{2}\ (\times 2)$

$$\begin{bmatrix} -348960 & -1440 & -2400 \\ -1440 & 4 & -4 \\ -2400 & -4 & -13 \end{bmatrix} \begin{bmatrix} -641 & -2 & -4 \\ -86400 & -271 & -540 \\ 145920 & 456 & 911 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 3 & 13 & 2 \\ -135 & -132 & 150 & 412 & 1770 & 271 \\ 225 & 224 & -240 & -688 & -2970 & -456 \end{bmatrix}$$

$L_{319.3} : [1^{1}2^{1}]_{0}64^{1}_{7},1^{2}3^{-},1^{-2}5^{-}\ \langle m\rangle$

$960^{s}_{2}8^{*}_{2}60^{l}_{2}2_{2}960_{2}1^{r}_{2}960^{*}_{2}8^{l}_{2}15_{2}2^{r}_{2}960^{s}_{2}4^{*}_{2}$

$$\begin{bmatrix} -7901760 & 11520 & 20160 \\ 11520 & -14 & -32 \\ 20160 & -32 & -49 \end{bmatrix} \quad \begin{bmatrix} 263 & 15 & 37 & 6 & 173 & 1 & 17 & -1 & -1 & 1 & 107 & 5 \\ 62640 & 3574 & 8820 & 1431 & 41280 & 239 & 4080 & -238 & -240 & 237 & 25440 & 1190 \\ 67200 & 3832 & 9450 & 1532 & 44160 & 255 & 4320 & -256 & -255 & 256 & 27360 & 1278 \end{bmatrix}$$

$L_{319.4} : 1^{1}_{7}4^{1}_{7}32^{1}_{1},1^{2}3^{-},1^{-2}5^{-}$ $15_{2}32_{2}60^{r}_{2}32^{s}_{2}60^{*}_{2}16^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} 759840 & -47520 & -2880 \\ -47520 & 2972 & 180 \\ -2880 & 180 & 11 \end{bmatrix} \begin{bmatrix} 8059 & -507 & -26 \\ 122760 & -7723 & -396 \\ 104160 & -6552 & -337 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & -7 & -19 & -77 & -23 \\ -15 & 16 & -105 & -288 & -1170 & -350 \\ -15 & 0 & -120 & -272 & -1050 & -304 \end{bmatrix}$$

W_{320} 8 lattices, $\chi = 48$ 12-gon: $2|2|2|2|2|2|2|2|2|2|2|2| \rtimes D_{12}$

$L_{320.1} : 1^1_1 4^1_1 16^1_1, 1^1 3^- 9^-, 1^{-2} 5^1 \ \langle 3 \rangle$

$1_2 720_2 4_2 45_2 16_2 180^r_2 4^s_2 720^l_2 4^r_2 180^s_2 16^l_2 180_2$

$$\begin{bmatrix} 769680 & 9360 & -1440 \\ 9360 & -444 & 168 \\ -1440 & 168 & -59 \end{bmatrix} \begin{bmatrix} 3 & 133 & 8 & 32 & 11 & 29 & 1 & -7 & -1 & -1 & 3 & 34 \\ -459 & -20340 & -1223 & -4890 & -1680 & -4425 & -152 & 1080 & 153 & 150 & -460 & -5205 \\ -1381 & -61200 & -3680 & -14715 & -5056 & -13320 & -458 & 3240 & 460 & 450 & -1384 & -15660 \end{bmatrix}$$

W_{321} 12 lattices, $\chi = 48$ 12-gon: $22|222|222|222|2 \rtimes D_4$

$L_{321.1} : 1^{-2}_{II} 4^1_1, 1^- 3^1 9^1, 1^1 7^1 49^1 \ \langle 23, 3, 2 \rangle$ $1764^r_2 2^b_2 252^b_2 98^l_2 36^r_2 14^l_2 \ (\times 2)$

$$\begin{bmatrix} -82908 & 24696 & -1764 \\ 24696 & -6846 & 483 \\ -1764 & 483 & -34 \end{bmatrix} \begin{bmatrix} 503 & -142 & 10 \\ 11844 & -3338 & 235 \\ 142884 & -40257 & 2834 \end{bmatrix} \begin{bmatrix} -65 & -1 & -1 & 1 & 1 & 0 \\ -1596 & -25 & -30 & 21 & 24 & 1 \\ -19404 & -305 & -378 & 245 & 288 & 14 \end{bmatrix}$$

W_{322} 12 lattices, $\chi = 24$ 8-gon: $22222222 \rtimes C_2$

$L_{322.1} : 1^{-2}_{II} 4^1_1, 1^2 3^-, 1^1 7^- 49^- \ \langle 2 \rangle$ $2^s_2 294^l_2 4^r_2 42^b_2 \ (\times 2)$

$$\begin{bmatrix} 105252 & -5292 & -2352 \\ -5292 & 266 & 119 \\ -2352 & 119 & 46 \end{bmatrix} \begin{bmatrix} 503 & -26 & -6 \\ 9324 & -482 & -111 \\ 1764 & -91 & -22 \end{bmatrix} \begin{bmatrix} 1 & 1 & -3 & -8 \\ 19 & 21 & -56 & -150 \\ 2 & 0 & -8 & -21 \end{bmatrix}$$

W_{323} 30 lattices, $\chi = 24$ 8-gon: $22|22|22|22| \rtimes D_4$

$L_{323.1} : 1^2_0 8^1_7, 1^1 3^- 9^1, 1^{-2} 5^-$ $360^s_2 4^*_2 60^*_2 36^s_2 40^l_2 9_2 15_2 1^r_2$

$$\begin{bmatrix} 2711160 & 720 & -15480 \\ 720 & -3 & -3 \\ -15480 & -3 & 88 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 7 & 17 & 8 & 7 & 1 \\ 60 & -62 & -70 & 414 & 1020 & 483 & 425 & 61 \\ 180 & -178 & -180 & 1242 & 3020 & 1422 & 1245 & 178 \end{bmatrix}$$

$L_{323.2} : [1^- 2^1]_2 16^-_5, 1^1 3^- 9^1, 1^{-2} 5^- \ \langle 2 \rangle$ $10_2 9^r_2 240^l_2 1_2 90^r_2 16^s_2 60^s_2 144^l_2$

$$\begin{bmatrix} 23253840 & 415440 & 5040 \\ 415440 & 7422 & 90 \\ 5040 & 90 & 1 \end{bmatrix} \begin{bmatrix} -7 & -5 & -11 & 0 & 4 & 1 & -3 & -17 \\ 395 & 282 & 620 & 0 & -225 & -56 & 170 & 960 \\ -250 & -171 & -360 & -1 & 90 & 8 & -150 & -648 \end{bmatrix}$$

$L_{323.3} : [1^1 2^1]_0 16^1_7, 1^1 3^- 9^1, 1^{-2} 5^- \ \langle m \rangle$ $40^l_2 9_2 240_2 1^r_2 360^*_2 16^l_2 15^r_2 144^*_2$

$$\begin{bmatrix} -3378960 & -51840 & 22320 \\ -51840 & -786 & 336 \\ 22320 & 336 & -143 \end{bmatrix} \begin{bmatrix} 9 & 5 & 21 & 1 & 7 & -1 & -1 & 5 \\ -1550 & -864 & -3640 & -174 & -1230 & 172 & 175 & -852 \\ -2240 & -1251 & -5280 & -253 & -1800 & 248 & 255 & -1224 \end{bmatrix}$$

$L_{323.4} : [1^1 2^1]_6 16^1_1, 1^1 3^- 9^1, 1^{-2} 5^- \ \langle m \rangle$ $10^r_2 36^*_2 240^*_2 4^l_2 90_2 16_2 15_2 144_2$

$$\begin{bmatrix} -3054960 & 2160 & 7920 \\ 2160 & 42 & -18 \\ 7920 & -18 & -17 \end{bmatrix} \begin{bmatrix} 8 & 7 & -1 & -1 & -1 & 3 & 6 & 29 \\ 815 & 714 & -100 & -102 & -105 & 304 & 610 & 2952 \\ 2860 & 2502 & -360 & -358 & -360 & 1072 & 2145 & 10368 \end{bmatrix}$$

$L_{323.5} : [1^- 2^1]_4 16^-_3, 1^1 3^- 9^1, 1^{-2} 5^-$ $40^*_2 36^s_2 240^s_2 4^*_2 360^s_2 16^*_2 60^*_2 144^s_2$

$$\begin{bmatrix} -1711440 & -25200 & 10080 \\ -25200 & -354 & 138 \\ 10080 & 138 & -53 \end{bmatrix} \begin{bmatrix} 7 & 7 & 13 & 1 & 1 & -1 & -1 & 5 \\ -1370 & -1374 & -2560 & -198 & -210 & 196 & 200 & -972 \\ -2240 & -2250 & -4200 & -326 & -360 & 320 & 330 & -1584 \end{bmatrix}$$

W_{324} 38 lattices, $\chi = 72$ 16-gon: $22|22|22|22|22|22|22|22| \rtimes D_8$

$L_{324.1} : 1^{-2}_4 8^-_3, 1^- 3^- 9^-, 1^2 5^1$ $24_2 5^r_2 72^s_2 20^*_2 24^*_2 180^s_2 8^l_2 45_2 \ (\times 2)$

$$\begin{bmatrix} 362520 & 1800 & -2160 \\ 1800 & -3 & -6 \\ -2160 & -6 & 11 \end{bmatrix} \begin{bmatrix} -379 & -14 & 7 \\ -36720 & -1361 & 680 \\ -93960 & -3480 & 1739 \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 & -1 & -1 & 7 & 3 & 16 \\ 488 & 195 & 96 & -100 & -100 & 660 & 288 & 1545 \\ 1248 & 500 & 252 & -250 & -252 & 1710 & 740 & 3960 \end{bmatrix}$$

$L_{324.2}$: $[1^{-}2^{-}]_{0}16^{1}_{7},1^{-}3^{-}9^{-},1^{2}5^{1}$ $\langle 2\rangle$ $24^{*}_{2}720^{s}_{2}8^{l}_{2}45_{2}6_{2}5^{r}_{2}72^{s}_{2}80^{*}_{2}$ $(\times 2)$

$$\begin{bmatrix} 18641520 & 442800 & -14400 \\ 442800 & 10518 & -342 \\ -14400 & -342 & 11 \end{bmatrix} \begin{bmatrix} -4429 & -105 & 3 \\ 191880 & 4549 & -130 \\ 177120 & 4200 & -121 \end{bmatrix} \begin{bmatrix} 5 & 91 & 9 & 29 & 5 & 8 & 19 & 17 \\ -214 & -3900 & -386 & -1245 & -215 & -345 & -822 & -740 \\ -108 & -2160 & -224 & -765 & -144 & -265 & -720 & -800 \end{bmatrix}$$

