

ROOT SYSTEMS FOR LORENTZIAN KAC-MOODY ALGEBRAS IN RANK 3

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ABSTRACT. Sometimes a hyperbolic Kac-Moody algebra admits an automorphic correction, meaning a generalized Kac-Moody algebra with the same real simple roots and whose denominator function has good automorphic properties; these for example allow one to work out the root multiplicities. Gritsenko and Nikulin have formalized this in their theory of Lorentzian Lie algebras and shown that the real simple roots must satisfy certain conditions in order for the algebra to admit an automorphic correction. We classify the hyperbolic root systems of rank 3 that satisfy their conditions and have only finite many simple roots, or equivalently a timelike Weyl vector. There are 994 of them, with as many as 24 simple roots. Patterns in the data suggest that some of the non-obvious cases may be the richest.

Kac and Moody introduced the Lie algebras that now bear their names in [17] and [19]. The main goal at the time was understanding the affine algebras—the central extensions of the algebras of loops in finite dimensional Lie algebras. But it was clear that the construction also yields hyperbolic algebras (among others). Here “affine” and “hyperbolic” refer to the action of the Weyl group W on Euclidean and hyperbolic space, and the elliptic case is the classical one of finite W . The hyperbolic algebras are the next step beyond the affine ones, but only a few examples are well-understood. The main difficulty is that only the best hyperbolic KMAs are likely to be interesting and it is not clear which ones are best. Various authors have tried to identify the gems, and our paper continues this process.

Borcherds has proposed the following criterion for a Kac-Moody algebra to be interesting: one must be able to know both the simple roots and the multiplicity of an arbitrary root (in some more concrete manner than the Peterson recursion formula). No known hyperbolic examples meet this criterion. But after a careful analysis of the case when the simple roots are those of the 26-dimensional even unimodular Lorentzian lattice, Borcherds discovered that his criterion could be met by enlarging the KMA slightly. This enlargement was not a KMA but something he called a generalized KMA, got by adjoining some imaginary simple roots [4]. This particular GKMA is now called the fake monster Lie algebra, and the same method allowed him to construct and analyze the monster Lie algebra, leading to his famous proof of the Conway-Norton moonshine conjectures [5]. The key to his analyses was that the denominator functions of these GKMA happen to be automorphic forms. Gritsenko and Nikulin developed these examples into their theory of Lorentzian Lie

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algebras [22][16], with the automorphic property at its heart. The essential axiom is that the denominator function be a reflective automorphic form.

They show that this can only happen if the set Π of real simple roots has certain properties. The Π with these properties fall into two classes, the “elliptic” ones with finitely many simple roots and the “parabolic” ones with infinitely many. We completely classify the elliptic ones in rank 3: there are 994 simple root systems, with as many as 24 simple roots. The parabolic case is also very interesting and we hope to classify those root systems too.

Carbone et. al., building on earlier tables, published extensive tables of hyperbolic Dynkin diagrams and associated data [7]. They address a different but related problem. They start with the idea that the Dynkin diagram for a root system in n -dimensional Minkowski space should have $n + 1$ nodes (and finite volume Weyl chamber). The assumption on the number of nodes is equivalent to the chamber being a simplex, and we think this restriction is not very natural. It is carried over from the classical case of finite Weyl group, where it just so happens that the chamber is always a spherical simplex. In hyperbolic space this isn’t true, and the simplicial chambers aren’t particularly special among all chambers.

On the other hand, Carbone et. al. treat the non-symmetrizable case too. Our formulation in terms of lattices restricts us to the symmetrizable case. Of the 123 rank 3 root systems in [7], 44 of them are symmetrizable. These are the 42 in tables 1 and 2 of Saçlıoğlu’s [23], plus two that Saçlıoğlu missed and Carbone et. al. caught. We checked that our $\Pi_{3,1}, \dots, \Pi_{3,44}$ correspond bijectively with this list. To save space we haven’t printed Dynkin diagrams for our root systems, but these can be read from what we have printed; see section 2.

To state our main result precisely we make the following mostly standard definitions.

A *lattice* L means a finitely generated free abelian group equipped with a symmetric \mathbb{Q} -valued bilinear pairing. We denote the pairing of $x, y \in L$ by $x \cdot y$ and write x^2 for the norm $x \cdot x$ of x . We call L *integral* if all inner products lie in \mathbb{Z} , and call L *unscaled* if it is integral and the gcd of all inner products is 1.