$L_{324.3}$: $[1^{1}2^{-}]_{4}16^{-}_{3},1^{-}3^{-}9^{-},1^{2}5^{1}$ $\langle m\rangle$ $24^{s}_{2}720^{*}_{2}8^{*}_{2}180^{l}_{2}6^{r}_{2}20^{*}_{2}72^{*}_{2}80^{s}_{2}$ $(\times 2)$

$$\begin{bmatrix} 195120 & 2880 & -2160 \\ 2880 & 42 & -30 \\ -2160 & -30 & 17 \end{bmatrix} \begin{bmatrix} 251 & 3 & 0 \\ -21000 & -251 & 0 \\ -5040 & -60 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 & 13 & 2 & 11 & 19 & 27 \\ 86 & -60 & -82 & -1080 & -167 & -920 & -1590 & -2260 \\ 24 & 0 & -20 & -270 & -42 & -230 & -396 & -560 \end{bmatrix}$$

$L_{324.4}$: $[1^{1}2^{1}]_{2}16^{1}_{1},1^{-}3^{-}9^{-},1^{2}5^{1}$ $\langle m\rangle$ $6^{r}_{2}720^{l}_{2}2^{r}_{2}180^{*}_{2}24^{*}_{2}20^{l}_{2}18^{r}_{2}80^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -49981680 & 1281600 & -18720 \\ 1281600 & -32862 & 480 \\ -18720 & 480 & -7 \end{bmatrix} \begin{bmatrix} -65089 & 1668 & -24 \\ -2549280 & 65329 & -940 \\ -650880 & 16680 & -241 \end{bmatrix} \begin{bmatrix} 2 & 119 & 7 & 107 & 23 & 49 & 37 & 93 \\ 79 & 4680 & 275 & 4200 & 902 & 1920 & 1449 & 3640 \\ 66 & 2520 & 128 & 1710 & 312 & 550 & 360 & 760 \end{bmatrix}$$

$L_{324.5}$: $1^{-}_{5}8^{1}_{1}64^{-}_{5},1^{-}3^{-}9^{-},1^{2}5^{1}$ $\langle 3,2\rangle$

$96^{*}_{2}20^{l}_{2}72_{2}320^{r}_{2}24^{b}_{2}2880^{l}_{2}8_{2}45^{r}_{2}$ $(\times 2)$

shares genus with its 2-dual $\cong$ 3-dual; isometric to its own 2.3-dual

$$\begin{bmatrix} -29338879680 & 752279040 & -1123200 \\ 752279040 & -19289208 & 28800 \\ -1123200 & 28800 & -43 \end{bmatrix} \begin{bmatrix} -1882369 & 48264 & -72 \\ -74510400 & 1910449 & -2850 \\ -734123520 & 18822960 & -28081 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -1 & 11 & 5 & 263 & 15 & 56 \\ 38 & -40 & -39 & 440 & 199 & 10440 & 595 & 2220 \\ -672 & -670 & 0 & 7360 & 2676 & 122400 & 6688 & 24075 \end{bmatrix}$$

W_{325} 12 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{325.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1},1^{2}9^{1},1^{1}5^{-}25^{-}$ $\langle 2\rangle$ $50^{l}_{2}36^{r}_{2}10^{l}_{2}4^{r}_{2}90^{b}_{2}$ $(\times 2)$

$$\begin{bmatrix} 6811107300 & -193193100 & -997200 \\ -193193100 & 5479810 & 28285 \\ -997200 & 28285 & 146 \end{bmatrix} \begin{bmatrix} 5702399 & -161766 & -891 \\ 201427200 & -5714099 & -31473 \\ -74880000 & 2124200 & 11699 \end{bmatrix} \begin{bmatrix} -1 & 1 & 3 & 331 & 1742 \\ -35 & 36 & 106 & 11692 & 61533 \\ -50 & -144 & -45 & -4348 & -22860 \end{bmatrix}$$

W_{326} 4 lattices, $\chi = 48$ 10-gon: $6222362223 \rtimes C_2$

$L_{326.1}$: $1^{-2}_{\mathrm{II}}16^{-}_{5},1^{-}3^{-}27^{-}$ $6_{6}2^{b}_{2}54^{s}_{2}2^{b}_{2}6^{-}_{3}$ $(\times 2)$

$$\begin{bmatrix} -13665456 & -44928 & 35856 \\ -44928 & -138 & 117 \\ 35856 & 117 & -94 \end{bmatrix} \begin{bmatrix} 664289 & 2365 & -1760 \\ 25895232 & 92191 & -68608 \\ 285523920 & 1016520 & -756481 \end{bmatrix} \begin{bmatrix} 2 & -1 & -2 & 4 & 21 \\ 79 & -39 & -81 & 155 & 817 \\ 861 & -430 & -864 & 1718 & 9024 \end{bmatrix}$$

W_{327} 32 lattices, $\chi = 36$ 10-gon: $222|22222|22 \rtimes D_2$

$L_{327.1}$: $[1^{-}2^{1}]_{6}32^{1}_{7},1^{2}3^{-},1^{2}7^{1}$ $7_{2}6^{r}_{2}224^{l}_{2}2_{2}224_{2}6^{r}_{2}28^{*}_{2}96^{*}_{2}56^{s}_{2}96^{l}_{2}$

$$\begin{bmatrix} -261408 & 0 & -672 \\ 0 & 22 & 18 \\ -672 & 18 & 13 \end{bmatrix} \begin{bmatrix} -1 & -1 & -5 & 0 & 9 & 2 & 5 & 5 & 1 & -1 \\ -315 & -315 & -1568 & 1 & 2912 & 645 & 1610 & 1608 & 322 & -312 \\ 385 & 384 & 1904 & -2 & -3584 & -792 & -1974 & -1968 & -392 & 384 \end{bmatrix}$$

$L_{327.2}$: $[1^{1}2^{1}]_{6}32^{-}_{3},1^{2}3^{-},1^{2}7^{1}$ $7^{r}_{2}24^{*}_{2}224^{s}_{2}8^{*}_{2}224^{s}_{2}24^{*}_{2}28^{s}_{2}96^{l}_{2}14_{2}96_{2}$

$$\begin{bmatrix} 1375584 & -449568 & -8736 \\ -449568 & 146926 & 2856 \\ -8736 & 2856 & 55 \end{bmatrix} \begin{bmatrix} 57 & 59 & 19 & -21 & -359 & -103 & -75 & 1 & 31 & 163 \\ 168 & 174 & 56 & -62 & -1064 & -306 & -224 & 0 & 91 & 480 \\ 329 & 336 & 112 & -116 & -1792 & -480 & -294 & 144 & 196 & 960 \end{bmatrix}$$

$L_{327.3}$: $1^{1}_{7}4^{1}_{1}32^{-}_{5},1^{2}3^{1},1^{2}7^{1}$ $224^{s}_{2}48^{*}_{2}28^{l}_{2}4_{2}7^{r}_{2}48^{*}_{2}224^{l}_{2}3^{r}_{2}112^{*}_{2}12^{*}_{2}$

$$\begin{bmatrix} -1907808 & -6720 & 7392 \\ -6720 & 4 & 16 \\ 7392 & 16 & -25 \end{bmatrix} \begin{bmatrix} 19 & 11 & 9 & 1 & 1 & -1 & -9 & -1 & -3 & 1 \\ 2548 & 1494 & 1232 & 139 & 140 & -138 & -1260 & -141 & -434 & 126 \\ 7168 & 4176 & 3430 & 384 & 385 & -384 & -3472 & -387 & -1176 & 366 \end{bmatrix}$$

$L_{327.4} : 1^{-}_{3}4^{1}_{7}32^{1}_{7},1^{2}3^{1},1^{2}7^{1}$ $\qquad 224_{2}12^{r}_{2}28^{*}_{2}16^{l}_{2}7_{2}12^{r}_{2}224^{s}_{2}12^{l}_{2}28_{2}3_{2}$

$$\begin{bmatrix} 16570848 & 2688 & -30240 \\ 2688 & -4 & -4 \\ -30240 & -4 & 55 \end{bmatrix} \quad \begin{bmatrix} 29 & 8 & 13 & 3 & 3 & 2 & 1 & -1 & -1 & 1 \\ 3136 & 873 & 1428 & 334 & 336 & 225 & 112 & -114 & -119 & 105 \\ 16128 & 4452 & 7238 & 1672 & 1673 & 1116 & 560 & -558 & -560 & 555 \end{bmatrix}$$

W_{328} 32 lattices, $\chi = 96$ $\qquad$ 20-gon: $2|22222|22222|22222|2222 \rtimes D_4$

$L_{328.1} : [1^{1}2^{1}]_{0}32^{-}_{5},1^{2}3^{-},1^{-2}7^{-}$

$672_{2}2^{r}_{2}672^{s}_{2}4^{*}_{2}24^{s}_{2}32^{*}_{2}168^{s}_{2}32^{*}_{2}24^{l}_{2}1_{2}\ (\times 2)$

$$\begin{bmatrix} 81312 & -2016 & 2016 \\ -2016 & 14 & -8 \\ 2016 & -8 & 1 \end{bmatrix} \begin{bmatrix} -2113 & 4 & 4 \\ -598752 & 1133 & 1134 \\ -517440 & 980 & 979 \end{bmatrix} \begin{bmatrix} -173 & -3 & -103 & -3 & -5 & -3 & -1 & 1 & 1 & 0 \\ -49056 & -851 & -29232 & -852 & -1422 & -856 & -294 & 280 & 282 & 0 \\ -42336 & -734 & -25200 & -734 & -1224 & -736 & -252 & 240 & 240 & -1 \end{bmatrix}$$

$L_{328.2} : [1^{1}2^{-}]_{4}32^{1}_{1},1^{2}3^{-},1^{-2}7^{-}$ $\qquad 672^{*}_{2}8^{s}_{2}672^{*}_{2}4^{l}_{2}6_{2}32_{2}42^{r}_{2}32^{l}_{2}6_{2}1^{r}_{2}\ (\times 2)$

$$\begin{bmatrix} 250656 & 2016 & -1344 \\ 2016 & -22 & -8 \\ -1344 & -8 & 7 \end{bmatrix} \begin{bmatrix} -937 & -21 & 6 \\ -13104 & -295 & 84 \\ -192192 & -4312 & 1231 \end{bmatrix}$$

$$\begin{bmatrix} 65 & 1 & -5 & -1 & -2 & -5 & -4 & -1 & 1 & 1 \\ 840 & 10 & -168 & -18 & -33 & -80 & -63 & -16 & 15 & 15 \\ 13104 & 192 & -1344 & -218 & -426 & -1056 & -840 & -208 & 210 & 209 \end{bmatrix}$$

$L_{328.3} : 1^{-}_{5}4^{1}_{1}32^{1}_{7},1^{2}3^{1},1^{-2}7^{-}$ $\qquad 84^{l}_{2}4_{2}21^{r}_{2}32^{*}_{2}48^{l}_{2}1_{2}84^{r}_{2}4^{*}_{2}48^{s}_{2}32^{*}_{2}\ (\times 2)$

$$\begin{bmatrix} -2250528 & 750624 & 4032 \\ 750624 & -250348 & -1336 \\ 4032 & -1336 & 1 \end{bmatrix} \begin{bmatrix} -1231201 & 409374 & 1026 \\ -3712800 & 1234505 & 3094 \\ 3964800 & -1318296 & -3305 \end{bmatrix}$$

$$\begin{bmatrix} -7507 & -834 & -5773 & -4169 & -6349 & -1026 & -8656 & -1731 & -4423 & -2243 \\ -22638 & -2515 & -17409 & -12572 & -19146 & -3094 & -26103 & -5220 & -13338 & -6764 \\ 24150 & 2684 & 18585 & 13424 & 20448 & 3305 & 27888 & 5578 & 14256 & 7232 \end{bmatrix}$$

$L_{328.4} : 1^{-}_{5}4^{1}_{7}32^{1}_{1},1^{2}3^{1},1^{-2}7^{-}$ $\qquad 21^{r}_{2}16^{*}_{2}84^{s}_{2}32^{l}_{2}12_{2}1^{r}_{2}336^{*}_{2}4^{l}_{2}12_{2}32_{2}\ (\times 2)$

$$\begin{bmatrix} -867552 & -672 & 2688 \\ -672 & 76 & -8 \\ 2688 & -8 & -7 \end{bmatrix} \begin{bmatrix} 13259 & 85 & -51 \\ 578760 & 3709 & -2226 \\ 4411680 & 28280 & -16969 \end{bmatrix}$$

$$\begin{bmatrix} 16 & 11 & 109 & 47 & 43 & 16 & 307 & 35 & 52 & 65 \\ 693 & 478 & 4746 & 2048 & 1875 & 698 & 13398 & 1528 & 2271 & 2840 \\ 5313 & 3656 & 36246 & 15632 & 14304 & 5323 & 102144 & 11646 & 17304 & 21632 \end{bmatrix}$$

W_{329} 8 lattices, $\chi = 144$ $\qquad$ 16-gon: $\infty\infty\infty 2|2\infty\infty\infty|\infty\infty\infty 2|2\infty\infty\infty| \rtimes D_4$

$L_{329.1} : 1^{2}_{0}4^{1}_{7},1^{-}7^{1}49^{1}$ $\qquad 28^{7,4}_{\infty z}7^{28,11}_{\infty}28^{7,1}_{\infty b}28_{2}49_{2}7^{28,15}_{\infty}28^{7,4}_{\infty a}28^{14,9}_{\infty}\ (\times 2)$

$$\begin{bmatrix} -143668 & -1960 & -84476 \\ -1960 & -21 & -1113 \\ -84476 & -1113 & -49400 \end{bmatrix} \begin{bmatrix} -883 & -7 & -484 \\ -10584 & -85 & -5808 \\ 1764 & 14 & 967 \end{bmatrix} \quad \begin{bmatrix} 173 & 90 & 381 & 499 & 267 & 125 & 49 & -69 \\ 2400 & 1243 & 5228 & 6812 & 3619 & 1669 & 596 & -988 \\ -350 & -182 & -770 & -1008 & -539 & -252 & -98 & 140 \end{bmatrix}$$

$L_{329.2} : 1^{2}_{\mathrm{II}}8^{1}_{7},1^{-}7^{1}49^{1}$ $\qquad 56^{7,2}_{\infty z}14^{28,11}_{\infty b}56^{7,2}_{\infty a}56^{r}_{2}98^{s}_{2}14^{28,15}_{\infty a}56^{7,1}_{\infty b}56^{7,2}_{\infty}\ (\times 2)$

$$\begin{bmatrix} -56840 & 18424 & 7448 \\ 18424 & -5964 & -2401 \\ 7448 & -2401 & -954 \end{bmatrix} \begin{bmatrix} 2057 & -686 & -301 \\ 8232 & -2745 & -1204 \\ -4704 & 1568 & 687 \end{bmatrix} \quad \begin{bmatrix} 127 & 85 & 475 & 741 & 484 & 312 & 319 & 53 \\ 516 & 344 & 1916 & 2984 & 1946 & 1252 & 1276 & 208 \\ -308 & -203 & -1120 & -1736 & -1127 & -721 & -728 & -112 \end{bmatrix}$$

W_{330} 8 lattices, $\chi = 36$ $\qquad$ 6-gon: $2\infty 2\infty\infty\infty$

$L_{330.1} : 1^{-2}_{\mathrm{II}}8^{-}_{5},1^{1}9^{-}81^{-}\ \langle 2\rangle$ $\qquad 72^{b}_{2}162^{12,1}_{\infty b}648^{b}_{2}18^{36,1}_{\infty b}72^{18,13}_{\infty z}18^{36,13}_{\infty a}$

$$\begin{bmatrix} -1577880 & 14256 & -166536 \\ 14256 & -90 & 1521 \\ -166536 & 1521 & -17570 \end{bmatrix} \quad \begin{bmatrix} 63 & 124 & -815 & -375 & -811 & -62 \\ 244 & 477 & -3168 & -1454 & -3140 & -239 \\ -576 & -1134 & 7452 & 3429 & 7416 & 567 \end{bmatrix}$$

W_{331} 12 lattices, $\chi = 20$ 7-gon: $\not{3}222|222 \rtimes D_2$

$L_{331.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1}{,}1^{1}3^{-}9^{1}{,}1^{-2}5^{1}{,}1^{2}11^{-}$ $\langle 2\rangle$ $6^{+}_{3}6^{b}_{2}22^{l}_{2}36^{r}_{2}330^{l}_{2}4^{r}_{2}198^{b}_{2}$

$$\begin{bmatrix} -100585980 & 132660 & -7815060 \\ 132660 & -174 & 10491 \\ -7815060 & 10491 & -572006 \end{bmatrix} \quad \begin{bmatrix} 99 & -98 & -657 & -1283 & -2668 & -165 & 196 \\ 57391 & -56810 & -380864 & -743760 & -1546655 & -95652 & 113619 \\ -300 & 297 & 1991 & 3888 & 8085 & 500 & -594 \end{bmatrix}$$

W_{332} 12 lattices, $\chi = 40$ 8-gon: $6|62|26|62|2 \rtimes D_4$

$L_{332.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1}{,}1^{-}3^{-}9^{-}{,}1^{-2}5^{1}{,}1^{2}11^{-}$ $\langle 2\rangle$ $18_{6}6_{6}2^{b}_{2}330^{b}_{2}\ (\times 2)$

$$\begin{bmatrix} -72232380 & -24025320 & 7611120 \\ -24025320 & -7991094 & 2531511 \\ 7611120 & 2531511 & -801646 \end{bmatrix} \begin{bmatrix} -45972829 & -15290131 & 4834932 \\ 142920360 & 47533969 & -15030840 \\ 14844060 & 4936995 & -1561141 \end{bmatrix}$$

$$\begin{bmatrix} -23051 & -11102 & -1994 & 5113 \\ 71661 & 34514 & 6199 & -15895 \\ 7443 & 3585 & 644 & -1650 \end{bmatrix}$$

W_{333} 12 lattices, $\chi = 72$ 16-gon: $2222|2222|2222|2222| \rtimes D_4$

$L_{333.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{1}{,}1^{-}3^{-}9^{-}{,}1^{2}5^{-}{,}1^{-2}11^{1}$ $\langle 2\rangle$

$132^{r}_{2}2^{b}_{2}60^{*}_{2}44^{b}_{2}6^{b}_{2}396^{*}_{2}60^{b}_{2}18^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} -137952540 & 146520 & 14172840 \\ 146520 & -138 & -18891 \\ 14172840 & -18891 & -620074 \end{bmatrix} \begin{bmatrix} -19556701 & 27683 & 503652 \\ -12750837000 & 18049129 & 328377720 \\ -58538700 & 82863 & 1507571 \end{bmatrix}$$

$$\begin{bmatrix} -21785 & -1480 & -13751 & -12561 & -885 & -8401 & -441 & 884 \\ -14203684 & -964951 & -8965570 & -8189698 & -577015 & -5477406 & -287530 & 576363 \\ -65208 & -4430 & -41160 & -37598 & -2649 & -25146 & -1320 & 2646 \end{bmatrix}$$

W_{334} 4 lattices, $\chi = 24$ 6-gon: $\not\infty 2|2\not\infty 2|2 \rtimes D_4$

$L_{334.1}$: $1^{1}_{7}8^{-}_{5}64^{1}_{7}{,}1^{-}3^{-}9^{1}$ $\langle 3\rangle$ $24^{24,7}_{\infty b}96^{*}_{2}576^{s}_{2}96^{48,1}_{\infty z}24^{b}_{2}36^{s}_{2}$

$$\begin{bmatrix} -300096 & 11520 & 576 \\ 11520 & -408 & -24 \\ 576 & -24 & -1 \end{bmatrix} \quad \begin{bmatrix} -3 & -5 & -1 & 1 & 0 & -1 \\ -43 & -70 & -12 & 14 & -1 & -15 \\ -732 & -1248 & -288 & 240 & 12 & -234 \end{bmatrix}$$