We call $\alpha \in L$ a *root* of L if α has positive norm and $L \cdot \alpha \subseteq \frac{1}{2}\alpha^2\mathbb{Z}$. In this case the reflection in α , negating α and fixing α^\perp pointwise, preserves L . A root need not be primitive in L . The reason for allowing imprimitive roots is that the “root lattice” in the construction of the associated KMA [18] is the free abelian group \mathbb{Z}^Π on Π . Our L is merely a quotient of \mathbb{Z}^Π . Although roots are obviously primitive in \mathbb{Z}^Π , there is no reason to expect their images in L to also be primitive. And there are known examples with imprimitive roots, like the 32nd entry in [14, table 1], which corresponds to our $\Pi_{8,11}$.

We call a set Π of spacelike (positive-norm) vectors in Minkowski space $\mathbb{R}^{n,1}$ a *root system* if $\alpha \cdot \alpha' \in \frac{1}{2}\alpha^2\mathbb{Z}$ for all $\alpha, \alpha' \in \Pi$; in this case they are roots of their integral span, called the root lattice L . The *Weyl group* W means the group generated by the reflections in the roots. To avoid degenerate cases we assume that L has full rank in $\mathbb{R}^{n,1}$, which implies $|W| = \infty$. We call a root system Π *simple* if its members’ pairwise inner products are nonpositive. In this case the *fundamental chamber*

$$(1) \quad C := \{x \in L \otimes \mathbb{R} \mid x \cdot \alpha \leq 0 \text{ for all } \alpha \in \Pi\}$$

meets just one of the two cones of negative norm vectors, which we call the *future cone*. Furthermore, C is a fundamental domain for the action of W on the *Tits cone* $T := \cup_{w \in W} wC$ in the sense that every point of T is W -equivalent to a unique point of C . The Tits cone always contains the future cone.

Now we can state the conditions of Gritsenko and Nikulin on the set Π of real simple roots of a GKMA, that are necessary for its denominator function to be an automorphic form. First, Π must be a simple root system spanning $\mathbb{R}^{n,1}$ for some n . Furthermore,

- (i) the interior T° of the Tits cone must coincide with the future cone;
- (ii) the normalizer of W must have finite index in $O(L)$;
- (iii) a Weyl vector must exist, meaning a vector $\rho \in \mathbb{R}^{n,1}$ with $\rho \cdot \alpha = -\alpha^2/2$ for all $\alpha \in \Pi$.

In the language of [21], the first condition is that Π has arithmetic type and the second refines this to restricted arithmetic type; also we say Weyl vector in place of [21]’s “lattice Weyl vector”. If it exists then ρ is unique and lies in C .

Under these conditions, it turns out that ρ must be timelike ($\rho^2 < 0$) or lightlike ($\rho^2 = 0$), in which cases Π is said to have elliptic or parabolic type. In the timelike case the Weyl chamber must be a finite-volume polytope in hyperbolic space (the projectivized future cone). We are abusing language here: we will say that C has finite area if its image in hyperbolic space does, and similarly for compactness. In the lightlike case the Weyl chamber has infinite area and infinitely many sides.

Theorem 1. *Suppose Π is a simple root system spanning $\mathbb{R}^{2,1}$ and satisfies (i)–(iii) with timelike Weyl vector ρ . Then up to scale, Π is isometric to exactly one of the 994 root systems appearing at the end of the paper.*

The Gritsenko-Nikulin conditions remain the same if one works even more generally, with generalized Kac-Moody superalgebras. The only difference is that a real root must satisfy $L \cdot \alpha \subseteq \alpha^2\mathbb{Z}$ before its parity can be set to odd. So we will say no more about them.

For fixed n there are only finitely many root systems satisfying (i)–(iii) with timelike ρ , by [21, thm. 1.3.2], and $n \leq 22$ by work of Esselmann [8]. For $n \leq 19$, there are infinitely many simple root systems in $\mathbb{R}^{n,1}$ with finite-volume chamber [1], and their number even grows exponentially with this volume (except perhaps for $n = 16, 17$). The essential condition that narrows this multitude down to finitely many is the existence of ρ . In the lightlike case Nikulin has proven a finiteness result [21, thm. 1.3.3] for fixed n , and the bound $n \leq 998$ [20].