W_{335} 8 lattices, $\chi = 24$ 6-gon: $22|22\infty|\infty \rtimes D_2$

$L_{335.1}$: $1^{1}_{1}8^{1}_{1}64^{-}_{5}{,}1^{-}3^{-}9^{1}$ $\langle 3\rangle$ $96^{l}_{2}9_{2}8^{r}_{2}36^{*}_{2}96^{48,7}_{\infty z}24^{24,13}_{\infty b}$

$$\begin{bmatrix} -241344 & 5760 & 59328 \\ 5760 & -120 & -1488 \\ 59328 & -1488 & -14287 \end{bmatrix} \quad \begin{bmatrix} 661 & 158 & 11 & -31 & -33 & 62 \\ 7910 & 1890 & 131 & -372 & -394 & 743 \\ 1920 & 459 & 32 & -90 & -96 & 180 \end{bmatrix}$$

W_{336} 4 lattices, $\chi = 12$ 6-gon: $2|222|22 \rtimes D_2$

$L_{336.1}$: $1^{-}_{3}8^{1}_{7}64^{1}_{1}{,}1^{2}3^{1}$ $64^{b}_{2}4^{l}_{2}64_{2}3^{r}_{2}32^{*}_{2}12^{s}_{2}$

$$\begin{bmatrix} 1478208 & 170112 & -4032 \\ 170112 & 19576 & -464 \\ -4032 & -464 & 11 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 3 & 1 & 1 & 1 \\ -8 & -1 & -32 & -9 & -6 & -6 \\ 32 & -42 & -256 & -15 & 112 & 114 \end{bmatrix}$$

W_{337} 16 lattices, $\chi = 36$ 10-gon: $2222|22222|2 \rtimes D_2$

$L_{337.1}$: $1^{-2}_{2}16^{-}_{3}{,}1^{2}3^{1}{,}1^{-2}5^{-}{,}1^{2}7^{-}$ $560_{2}3^{r}_{2}140^{*}_{2}48^{b}_{2}10^{l}_{2}48_{2}35^{r}_{2}12^{*}_{2}560^{b}_{2}2^{l}_{2}$

$$\begin{bmatrix} -132714960 & 25200 & 65520 \\ 25200 & -1 & -16 \\ 65520 & -16 & -29 \end{bmatrix} \quad \begin{bmatrix} 183 & 8 & 81 & 35 & 2 & 1 & -2 & -1 & 13 & 1 \\ 255920 & 11187 & 113260 & 48936 & 2795 & 1392 & -2800 & -1398 & 18200 & 1399 \\ 272160 & 11898 & 120470 & 52056 & 2975 & 1488 & -2975 & -1488 & 19320 & 1487 \end{bmatrix}$$

W_{338} 12 lattices, $\chi = 24$ 8-gon: $2222|2222| \rtimes D_2$

$L_{338.1}$: $1^{-2}_{\mathrm{II}}4^{1}_{7},1^{1}3^{1}9^{1},1^{-2}5^{-},1^{-2}13^{1}$ $\langle 2\rangle$ $156^{r}_{2}10^{b}_{2}468^{*}_{2}4^{b}_{2}390^{b}_{2}36^{*}_{2}52^{b}_{2}90^{l}_{2}$

$$\begin{bmatrix} 73848060 & -24605100 & -65520 \\ -24605100 & 8198058 & 21831 \\ -65520 & 21831 & 58 \end{bmatrix} \quad \begin{bmatrix} 861 & 49 & -79 & -25 & 22 & 221 & 615 & 517 \\ 2548 & 145 & -234 & -74 & 65 & 654 & 1820 & 1530 \\ 13572 & 775 & -1170 & -388 & 390 & 3492 & 9698 & 8145 \end{bmatrix}$$

W_{339} 32 lattices, $\chi = 36$ 10-gon: $2222|22222|2 \rtimes D_2$

$L_{339.1}$: $1^{2}_{6}8^{1}_{7},1^{1}3^{-}9^{1},1^{-2}5^{1},1^{2}7^{-}$ $\langle 2\rangle$

$70^{s}_{2}6^{b}_{2}2520^{*}_{2}4^{s}_{2}168^{s}_{2}36^{*}_{2}280^{b}_{2}6^{s}_{2}630^{b}_{2}24^{b}_{2}$

$$\begin{bmatrix} 3903480 & -652680 & 148680 \\ -652680 & 109131 & -24864 \\ 148680 & -24864 & 5539 \end{bmatrix} \quad \begin{bmatrix} 15297 & 5387 & 298583 & 3581 & 2655 & 1 & -731 & 16 & 8294 & 2717 \\ 90825 & 31985 & 1772820 & 21262 & 15764 & 6 & -4340 & 95 & 49245 & 16132 \\ -2905 & -1023 & -56700 & -680 & -504 & 0 & 140 & -3 & -1575 & -516 \end{bmatrix}$$

$L_{339.2}$: $1^{-2}_{6}8^{-}_{3},1^{1}3^{-}9^{1},1^{-2}5^{1},1^{2}7^{-}$ $\langle m\rangle$

$70^{b}_{2}6^{l}_{2}2520_{2}1^{r}_{2}168^{l}_{2}9_{2}280^{r}_{2}6^{b}_{2}630^{l}_{2}24^{r}_{2}$

$$\begin{bmatrix} 4236120 & -5040 & -2520 \\ -5040 & 6 & 3 \\ -2520 & 3 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 & 0 & -1 & -2 & -37 & -2 & -17 & -1 \\ -805 & 1 & 840 & 0 & -812 & -1620 & -29960 & -1619 & -13755 & -808 \\ -70 & 0 & 0 & -1 & -84 & -153 & -2800 & -150 & -1260 & -72 \end{bmatrix}$$

W_{340} 16 lattices, $\chi = 72$ 16-gon: $2|2222|2222|2222|222 \rtimes D_4$

$L_{340.1}$: $1^{-2}_{\mathrm{II}}8^{-}_{5},1^{1}3^{-}9^{1},1^{2}5^{-},1^{2}7^{-}$ $\langle 2\rangle$ $360^{r}_{2}42^{l}_{2}40^{r}_{2}6^{b}_{2}90^{l}_{2}168^{r}_{2}10^{b}_{2}6^{l}_{2}$ $(\times 2)$

$$\begin{bmatrix} -11473560 & -5040 & 4919040 \\ -5040 & 6 & 393 \\ 4919040 & 393 & -1728470 \end{bmatrix} \begin{bmatrix} 352031 & 386 & -200720 \\ 226695840 & 248569 & -129256400 \\ 1053360 & 1155 & -600601 \end{bmatrix}$$

$$\begin{bmatrix} 6617 & 779 & 909 & 0 & -391 & -1179 & -132 & -3 \\ 4261320 & 501676 & 585400 & 1 & -251805 & -759304 & -85015 & -1943 \\ 19800 & 2331 & 2720 & 0 & -1170 & -3528 & -395 & -9 \end{bmatrix}$$

W_{341} 4 lattices, $\chi = 48$ 8-gon: $2\infty|\infty 2|2\infty|\infty 2| \rtimes D_4$

$L_{341.1}$: $1^{1}_{7}8^{1}_{1}256^{1}_{1}$ $256^{*}_{2}32^{32,17}_{\infty z}8^{32,9}_{\infty}32^{s}_{2}$ $(\times 2)$

shares genus with $L_{342.1}$

$$\begin{bmatrix} -239360 & -11008 & -11520 \\ -11008 & -504 & -528 \\ -11520 & -528 & -553 \end{bmatrix} \begin{bmatrix} -1153 & -50 & -53 \\ -56448 & -2451 & -2597 \\ 78336 & 3400 & 3603 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 2 & 13 \\ 112 & 202 & 115 & 650 \\ -128 & -256 & -152 & -896 \end{bmatrix}$$

$L_{341.2}$: $1^{1}_{1}8^{1}_{1}256^{1}_{7}$ $1_{2}8^{32,31}_{\infty}32^{32,7}_{\infty z}8^{r}_{2}4^{*}_{2}32^{32,15}_{\infty z}8^{32,7}_{\infty}32^{l}_{2}$

$$\begin{bmatrix} -1272064 & 8960 & 8960 \\ 8960 & -56 & -64 \\ 8960 & -64 & -63 \end{bmatrix} \quad \begin{bmatrix} 10 & 29 & 11 & 1 & -1 & -1 & 4 & 55 \\ 154 & 445 & 166 & 13 & -16 & -14 & 63 & 850 \\ 1263 & 3664 & 1392 & 128 & -126 & -128 & 504 & 6944 \end{bmatrix}$$

W_{342} 2 lattices, $\chi = 24$ 6-gon: $2|22\infty|\infty 2 \rtimes D_2$

$L_{342.1}$: $1^{1}_{7}8^{1}_{1}256^{1}_{1}$ $256^{r}_{2}4^{b}_{2}256^{l}_{2}8^{32,17}_{\infty}32^{32,25}_{\infty z}8_{2}$

shares genus with $L_{341.1}$

$$\begin{bmatrix} 13951232 & 6751744 & 425216 \\ 6751744 & 3267528 & 205784 \\ 425216 & 205784 & 12959 \end{bmatrix} \quad \begin{bmatrix} -209 & 84 & 8929 & 1411 & 209 & -112 \\ 448 & -181 & -19232 & -3039 & -450 & 241 \\ -256 & 118 & 12416 & 1960 & 288 & -152 \end{bmatrix}$$

W_{343} 2 lattices, $\chi = 24$ 7-gon: $222{\mid\!\!\!\!\infty}222| \rtimes D_2$

$$L_{343.1} : 1_1^1 8_7^1 256_1^1 \qquad 256^{l}_{2}1_{2}256^{r}_{2}8^{16,1}_{\infty a}8^{b}_{2}256^{s}_{2}4^{*}_{2}$$

$$\begin{bmatrix} 385280 & 8448 & -2560 \\ 8448 & 184 & -56 \\ -2560 & -56 & 17 \end{bmatrix} \qquad \begin{bmatrix} -3 & 0 & 1 & 0 & -1 & -15 & -1 \\ -16 & 1 & 32 & 1 & -9 & -128 & -8 \\ -512 & 3 & 256 & 4 & -180 & -2688 & -178 \end{bmatrix}$$

W_{344} 32 lattices, $\chi = 96$ 14-gon: $2222\infty\infty\infty2222\infty\infty\infty \rtimes C_2$

$$L_{344.1} : 1_{\mathrm{II}}^{-2} 8_1^1, 1^1 3^- 9^-, 1^- 7^- 49^1 \ \langle 23, 3, 2\rangle$$

$$42^{l}_{2}3528^{r}_{2}6^{s}_{2}882^{b}_{2}168^{42,1}_{\infty z}42^{84,37}_{\infty b}168^{42,37}_{\infty z} \ (\times 2)$$

$$\begin{bmatrix} -49106232 & 190512 & -1090152 \\ 190512 & -714 & 4389 \\ -1090152 & 4389 & -23186 \end{bmatrix} \begin{bmatrix} -1834561 & 7176 & -40352 \\ -248900400 & 973589 & -5474680 \\ 39143160 & -153111 & 860971 \end{bmatrix}$$

$$\begin{bmatrix} -6174 & -96233 & -900 & -6676 & -3807 & -250 & -689 \\ -837638 & -13056120 & -122105 & -905751 & -516508 & -33919 & -93484 \\ 131733 & 2053296 & 19203 & 142443 & 81228 & 5334 & 14700 \end{bmatrix}$$

W_{345} 16 lattices, $\chi = 48$ 12-gon: $222|222|222|222| \rtimes D_4$

$$L_{345.1} : 1_{\mathrm{II}}^{-2} 8_1^1, 1^- 3^1 9^-, 1^1 7^1 49^1 \ \langle 23, 3, 2*\rangle \qquad 14^{l}_{2}72^{r}_{2}98^{b}_{2}126^{b}_{2}2^{l}_{2}3528^{r}_{2} \ (\times 2)$$

shares genus with its 3-dual; isometric to its own 7-dual

$$\begin{bmatrix} 5761224 & 1481760 & -3528 \\ 1481760 & 380982 & -903 \\ -3528 & -903 & 2 \end{bmatrix} \begin{bmatrix} 16463 & 4151 & -7 \\ -68208 & -17198 & 29 \\ -1728720 & -435855 & 734 \end{bmatrix} \qquad \begin{bmatrix} -6 & -23 & -5 & 8 & 7 & 1297 \\ 25 & 96 & 21 & -33 & -29 & -5376 \\ 707 & 2808 & 686 & -756 & -734 & -137592 \end{bmatrix}$$

W_{346} 12 lattices, $\chi = 60$ 14-gon: $22|222{\mid\!\!\!2}222|222{\mid\!\!\!2}2 \rtimes D_4$

$$L_{346.1} : 1_{\mathrm{II}}^{-2} 4_3^-, 1^- 3^1 9^-, 1^2 7^1, 1^2 11^1 \ \langle 2\rangle \qquad 396^{r}_{2}14^{l}_{2}12^{r}_{2}126^{l}_{2}44^{r}_{2}18^{b}_{2}2^{l}_{2} \ (\times 2)$$

$$\begin{bmatrix} -84537684 & -42139944 & 63756 \\ -42139944 & -21005718 & 31779 \\ 63756 & 31779 & -46 \end{bmatrix} \begin{bmatrix} -6674977 & -3326824 & 4472 \\ 13408164 & 6682660 & -8983 \\ 11467764 & 5715561 & -7684 \end{bmatrix}$$

$$\begin{bmatrix} -6637 & 115 & 229 & -1150 & -6199 & -2869 & -1262 \\ 13332 & -231 & -460 & 2310 & 12452 & 5763 & 2535 \\ 11484 & -196 & -396 & 1953 & 10604 & 4914 & 2164 \end{bmatrix}$$

W_{347} 4 lattices, $\chi = 24$ 8-gon: $22|22|22|22| \rtimes D_4$

$$L_{347.1} : 1_2^{-2} 16_5^-, 1^- 3^1 9^-, 1^{-2} 5^1 \qquad 18^{l}_{2}80_{2}3_{2}720^{r}_{2}2^{b}_{2}720^{*}_{2}12^{*}_{2}80^{b}_{2}$$

$$\begin{bmatrix} -137520 & 2160 & -720 \\ 2160 & -6 & -9 \\ -720 & -9 & 11 \end{bmatrix} \qquad \begin{bmatrix} 1 & 11 & 1 & 23 & 0 & -13 & -1 & -1 \\ 117 & 1280 & 116 & 2640 & -1 & -1560 & -118 & -120 \\ 162 & 1760 & 159 & 3600 & -2 & -2160 & -162 & -160 \end{bmatrix}$$

W_{348} 12 lattices, $\chi = 32$ 9-gon: $222|2222{\mid\!\!\!2}2 \rtimes D_2$

$$L_{348.1} : 1_{\mathrm{II}}^{-2} 4_3^-, 1^1 3^- 9^1, 1^{-2} 5^-, 1^{-2} 17^- \ \langle 2\rangle$$

$$340^{*}_{2}36^{b}_{2}10^{l}_{2}204^{r}_{2}90^{b}_{2}4^{*}_{2}3060^{b}_{2}6^{+}_{3}6^{b}_{2}$$

$$\begin{bmatrix} -23993460 & -1407600 & 18360 \\ -1407600 & -82578 & 1077 \\ 18360 & 1077 & -14 \end{bmatrix} \qquad \begin{bmatrix} -29 & -1 & 2 & 27 & 11 & 3 & 83 & -1 & -2 \\ 510 & 18 & -35 & -476 & -195 & -54 & -1530 & 17 & 35 \\ 1190 & 72 & -70 & -1224 & -585 & -226 & -9180 & -6 & 69 \end{bmatrix}$$

W_{349} 30 lattices, $\chi = 120$ 24-gon: $22|222|222|222|222|222|222|222|2 \rtimes D_8$

$L_{349.1}$: $1_0^2 8_1^1,1^-3^-9^-,1^2 11^1$

$$99_2 8_2 33_2 72_2 11_2^r 24_2^s 44_2^* 72_2^* 132_2^* 8_2^* 396_2^s 24_2^l \ (\times 2)$$

$$\begin{bmatrix} 1016136 & -3960 & -1584 \\ -3960 & 15 & 9 \\ -1584 & 9 & -16 \end{bmatrix} \begin{bmatrix} -1099 & 5 & -3 \\ -241560 & 1099 & -660 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -23 & -5 & -9 & -11 & -4 & -1 & 3 & 7 & 15 & 5 & 53 & 7 \\ -5643 & -1232 & -2233 & -2760 & -1023 & -284 & 594 & 1560 & 3454 & 1168 & 12474 & 1660 \\ -990 & -224 & -429 & -576 & -242 & -108 & -110 & 36 & 264 & 116 & 1386 & 204 \end{bmatrix}$$

$L_{349.2}$: $[1^1 2^1]_2 16_7^1,1^-3^-9^-,1^2 11^1$ $\langle 2 \rangle$

$$1584_2^l 2_2 33_2 18_2^r 176_2^l 6_2 11_2^r 72_2^* 528_2^* 8_2^l 99_2 6_2^r \ (\times 2)$$

$$\begin{bmatrix} -73806480 & -34848 & 52272 \\ -34848 & 6 & 24 \\ 52272 & 24 & -37 \end{bmatrix} \begin{bmatrix} -20701 & -21 & 15 \\ -1062600 & -1079 & 770 \\ -30056400 & -30492 & 21779 \end{bmatrix}$$

$$\begin{bmatrix} -131 & -4 & -17 & -13 & -51 & -4 & -10 & -17 & -35 & -3 & -8 & 0 \\ -6072 & -187 & -803 & -621 & -2464 & -197 & -506 & -882 & -1892 & -174 & -528 & -17 \\ -189288 & -5782 & -24585 & -18810 & -73832 & -5796 & -14509 & -24696 & -50952 & -4384 & -11781 & -24 \end{bmatrix}$$

$L_{349.3}$: $[1^- 2^1]_6 16_3^-,1^-3^-9^-,1^2 11^1$ $\langle m \rangle$

$$1584_2 2_2^r 132_2^l 18_2 176_2 6_2^r 44_2^* 72_2^s 528_2^s 8_2^* 396_2^l 6_2 \ (\times 2)$$

$$\begin{bmatrix} -2659536 & -4752 & -7920 \\ -4752 & 6 & -6 \\ -7920 & -6 & -19 \end{bmatrix} \begin{bmatrix} -5125 & -7 & -14 \\ -1465464 & -2003 & -4004 \\ 2608848 & 3564 & 7127 \end{bmatrix}$$

$$\begin{bmatrix} 115 & 4 & 39 & 17 & 75 & 7 & 43 & 43 & 111 & 13 & 107 & 5 \\ 33000 & 1147 & 11176 & 4869 & 21472 & 2003 & 12298 & 12294 & 31724 & 3714 & 30558 & 1427 \\ -58608 & -2038 & -19866 & -8658 & -38192 & -3564 & -21890 & -21888 & -56496 & -6616 & -54450 & -2544 \end{bmatrix}$$

$L_{349.4}$: $[1^- 2^1]_4 16_5^-,1^-3^-9^-,1^2 11^1$ $\langle m \rangle$

$$1584_2^s 8_2^* 132_2^* 72_2^s 176_2^* 24_2^l 11_2 18_2^r 528_2^l 2_2 99_2^r 24_2^* \ (\times 2)$$

$$\begin{bmatrix} -1699632 & -9504 & 4752 \\ -9504 & -30 & 24 \\ 4752 & 24 & -13 \end{bmatrix} \begin{bmatrix} 815 & 2 & -2 \\ 44880 & 109 & -110 \\ 376992 & 924 & -925 \end{bmatrix}$$

$$\begin{bmatrix} 91 & 5 & 19 & 13 & 23 & 3 & 3 & 2 & 5 & 0 & -2 & -1 \\ 4884 & 270 & 1034 & 714 & 1276 & 170 & 176 & 123 & 352 & 5 & -66 & -46 \\ 41976 & 2308 & 8778 & 6012 & 10648 & 1392 & 1397 & 936 & 2376 & 4 & -891 & -456 \end{bmatrix}$$