These necessary conditions (i)–(iii) for the denominator function to be “correctable” to a reflective automorphic form have also proven sufficient in every case investigated so far. That is, given such a Π , it has proven possible to “automorphically correct” [11, (3.3)] the KMA with root system Π to a GKMA (possibly a superalgebra) with the same real root system but also some imaginary simple roots, whose denominator function is a reflective automorphic form. That this should always be possible is hinted at by Borchers’ suggestion [6, §12] that “good” reflection groups and “good” automorphic forms should correspond to each other, and formalized by the arithmetic mirror-symmetry conjecture of Gritsenko and Nikulin [14, conj. 2.2.4]. Specific examples of automorphic corrections are worked out in [11], [12], [13] and [15]; see section 3 for more details of the correspondences between those root systems and ours. Gritsenko’s paper [10] promises further automorphic corrections in a forthcoming paper with Nikulin.

Open problems: The obvious open problem is to investigate the possible automorphic corrections for these 994 root systems. For a given root system Π this amounts to seeking reflective automorphic forms for subgroups Γ of $O(M)$, where M is a lattice of signature $(2, 3)$ that possesses an isotropic vector orthogonal to a copy of Π . (Π should consist of roots of M and Γ should contain W .) See [15] for the largest investigation of this sort.

Surely only some of our root systems are the gems we seek, and our results suggest that certain cases are worth investigating first. One very surprising outcome is that 390 of the 994 cases share a single root lattice $L_{16.13} \cong \langle -1 \rangle \oplus A_2[15]$. (See section 2 for lattice names.) This suggests that $O(L_{16.13} \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})$ and its finite-index subgroups may admit an extraordinarily rich family of reflective automorphic forms. Or perhaps there is a single automorphic form for some many-cusped subgroup, whose Fourier expansions at various cusps give automorphic corrections to the KMAs associated to many of the root systems with root lattice $L_{16.13}$? A few other lattices account also occur very frequently, but none of them are “obvious” lattices. For example, $L_{142.20}$ is an index 2 superlattice of $\langle -4, 36, 36 \rangle$ and occurs 42 times. This is the only one of the most-frequently-occurring lattices which is isotropic. Interestingly, $O(L_{142.20})$ is a subgroup of $O(\mathbb{Z}^{2,1}) = \mathrm{PGL}_2\mathbb{Z} \times \{\pm 1\}$, whose root system defines the first- and most-studied hyperbolic KMA, introduced by Feingold-Frenkel in [9]. See section 2 for more about the frequently-occurring root lattices.

In section 1 we prove the main theorem, and in section 2 we explain how to read the table. We have presented each root system so that its essential properties are visible, including its symmetry group, root norms, Weyl vector, the angles of its Weyl chamber, and the isomorphism type of the root lattice. We also provide additional information like largest and smallest Weyl chambers and the number of chambers with interesting properties like compactness or right-angledness. Finally, section 3 is devoted to correlating our classification with the extensive work of Gritsenko and Nikulin, and contains further references to the literature. This includes proving conjecture 1.2.2 of their paper [14]. The table itself appears at the very end of the paper. All information in the table appears in computer-readable form at the end of the $\mathrm{T}_\mathrm{E}\mathrm{X}$ file, commented out.

1. PROOF OF THE CLASSIFICATION

We begin by supposing that Π is a simple root system spanning $\mathbb{R}^{2,1}$, with finite-area Weyl chamber. We write L for the root lattice. The chamber has finitely many sides (since it has finite area), so it is a polygon. The roots correspond to these sides, so they have a natural cyclic ordering, up to reversal. We suppose $\Pi = \{\alpha_1, \dots, \alpha_n\}$ with the α_i ordered in this manner.