$L_{349.5}$: $[1^1 2^1]_0 16_1^1,1^-3^-9^-,1^2 11^1$

$$1584_2^* 8_2^l 33_2^r 72_2^* 176_2^s 24_2^* 44_2^l 18_2 528_2 2_2^r 396_2^* 24_2^s \ (\times 2)$$

$$\begin{bmatrix} -3923568 & -39600 & 11088 \\ -39600 & -354 & 108 \\ 11088 & 108 & -31 \end{bmatrix} \begin{bmatrix} 12959 & 170 & -40 \\ 541728 & 7105 & -1672 \\ 6500736 & 85272 & -20065 \end{bmatrix}$$

$$\begin{bmatrix} 367 & 21 & 42 & 61 & 115 & 17 & 41 & 17 & 69 & 3 & 35 & 1 \\ 15444 & 882 & 1760 & 2550 & 4796 & 706 & 1694 & 699 & 2816 & 121 & 1386 & 34 \\ 184536 & 10552 & 21087 & 30600 & 57640 & 8508 & 20482 & 8478 & 34320 & 1486 & 17226 & 468 \end{bmatrix}$$

W_{350} 12 lattices, $\chi = 84$ 18-gon: $2\dagger 2|2\dagger 2|2\dagger 2|2\dagger 2|2\dagger 2|2\dagger 2| \rtimes D_{12}$

$L_{350.1}$: $1_2^2 16_1^1,1^1 3^- 9^1,1^2 13^1$

$$1872_2^l 1_2 144_2 13_2^r 36_2^* 4_2^l 117_2 16_2 9_2^r 208_2^s 36_2^* 16_2^* 468_2^l 1_2 9_2^r 52_2^* 144_2^* 4_2^s$$

$$\begin{bmatrix} -57528432 & 48672 & 44928 \\ 48672 & -30 & -39 \\ 44928 & -39 & -35 \end{bmatrix} \begin{bmatrix} 443 & 8 & 97 & 38 & 41 & 11 & 62 & 11 & 4 & 9 \\ 45552 & 823 & 9984 & 3913 & 4224 & 1134 & 6396 & 1136 & 414 & 936 \\ 517608 & 9347 & 113328 & 44395 & 47898 & 12850 & 72423 & 12848 & 4671 & 10504 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 7 & 1 & 7 & 37 & 61 & 13 \\ -102 & -104 & 702 & 102 & 717 & 3796 & 6264 & 1336 \\ -1170 & -1168 & 8190 & 1169 & 8181 & 43238 & 71280 & 15190 \end{bmatrix}$$

$L_{350.2}$: $1_1^1 4_1^1 16_1^1,1^1 3^- 9^1,1^2 13^1$ $\langle 3\rangle$

$$52_2^s 144_2^s 4_2^s 1872_2^l 4_2 144_2 52_2^r 36_2^l 4_2 117_2 16_2 9_2 208_2 36_2^r 16_2^l 468_2 1_2 36_2^r$$

shares genus with its 2-dual $\cong$ 3-dual; isometric to its own 2.3-dual

$$\begin{bmatrix} -57528432 & -172224 & 44928 \\ -172224 & -444 & 132 \\ 44928 & 132 & -35 \end{bmatrix} \begin{bmatrix} 37 & 61 & 13 & 443 & 16 & 97 & 76 & 41 & 11 \\ 1898 & 3132 & 668 & 22776 & 823 & 4992 & 3913 & 2112 & 567 \\ 54626 & 90072 & 19198 & 654264 & 23632 & 143280 & 112268 & 60570 & 16252 \end{bmatrix}$$

$$\begin{bmatrix} 62 & 11 & 4 & 9 & -1 & -1 & 7 & 1 & 14 \\ 3198 & 568 & 207 & 468 & -51 & -52 & 351 & 51 & 717 \\ 91611 & 16256 & 5913 & 13312 & -1476 & -1480 & 10296 & 1475 & 20664 \end{bmatrix}$$

W_{351} 4 lattices, $\chi = 48$ 12-gon: $222|222|222|222| \rtimes D_4$

$L_{351.1}$: $1_5^- 8_7^1 64_1^1,1^{-2} 5^1$ $8_2^b 64_2^s 20_2^* 32_2^l 5_2 64_2^r$ $(\times 2)$

$$\begin{bmatrix} 11375680 & -733440 & 12160 \\ -733440 & 47288 & -784 \\ 12160 & -784 & 13 \end{bmatrix} \begin{bmatrix} 3759 & -242 & 4 \\ 56400 & -3631 & 60 \\ -120320 & 7744 & -129 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 & 3 & 4 & 9 \\ -1 & 16 & 50 & 50 & 65 & 144 \\ -60 & 32 & 210 & 208 & 175 & 256 \end{bmatrix}$$

W_{352} 12 lattices, $\chi = 72$ 16-gon: $2|2222|2222|2222|222 \rtimes D_4$

$L_{352.1}$: $1_{\mathrm{II}}^{-2} 4_1^1,1^1 3^1 9^1,1^{-2} 5^1,1^2 19^1$ $\langle 2\rangle$

$$684_2^* 12_2^* 76_2^b 30_2^l 4_2^r 570_2^l 36_2^r 30_2^b \ (\times 2)$$

$$\begin{bmatrix} 181639620 & -30595320 & 160740 \\ -30595320 & 5153466 & -27075 \\ 160740 & -27075 & 142 \end{bmatrix} \begin{bmatrix} 1231199 & -207384 & 1044 \\ 7284600 & -1227023 & 6177 \\ -4924800 & 829536 & -4177 \end{bmatrix}$$

$$\begin{bmatrix} 2155 & 59 & 333 & 123 & 31 & -16 & -95 & -112 \\ 12768 & 350 & 1976 & 730 & 184 & -95 & -564 & -665 \\ -5130 & -54 & -190 & -45 & -8 & 0 & 0 & -15 \end{bmatrix}$$

W_{353} 24 lattices, $\chi = 36$ 10-gon: $2222222222 \rtimes C_2$

$L_{353.1}$: $1_{\mathrm{II}}^{-2} 4_5^-,1^2 3^1,1^1 5^- 25^-,1^2 7^-$ $\langle 2\rangle$ $84_2^r 50_2^b 140_2^* 300_2^b 10_2^l$ $(\times 2)$

$$\begin{bmatrix} -2674574700 & -18958800 & -277200 \\ -18958800 & -134390 & -1965 \\ -277200 & -1965 & -26 \end{bmatrix} \begin{bmatrix} -9154741 & -64894 & -923 \\ 1291076220 & 9151881 & 130169 \\ 28366800 & 201080 & 2859 \end{bmatrix}$$

$$\begin{bmatrix} 6073 & 2164 & 1947 & 1799 & -1 \\ -856464 & -305185 & -274582 & -253710 & 141 \\ -18816 & -6700 & -6020 & -5550 & 5 \end{bmatrix}$$

W_{354} 12 lattices, $\chi = 24$ 8-gon: $222|2222|2 \rtimes D_2$

$L_{354.1}$: $1_{\mathrm{II}}^{-2} 4_5^-,1^2 3^1,1^- 5^1 25^-,1^{-2} 7^1$ $\langle 2\rangle$ $700_2^* 12_2^b 50_2^l 20_2^r 2_2^b 300_2^* 28_2^b 30_2^b$

$$\begin{bmatrix} -48300 & 21000 & 2100 \\ 21000 & -9130 & -905 \\ 2100 & -905 & 58 \end{bmatrix} \begin{bmatrix} 2241 & 257 & 132 & -19 & -16 & 13 & 109 & 109 \\ 5180 & 594 & 305 & -44 & -37 & 30 & 252 & 252 \\ -350 & -42 & -25 & 0 & 2 & 0 & -14 & -15 \end{bmatrix}$$

W_{355} 8 lattices, $\chi = 120$ 14-gon: $\infty\infty\infty\infty\infty 22\infty\infty\infty\infty\infty 22 \rtimes C_2$

$L_{355.1}$: $1^{-2}_{\mathrm{II}}8^1_7,1^1 11^- 121^-$ $\langle 2\rangle 88^{22,1}_{\infty z}22^{44,3}_{\infty b}88^{22,5}_{\infty z}22^{44,27}_{\infty b}88^{22,3}_{\infty z}22^s_2 242^b_2\ (\times 2)$

$$\begin{bmatrix} -6018056 & 31944 & -303952 \\ 31944 & -154 & 1727 \\ -303952 & 1727 & -14522 \end{bmatrix} \begin{bmatrix} -3926209 & 17160 & -225160 \\ -321231768 & 1403984 & -18421985 \\ 43981080 & -192225 & 2522224 \end{bmatrix}$$

$$\begin{bmatrix} -22935 & -2234 & -3543 & -382 & -711 & -110 & 108 \\ -1876480 & -182781 & -289884 & -31256 & -58180 & -9003 & 8833 \\ 256916 & 25025 & 39688 & 4279 & 7964 & 1232 & -1210 \end{bmatrix}$$

W_{356} 4 lattices, $\chi = 32$ 9-gon: $222\mathrlap{|}22222|2 \rtimes D_2$

$L_{356.1}$: $1^{-2}_{\mathrm{II}}16^1_1,1^1 3^- 9^1,1^{-2}7^-$ $144^r_2 42^b_2 16^b_2 6^-_3 6^b_2 144^b_2 42^l_2 16^r_2 6^l_2$

$$\begin{bmatrix} -7917647472 & 2568704544 & 24677856 \\ 2568704544 & -833359026 & -8006175 \\ 24677856 & -8006175 & -76910 \end{bmatrix}$$

$$\begin{bmatrix} -282263 & -75612 & -14425 & -3309 & 2969 & 6949 & -31666 & -60605 & -17823 \\ -878352 & -235291 & -44888 & -10297 & 9239 & 21624 & -98539 & -188592 & -55462 \\ 865872 & 231945 & 44248 & 10149 & -9108 & -21312 & 97146 & 185920 & 54675 \end{bmatrix}$$

W_{357} 4 lattices, $\chi = 48$ 12-gon: $22|222|222|222|2 \rtimes D_4$

$L_{357.1}$: $1^1_1 8^1_7 128^-_3,1^2 3^-$ $384_2 1^r_2 96^*_2 4^s_2 384^b_2 8^l_2\ (\times 2)$

$$\begin{bmatrix} 384 & 0 & 0 \\ 0 & -8 & 8 \\ 0 & 8 & -7 \end{bmatrix} \begin{bmatrix} -97 & -18 & 14 \\ -672 & -127 & 98 \\ -1536 & -288 & 223 \end{bmatrix} \begin{bmatrix} -97 & -4 & -11 & -5 & -49 & 0 \\ -672 & -29 & -90 & -46 & -480 & -3 \\ -1536 & -65 & -192 & -94 & -960 & -4 \end{bmatrix}$$