Lemma 2. *Some 3, 4 or 5 consecutive members of Π have inner product matrix equal to one of those given below in (2)–(6), up to multiplication by a rational number.*

In each case A, B, A', B', C, C' are positive integers, except in cases (2) and (4) where A' and B' are both taken to be 0. In cases (4)–(6), k is a positive integer, and in case (6) so is k' . In every case we require

$$AB'C' = A'BC$$

and write β for the common value. In every case we also require $CC' < 4K^2$ where

$$K := 1 + \frac{\sqrt{AB}}{2} + \frac{\sqrt{A'B'}}{2} + \sqrt{(2 + \sqrt{AB})(2 + \sqrt{A'B'})}$$

In cases (4)–(6), we make the further requirement on A, B, A', B', C, C' that

$$N := 4 + 4 \frac{CC' + \beta + A'B'}{AB - 4}$$

is an integer, and require k to divide it. In case (6) we define N' analogously with primed and unprimed letters exchanged, and require that it is an integer and that k' divides it. Finally, in case (6) we also require that $\gamma k^2 / A'BN$ and $\gamma k'^2 / AB'N'$ are integers, where

$$\gamma := \frac{2\beta}{kk'} \left(2 + \frac{\beta}{CC'} - \frac{(2CC' + \beta)^3}{(AB - 4)(A'B' - 4)C^2C'^2} \right).$$

The inner product matrices are:

$$(2) \quad \begin{pmatrix} 2AC & -ABC & -ACC' \\ -ABC & 2BC & 0 \\ -ACC' & 0 & 2AC' \end{pmatrix} \quad AB \leq 4;$$

$$(3) \quad \begin{pmatrix} 2AB' & -ABB' & -\beta \\ -ABB' & 2BB' & -A'B'B \\ -\beta & -A'B'B & 2A'B \end{pmatrix} \quad \begin{array}{l} AB \leq 4 \\ A'B' \leq 4; \end{array}$$

$$(4) \quad \begin{pmatrix} 2AC & 0 & -ABC & -ACC' \\ 0 & 2AC'N/k^2 & 0 & -AC'N/k \\ -ABC & 0 & 2BC & 0 \\ -ACC' & -AC'N/k & 0 & 2AC' \end{pmatrix} \quad \begin{array}{l} 4 < AB < 36 \\ 4 < CC'; \end{array}$$

$$(5) \quad \begin{pmatrix} 2AB' & 0 & -ABB' & -\beta \\ 0 & 2A'BN/k^2 & 0 & -A'BN/k \\ -ABB' & 0 & 2BB' & -A'B'B \\ -\beta & -A'BN/k & -A'B'B & 2A'B \end{pmatrix} \quad \begin{array}{l} 4 < AB < 36 \\ A'B' \leq 4 \\ 4 < CC'; \end{array}$$

$$(6) \quad \begin{pmatrix} 2AB' & 0 & -ABB' & -AB'N'/k' & -\beta \\ 0 & 2A'BN/k^2 & 0 & \gamma & -A'BN/k \\ -ABB' & 0 & 2BB' & 0 & -A'B'B \\ -AB'N'/k' & \gamma & 0 & 2AB'N'/k'^2 & 0 \\ -\beta & -A'BN/k & -A'B'B & 0 & 2A'B \end{pmatrix} \quad \begin{array}{l} 4 < AB < 36 \\ 4 < A'B' < 36 \\ 4 < CC'. \end{array}$$

Proof. All the hard work was done in [2], which used a refinement of Nikulin's method of narrow parts of polygons. First, theorem 1 of that paper uses the finiteness of the chamber to show that the chamber has one of three features: a “short edge” orthogonal to at most one of its neighbors (which are not orthogonal to each other), or at least 5 edges and a “short pair”, or at least 6 edges and a “close pair”. Second, lemmas 7–9 of the same paper show that in each of these cases there exist 3, 4 or 5 consecutive roots with one of the inner product matrices (2)–(6). We remark that the notion of root used in this paper is the same as that of a weight 2 quasiroot in [2], except that [2] assumes all quasiroots are primitive.

Since primitivity played no role in the proofs of these lemmas, they apply to the current situation. \square

Proof of theorem 1. We first explain how we assembled the list, using a refinement of the proof of Gritsenko-Nikulin's [14, thm. 1.2.1] We began by constructing all the matrices from lemma 2, using a computer. There are 317906 of them, the same set of matrices we started with in the proof of [2, thm. 13]. Then we discarded the ones that are positive definite (5 of them) or have rank 2 (9 of them). For each remaining matrix M , we considered the lattice D got as the quotient of $\mathbb{Z}^{k=3,4 \text{ or } 5}$ by the kernel of M , and the vectors $\alpha_1, \dots, \alpha_k$ which are the images of the standard basis vectors for \mathbb{Z}^k . In every case $\alpha_1, \dots, \alpha_k$ is a simple root system in D . In each case we sought a Weyl vector. This was easy: since $k \geq 3$, the conditions $\rho \cdot \alpha_i = -\alpha_i^2/2$ determine (or overdetermine) ρ . We discarded $(D, \alpha_1, \dots, \alpha_k)$ if no Weyl vector existed, or if a Weyl vector existed but was not timelike. This accounted for 310179 of the cases, leaving 7713.

For each remaining tuple $(D, \alpha_1, \dots, \alpha_k)$ we computed the reflective hull of $\alpha_1, \dots, \alpha_k$. This means the set of vectors $x \in D \otimes \mathbb{Q}$ satisfying $x \cdot \alpha_i \in \frac{1}{2}\alpha_i^2\mathbb{Z}$ for all $i = 1, \dots, k$. It is the unique maximal-under-inclusion lattice in which $\alpha_1, \dots, \alpha_k$ are roots. Because $\alpha_1, \dots, \alpha_k$ span $D \otimes \mathbb{Q}$, the hull contains D of finite index. We enumerated all lattices E lying between D and the hull. There were a total of 61811 of these enlargements. We rescaled each to be unscaled.

The following definition is useful for further analysis; it is a codification of the properties possessed by our 61811 tuples $(E, \alpha_1, \dots, \alpha_k)$. If E is an integral lattice of signature $(2, 1)$ then a *chain* in E means a sequence $\alpha_1, \dots, \alpha_{m \geq 3}$ of roots of E , which form a simple root system, admit a timelike Weyl vector ρ , and have the property that each span $\langle \alpha_1, \alpha_2 \rangle, \dots, \langle \alpha_{m-1}, \alpha_m \rangle$ is positive-definite or positive-semidefinite. The Weyl vector is uniquely determined since $m \geq 3$. The geometric meaning of the last condition is that the mirrors of any two consecutive roots meet in H^2 or its boundary. We say that the chain is *closed* if $\langle \alpha_m, \alpha_1 \rangle$ satisfies the same condition. An *extension* of the chain means a root α_{m+1} of E such that $\alpha_1, \dots, \alpha_{m+1}$ is also a chain in E .

Now we address the question: what are all the possible extensions of a non-closed chain $\alpha_1, \dots, \alpha_m$ in an integral lattice E ? In particular, are there any? Our method is a refinement of one used by Gritsenko and Nikulin [14, thm. 1.2.1]. One can restrict the norm of a root α of E , using the fact that one of the following holds: (i) α is imprimitive, in which case $\alpha/2$ spans an orthogonal summand of E ; (ii) α is primitive and spans an orthogonal summand of E ; or (iii) α is primitive and spans an orthogonal summand of an index 2 sublattice of E . The result is that α^2 divides $4e$, where e is the largest elementary divisor of the discriminant group E^*/E of E .

So suppose N a positive divisor of $4e$. If an extension α_{m+1} has norm N , then its inner product I with α_m must be one of a few possibilities. This uses the fact that α_m and α_{m+1} are simple roots for a finite or affine Coxeter group. For example, if $\alpha_m^2 = \alpha_{m+1}^2 = N$ then α_m and α_{m+1} must be simple roots for an A_1^2 , A_2 or \tilde{A}_1 root system, so $I \in \{0, -N/2, -N\}$. Except in the cases $\alpha_m^2/N \in \{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4\}$, the only possibility is $I = 0$.

After fixing such a pair (N, I) , one can seek extensions α_{m+1} with $\alpha_{m+1}^2 = N$ and $\alpha_m \cdot \alpha_{m+1} = I$. If they exist then we record them, and if not then we continue to the next pair (N, I) . Here are the details. The conditions $\alpha_{m+1} \cdot \rho = -N/2$

and $\alpha_{m+1} \cdot \alpha_m = I$ restrict α_{m+1} to a line in $\mathbb{R}^{2,1}$, and imposing the condition $\alpha_{m+1}^2 = N$ restricts α_{m+1} to one or two possibilities. Finding them amounts to solving a quadratic equation. If solving this gives non-rational vectors (the most common result) then the sought extension doesn't exist. If the solution or solutions are rational then one checks which have nonpositive inner products with $\alpha_1, \dots, \alpha_m$. One can show (or just check) that at most one does. If none do then the sought extension doesn't exist. So suppose exactly one does. If it is a root of E then it is the extension sought; otherwise the sought extension doesn't exist.