W_{358} 12 lattices, $\chi = 88$ 16-gon: $262|2622|2262|2622|2 \rtimes D_4$

$L_{358.1}$: $1^{-2}_{\mathrm{II}}4^-_5,1^- 3^- 9^-,1^{-2}5^-,1^2 23^1$ $\langle 2\rangle$

$828^b_2 6_6 18^l_2 276^r_2 2_6 6^b_2 92^*_2 60^*_2\ (\times 2)$

$$\begin{bmatrix} -4899644460 & 14092560 & -1184040 \\ 14092560 & -36786 & 2769 \\ -1184040 & 2769 & -178 \end{bmatrix} \begin{bmatrix} 119343089 & -267280 & 15934 \\ 82102473480 & -183876161 & 10961848 \\ 483342300720 & -1082490240 & 64533071 \end{bmatrix}$$

$$\begin{bmatrix} -7955 & -788 & -875 & -9147 & -492 & -2591 & -11867 & -2487 \\ -5472666 & -542107 & -601959 & -6292708 & -338473 & -1782487 & -8163942 & -1710940 \\ -32217894 & -3191415 & -3543768 & -37045548 & -1992611 & -10493610 & -48061628 & -10072410 \end{bmatrix}$$

W_{359} 3 lattices, $\chi = 24$ 8-gon: $2|2|2|2|2|2|2|2| \rtimes D_8$

$L_{359.1}$: $1^{-2}_{\mathrm{II}}4^-_3,1^1 3^1 9^1,1^- 5^- 25^-$ $\langle 2\rangle$ $300^r_2 10^l_2 12^r_2 90^l_2\ (\times 2)$

$$\begin{bmatrix} 146700 & -47700 & -1800 \\ -47700 & 15510 & 585 \\ -1800 & 585 & 22 \end{bmatrix} \begin{bmatrix} 89 & -29 & -1 \\ 180 & -59 & -2 \\ 2700 & -870 & -31 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 & -1 \\ 20 & 4 & 4 & -3 \\ -300 & -25 & -24 & 0 \end{bmatrix}$$

W_{360} 12 lattices, $\chi = 54$ 13-gon: $2\mathrlap{|}2222222|22222 \rtimes D_2$

$L_{360.1}$: $1^{-2}_{\mathrm{II}}4^-_3,1^1 3^- 9^1,1^2 7^-,1^2 19^1$ $\langle 2\rangle$

$42^b_2 36^*_2 4^b_2 42^l_2 76^r_2 6^b_2 532^*_2 36^b_2 114^b_2 4^*_2 4788^b_2 6^l_2 684^r_2$

$$\begin{bmatrix} -66566189844 & 146268612 & -2322180 \\ 146268612 & -321402 & 5103 \\ -2322180 & 5103 & -50 \end{bmatrix} \begin{bmatrix} -2896 & -1795 & -415 & -971 & -1531 & -1 & 957 & 139 & -134 & -687 & -70279 & -826 & -25493 \\ -1317701 & -816738 & -188828 & -441812 & -696616 & -455 & 435442 & 63246 & -60971 & -312590 & -31977456 & -375836 & -11599500 \\ 16086 & 9972 & 2306 & 5397 & 8512 & 6 & -5320 & -774 & 741 & 3814 & 390222 & 4587 & 141588 \end{bmatrix}$$

W_{361} 12 lattices, $\chi = 112$ 20-gon: $22226|62222|22226|62222| \rtimes D_4$

$L_{361.1} : 1^{-2}_{\mathrm{II}} 4^{1}_{7}, 1^{-}3^{-}9^{-}, 1^{-2}5^{1}, 1^{-2}29^{1}\ \langle 2\rangle$

$870^{s}_{2} 2^{b}_{2} 1044^{*}_{2} 20^{b}_{2} 18_{6} 6_{6} 2^{b}_{2} 180^{*}_{2} 116^{b}_{2} 18^{s}_{2}\ (\times 2)$

$$\begin{bmatrix} -37618707780 & 31246920 & -1237140 \\ 31246920 & -25878 & 1005 \\ -1237140 & 1005 & -34 \end{bmatrix} \begin{bmatrix} 84550949 & -66804 & 1767 \\ 117519887100 & -92852873 & 2456006 \\ 397247063400 & -313866288 & 8301923 \end{bmatrix} \begin{bmatrix} -7664 & -386 & -19585 & -1711 & -1381 & -56 & -124 & -1097 & -1147 & -149 \\ -10652425 & -536513 & -27221778 & -2378170 & -1919493 & -77836 & -172351 & -1524750 & -1594246 & -207099 \\ -36007995 & -1813553 & -92016594 & -8038820 & -6488370 & -263103 & -582580 & -5153940 & -5388838 & -700029 \end{bmatrix}$$

W_{362} 4 lattices, $\chi = 48$ 7-gon: $\infty\infty|\infty\infty2\rlap{|}\infty2 \rtimes D_2$

$L_{362.1} : 1^{1}_{7} 16^{-}_{5} 256^{-}_{5}$ $64^{32,17}_{\infty z} 16^{16,13}_{\infty b} 64^{32,9}_{\infty z} 16^{16,5}_{\infty a} 64^{s}_{2} 256^{16,1}_{\infty z} 256^{*}_{2}$

$$\begin{bmatrix} -58112 & -3328 & 256 \\ -3328 & -176 & 16 \\ 256 & 16 & -1 \end{bmatrix} \qquad \begin{bmatrix} -27 & -7 & -5 & 0 & 1 & -1 & -15 \\ 278 & 73 & 54 & 1 & -10 & 8 & 152 \\ -2720 & -696 & -480 & 8 & 96 & -128 & -1536 \end{bmatrix}$$

$L_{362.2} : 1^{1}_{1} 16^{-}_{5} 256^{-}_{3}$ $64^{32,31}_{\infty z} 16^{16,11}_{\infty a} 64^{32,23}_{\infty z} 16^{16,3}_{\infty b} 64^{*}_{2} 4^{8,3}_{\infty z} 1^{r}_{2}$

$$\begin{bmatrix} -1131776 & 8448 & 8448 \\ 8448 & -48 & -64 \\ 8448 & -64 & -63 \end{bmatrix} \qquad \begin{bmatrix} 47 & 16 & 19 & 4 & -1 & -1 & 1 \\ 314 & 111 & 138 & 31 & -6 & -8 & 6 \\ 5952 & 2024 & 2400 & 504 & -128 & -126 & 127 \end{bmatrix}$$

W_{363} 12 lattices, $\chi = 72$ 16-gon: $2|2222|2222|2222|222 \rtimes D_4$

$L_{363.1} : 1^{-2}_{\mathrm{II}} 4^{1}_{1}, 1^{2}3^{-}, 1^{1}5^{-}25^{1}, 1^{-2}11^{1}\ \langle 2\rangle$

$4^{r}_{2} 10^{l}_{2} 100^{r}_{2} 6^{b}_{2} 1100^{*}_{2} 60^{*}_{2} 44^{b}_{2} 150^{l}_{2}\ (\times 2)$

$$\begin{bmatrix} -12035100 & 19800 & 13200 \\ 19800 & -10 & -25 \\ 13200 & -25 & -14 \end{bmatrix} \begin{bmatrix} 102299 & -185 & -110 \\ 11130240 & -20129 & -11968 \\ 76418100 & -138195 & -82171 \end{bmatrix} \begin{bmatrix} 39 & 3 & 13 & 2 & 17 & -1 & -1 & 1 \\ 4244 & 327 & 1420 & 219 & 1870 & -108 & -110 & 105 \\ 29132 & 2240 & 9700 & 1491 & 12650 & -750 & -748 & 750 \end{bmatrix}$$

W_{364} 4 lattices, $\chi = 32$ 8-gon: $2622|2262| \rtimes D_2$

$L_{364.1} : 1^{-2}_{\mathrm{II}} 32^{-}_{3}, 1^{-}3^{-}9^{-}, 1^{-2}5^{-}$ $96^{b}_{2} 18_{6} 6^{b}_{2} 2^{l}_{2} 96^{r}_{2} 18^{b}_{2} 6_{6} 2^{b}_{2}$

$$\begin{bmatrix} 3627360 & -1248480 & -642240 \\ -1248480 & 429018 & 202803 \\ -642240 & 202803 & -369406 \end{bmatrix} \qquad \begin{bmatrix} -52331 & -20911 & 24423 & 37202 & 645133 & 167369 & 80186 & 30205 \\ -155056 & -61959 & 72365 & 110229 & 1911520 & 495912 & 237590 & 89497 \\ 5856 & 2340 & -2733 & -4163 & -72192 & -18729 & -8973 & -3380 \end{bmatrix}$$

W_{365} 12 lattices, $\chi = 144$ 28-gon: $2\rlap{|}2222|222\rlap{|}2222|222\rlap{|}2222|222\rlap{|}2222|22 \rtimes D_8$

$L_{365.1} : 1^{2}_{2} 32^{1}_{1}, 1^{-}3^{-}9^{-}, 1^{-}5^{1}25^{-}\ \langle 5,3\rangle$

$800^{r}_{2} 18^{b}_{2} 50^{l}_{2} 288_{2} 5^{r}_{2} 7200^{s}_{2} 20^{*}_{2} 288^{b}_{2} 50^{s}_{2} 18^{b}_{2} 800^{*}_{2} 180^{s}_{2} 32^{l}_{2} 45_{2}\ (\times 2)$

shares genus with its 3-dual $\cong$ 5-dual; isometric to its own 3.5-dual

$$\begin{bmatrix} -1635199200 & -331164000 & 410400 \\ -331164000 & -67068030 & 83115 \\ 410400 & 83115 & -103 \end{bmatrix} \begin{bmatrix} -2864431 & -580199 & 721 \\ 15128640 & 3064351 & -3808 \\ 794253600 & 160878480 & -199921 \end{bmatrix} \begin{bmatrix} -3121 & -313 & -514 & -1799 & -95 & -2497 & -135 & -1007 & -239 & -115 & -921 & -121 & -15 & -1 \\ 16480 & 1653 & 2715 & 9504 & 502 & 13200 & 714 & 5328 & 1265 & 609 & 4880 & 642 & 80 & 6 \\ 862400 & 86688 & 142750 & 500832 & 26545 & 702000 & 38230 & 286848 & 68450 & 33192 & 268000 & 35910 & 4784 & 855 \end{bmatrix}$$