We began with our 61811 chains, checked which chains were closed (722 of them), and found all the extensions (10178 of them) of the non-closed chains. We set the closed ones aside for later analysis and repeated the process with the 10178 new chains, finding 2446 closed chains and 5354 extensions. Continuing in this manner, on the 21st iteration we found 280 closed chains and no extensions. This left us with 21831 tuples $(E, \alpha_1, \dots, \alpha_m)$ with E an integral lattice and $\alpha_1, \dots, \alpha_m$ a closed chain in E . The real content of the theorem is that (L, Π) is isometric to one of them, up to dihedral rearrangement of the simple roots.

Given our preparations, this is easy: by lemma 2, some 3, 4 or 5 consecutive members of Π have one of the inner product matrices given there, up to scale. Call them $\alpha_1, \dots, \alpha_k$. Since L lies in the reflective hull of $\alpha_1, \dots, \alpha_k$, the tuple $(L, \alpha_1, \dots, \alpha_k)$ occurs on the list of 61811 chains. Obviously each of $\alpha_{k+1}, \dots, \alpha_n$ is an extension of the chain of its predecessors, and $\alpha_1, \dots, \alpha_n$ is closed. So $(L, \alpha_1, \dots, \alpha_n)$ must occur on our final list of 21831 closed chains.

We have established the completeness of the enumeration, but more work was required to weed out redundancy. One source of redundancy is the dihedral reorderings of roots. The second is that in one of our 21831 tuples $(E, \alpha_1, \dots, \alpha_n)$, E may be strictly larger than $\langle \alpha_1, \dots, \alpha_n \rangle$. We eliminated this by computing the inner product matrix of the roots, effectively forgetting E , and then rescaling to make the root lattice unscaled. We eliminated the dihedral reordering redundancy in the obvious way: whenever several of these inner product matrices were equal up to dihedral permutation, we kept only one. The result was the list of 994 root systems that appears at the end of the paper. \square

2. HOW TO READ THE TABLE

The simple root systems are named $\Pi_{i,j}$ where i indicates the number of roots. For given i , the root systems are organized according to their symmetry groups, but those with a given symmetry group appear in no special order. For each Π we have in mind its simple roots $\alpha_1, \dots, \alpha_n$ in cyclic order, although not all of them are listed when symmetries are present. See below for how to recover the full root system from the displayed roots.

Other names: Some of the root systems have already been named by Gritsenko-Nikulín. This accounts for names like “ $A_{1,II}$ ” and “ GN_{25} ”. See the next section for details and further references.

Root lattice: We chose the scale at which the root lattice L is unscaled. Because the Weyl group of Π has finite area chamber, L is reflective, meaning that its reflections generate a finite index subgroup of $O(L)$. Therefore L is one of the lattices $L_{k,l}$ from the classification of these lattices given in [3] (and proven in [2]). We indicate which one; see the end of this section for some comments on how

we worked this out. The reader may consult [3] for further information like the Conway-Sloane symbol for the genus of L .

Some of the $L_{k,l}$ occurred many times. The ten most frequently appearing lattices are the following, where we have written $A_2[m]$ for the triangular lattice with inner product scaled by m , so that its six minimal vectors have norm $2m$.

$L_{16.13} \times 390$	$\langle -1 \rangle \oplus A_2[15]$	$L_{251.3} \times 74$	$\langle -5 \rangle \oplus A_2[6]$
$L_{142.20} \times 42$	$\langle -4, 36, 36 \rangle$, glued	$L_{123.8} \times 42$	$\langle -1, 6, 6 \rangle$
$L_{155.1} \times 19$	$\langle -4 \rangle \oplus A_2[3]$	$L_{7.7} \times 14$	$\langle -1 \rangle \oplus A_2[2]$
$L_{31.7} \times 12$	$\langle -1 \rangle \oplus A_2[10]$	$L_{22.4} \times 12$	$\langle -1 \rangle \oplus A_2[7]$
$L_{16.9} \times 12$	$\langle -1 \rangle \oplus A_2[5]$	$L_{16.7} \times 12$	$\langle -5 \rangle \oplus A_2[3]$

We found the stated descriptions of these lattices by referring to the ‘‘corner symbols’’ in [3] (explained in [2]), which made the direct sum decompositions visible. The gluing in the description of $L_{142.20}$ means the adjunction of the vector $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.