W_{366} 8 lattices, $\chi = 84$ 12-gon: $\infty\infty\infty222\infty\infty\infty222 \rtimes C_2$

$L_{366.1}$: $1_{\mathrm{II}}^{-2}8_1^1,1^-13^-169^-$ $\langle 2* \rangle$ $26^{52,25}_{\infty a}104^{26,9}_{\infty z}26^{52,17}_{\infty a}104^b_2 338^l_2 8^r_2$ $(\times 2)$

shares genus with its 13-dual

$$\begin{bmatrix} -3081208 & 28392 & -163592 \\ 28392 & -234 & 1573 \\ -163592 & 1573 & -8530 \end{bmatrix} \begin{bmatrix} -192193 & 1463 & -10934 \\ -6090240 & 46359 & -346480 \\ 2563392 & -19513 & 145833 \end{bmatrix}$$

$$\begin{bmatrix} -272 & -581 & -193 & -1579 & -1077 & -529 \\ -8626 & -18420 & -6117 & -50036 & -34125 & -16760 \\ 3627 & 7748 & 2574 & 21060 & 14365 & 7056 \end{bmatrix}$$

W_{367} 12 lattices, $\chi = 48$ 12-gon: $222|222|222|222| \rtimes D_4$

$L_{367.1}$: $1_{\mathrm{II}}^{-2}4_5^-,1^2 3^1,1^{-2}5^-,1^-7^-49^-$ $\langle 2 \rangle$ $84^r_2 10^b_2 588^*_2 140^*_2 12^b_2 490^l_2$ $(\times 2)$

$$\begin{bmatrix} -38634540 & -1073100 & 8820 \\ -1073100 & -29806 & 245 \\ 8820 & 245 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -14280 & -398 & 3 \\ -1899240 & -52801 & 398 \end{bmatrix} \begin{bmatrix} -1 & -1 & -11 & -3 & -1 & -2 \\ 36 & 35 & 378 & 100 & 30 & 35 \\ 0 & -130 & -2352 & -1050 & -792 & -4900 \end{bmatrix}$$

W_{368} 16 lattices, $\chi = 36$ 10-gon: $2222|22222|2 \rtimes D_2$

$L_{368.1}$: $1_{\mathrm{II}}^{-2}8_5^-,1^2 3^-,1^-5^-25^-,1^2 7^-$ $\langle 2 \rangle$

$168^r_2 50^b_2 8^b_2 1050^l_2 40^r_2 42^b_2 200^b_2 2^l_2 4200^r_2 10^l_2$

$$\begin{bmatrix} -509338200 & 97032600 & 483000 \\ 97032600 & -18485410 & -92015 \\ 483000 & -92015 & -458 \end{bmatrix} \begin{bmatrix} 257 & 1 & -13 & -61 & 41 & 104 & 267 & 50 & 5933 & 30 \\ 1344 & 5 & -68 & -315 & 216 & 546 & 1400 & 262 & 31080 & 157 \\ 1008 & 50 & -48 & -1050 & -160 & -21 & 300 & 91 & 12600 & 95 \end{bmatrix}$$

W_{369} 8 lattices, $\chi = 24$ 8-gon: $22|2222|22 \rtimes D_2$

$L_{369.1}$: $1_1^1 8_7^1 64_7^1,1^2 3^-,1^{-2}5^-$ $960^b_2 8^s_2 60^b_2 8^l_2 960_2 1^r_2 160^*_2 4^s_2$

$$\begin{bmatrix} 807360 & 960 & -960 \\ 960 & -8 & 0 \\ -960 & 0 & 1 \end{bmatrix} \begin{bmatrix} -29 & -1 & -1 & 0 & 1 & 0 & -1 & -1 \\ -3120 & -107 & -105 & 1 & 120 & 0 & -110 & -108 \\ -27360 & -940 & -930 & 4 & 960 & -1 & -960 & -946 \end{bmatrix}$$

W_{370} 4 lattices, $\chi = 24$ 8-gon: $2|22|22|22|2 \rtimes D_4$

$L_{370.1}$: $1_7^1 8_1^1 64_7^1,1^2 3^-,1^{-2}5^-$ $960^l_2 8_2 15^r_2 64^*_2 60^l_2 8_2 960^r_2 4^b_2$

$$\begin{bmatrix} 960 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -11 & -1 & -2 & -1 & -1 & 0 & 1 & 0 \\ -120 & -9 & -15 & -4 & 0 & 1 & 0 & -1 \\ -480 & -40 & -75 & -32 & -30 & 0 & 0 & -2 \end{bmatrix}$$

W_{371} 4 lattices, $\chi = 144$ 20-gon:
$\infty|\infty222\infty|\infty222\infty|\infty222\infty|\infty222 \rtimes D_4$

$L_{371.1}$: $1_{\mathrm{II}}^{-2}8_5^-,1^1 17^1 289^1$ $\langle 2 \rangle$

$136^{34,1}_{\infty z}34^{68,13}_{\infty a}136^b_2 2^b_2 2312^b_2 34^{68,1}_{\infty b}136^{34,21}_{\infty z}34^b_2 8^b_2 578^b_2$ $(\times 2)$

$$\begin{bmatrix} -37498328 & 136408 & -1583720 \\ 136408 & -442 & 6001 \\ -1583720 & 6001 & -65826 \end{bmatrix} \begin{bmatrix} -15181273 & 47637 & -674739 \\ -1151705528 & 3613912 & -51188111 \\ 260259528 & -816663 & 11567360 \end{bmatrix}$$

$$\begin{bmatrix} 119 & -118 & -4379 & -383 & -14565 & -2961 & -7937 & -5687 & -2781 & -36497 \\ 9028 & -8951 & -332196 & -29055 & -1104932 & -224629 & -602124 & -431434 & -210976 & -2768790 \\ -2040 & 2023 & 75072 & 6566 & 249696 & 50762 & 136068 & 97495 & 47676 & 625685 \end{bmatrix}$$

W_{372} 8 lattices, $\chi = 72$ 16-gon: $22|2222|2222|2222|22 \rtimes D_4$

$L_{372.1}$: $1^{1}_{1}8^{-}_{5}64^{1}_{7}$,$1^{2}3^{1}$,$1^{2}7^{1}$ $448^{r}_{2}12^{b}_{2}56^{s}_{2}12^{b}_{2}448^{s}_{2}4^{*}_{2}64^{l}_{2}1_{2}$ $(\times 2)$

$$\begin{bmatrix} 1735104 & -577920 & -1344 \\ -577920 & 192488 & 448 \\ -1344 & 448 & 1 \end{bmatrix} \begin{bmatrix} -37633 & 12404 & 42 \\ -110208 & 36325 & 123 \\ -1171968 & 386296 & 1307 \end{bmatrix}$$

$$\begin{bmatrix} -1587 & -169 & -184 & -128 & -1013 & -43 & -15 & 0 \\ -4648 & -495 & -539 & -375 & -2968 & -126 & -44 & 0 \\ -49280 & -5238 & -5684 & -3942 & -31136 & -1318 & -448 & -1 \end{bmatrix}$$

W_{373} 8 lattices, $\chi = 72$ 16-gon: $2222|2222|2222|2222| \rtimes D_4$

$L_{373.1}$: $1^{-2}_{2}16^{-}_{3}$,$1^{-}3^{1}9^{-}$,$1^{-2}5^{-}$,$1^{2}7^{-}$

$18^{b}_{2}560^{*}_{2}12^{l}_{2}315_{2}48_{2}35^{r}_{2}12^{*}_{2}5040^{b}_{2}2^{l}_{2}5040_{2}3^{r}_{2}140^{*}_{2}48^{*}_{2}1260^{l}_{2}3_{2}560^{r}_{2}$

$$\begin{bmatrix} -362633040 & 120960 & 201600 \\ 120960 & -33 & -72 \\ 201600 & -72 & -109 \end{bmatrix} \left[\begin{matrix} 19 & 311 & 15 & 53 & 3 & -1 & -1 & 37 & 1 & 607 \\ 15927 & 260680 & 12572 & 44415 & 2512 & -840 & -838 & 31080 & 839 & 509040 \\ 24615 & 402920 & 19434 & 68670 & 3888 & -1295 & -1296 & 47880 & 1295 & 786240 \end{matrix}\right.$$

$$\left.\begin{matrix} 9 & 93 & 41 & 391 & 17 & 501 \\ 7547 & 77980 & 34376 & 327810 & 14252 & 420000 \\ 11658 & 120470 & 53112 & 506520 & 22023 & 649040 \end{matrix}\right]$$

W_{374} 4 lattices, $\chi = 72$ 15-gon: $222222|2222222\infty\!\!\!|\,2 \rtimes D_2$

$L_{374.1}$: $1^{1}_{1}8^{1}_{7}256^{1}_{1}$,$1^{2}9^{1}$

$2304_{2}1^{r}_{2}256^{*}_{2}36^{s}_{2}256^{s}_{2}4^{*}_{2}2304^{l}_{2}1_{2}256_{2}9^{r}_{2}256^{*}_{2}4^{s}_{2}2304^{b}_{2}8^{48,1}_{\infty b}8^{l}_{2}$

$$\begin{bmatrix} -96876288 & 82944 & 82944 \\ 82944 & -8 & -72 \\ 82944 & -72 & -71 \end{bmatrix} \left[\begin{matrix} 2737 & 47 & 607 & 179 & 363 & 33 & 541 & 7 & 59 & 4 \\ 48672 & 836 & 10800 & 3186 & 6464 & 588 & 9648 & 125 & 1056 & 72 \\ 3147264 & 54045 & 697984 & 205830 & 417408 & 37946 & 622080 & 8049 & 67840 & 4599 \end{matrix}\right.$$

$$\left.\begin{matrix} -1 & -1 & 1 & 2 & 21 \\ -16 & -18 & 0 & 35 & 373 \\ -1152 & -1150 & 1152 & 2300 & 24148 \end{matrix}\right]$$

Department of Mathematics, U.T. Austin
E-mail address: allcock@math.utexas.edu
URL: http://www.math.utexas.edu/~allcock