It is remarkable that $L_{16.13}$ occurs so frequently—by itself it accounts for more than half of the table entries with compact chamber. Looking at the lists of roots in these systems also reveals a certain sameness: many of the root systems differ only slightly, by ‘‘reordering’’ the roots or other slight alterations. These ten lattices are anisotropic (hence have compact Weyl chambers) except $L_{142.20}$. We mentioned in the introduction that $O(L_{142.20}) \subseteq O(\mathbb{Z}^{2,1})$; this is because 3-filling ([2, §1.1]) the even sublattice of $L_{142.20}$ and then rescaling yields $L_{4,1} = \mathbb{Z}^{2,1}$. Simple roots for this lattice form the root system $\Pi_{3,27}$, introduced by Feingold-Frenkel [9].

Chamber angles: We give the list of angles $\pi/(2, 3, 4, 6$ or $\infty)$ at the vertices of the chamber in cyclic order: the first vertex is $\alpha_1^\perp \cap \alpha_2^\perp$ and the last is $\alpha_n^\perp \cap \alpha_1^\perp$. Of the 994 Weyl chambers listed, 9 have all ideal vertices ($\Pi_{3,1}, \Pi_{3,14}, \Pi_{3,15}, \Pi_{4,2}, \Pi_{4,6}, \Pi_{4,17}, \Pi_{6,3}, \Pi_{6,16}$ and $\Pi_{6,17}$), 101 have all right angles, 766 are compact, and 228 are noncompact. Also, 9 of them are regular ($\Pi_{3,1}, \Pi_{3,14}, \Pi_{3,15}, \Pi_{4,1}, \Pi_{4,2}, \Pi_{4,17}, \Pi_{6,1}, \Pi_{6,2}$ and $\Pi_{6,3}$), and the regular chambers are the ideal triangle (3 times), square (twice) and hexagon, the square with angles $\pi/3$, and the hexagons with angles $\pi/2$ and $\pi/3$. In four of these cases, $\text{Aut } \Pi$ is strictly smaller than the isometry group of the chamber.

The (orbifold) Euler characteristic and therefore the area of the chamber are easy to find from the angles. The smallest Weyl chamber is shared by the Weyl groups of $\Pi_{3,25}, \Pi_{3,28}, \Pi_{3,31}$ and $\Pi_{3,40}$, with Euler characteristic $-1/24$, and the largest chamber is that of $\Pi_{24,1}$, whose Weyl group has Euler characteristic -8 .

Chamber isometries: If Π has nontrivial isometries then the list of angles is followed by ‘‘ $\times C_m$ ’’ or ‘‘ $\times D_m$ ’’. The subscript m is $|\text{Aut } \Pi|$ and we write D (resp. C) when there are (resp. are not) reflections in this finite group. Note the distinction between C_2 and D_2 . In the dihedral case, we have also indicated how the mirrors of the reflections meet the boundary of the chamber. A $|$ through a digit (i.e., \ddagger, \S, \wp) means that a mirror passes through that vertex, and a $|$ between two digits means that a mirror bisects the edge joining the two corresponding vertices. Because of the cyclic ordering, a $|$ before the string of digits means that a mirror bisects the edge α_1^\perp .

Shared Weyl chambers: Sometimes the Weyl groups of distinct root systems are conjugate in $O(2,1)$. When this happens we include the note ‘‘(shared)’’ in the table. We have numbered the roots for our root systems so that the roots of the two systems correspond in the obvious way. That is, if $\alpha_1, \dots, \alpha_n \in \mathbb{R}^{2,1}$ and $\alpha'_1, \dots, \alpha'_n \in \mathbb{R}^{2,1}$ have isometric Weyl chambers, then there is an isometry of

3 : 1 ~ 14 ~ 15	3 : 3 ~ 6
3 : 4 ~ 8	3 : 5 ~ 7 ~ 9
3 : 10 ~ 11 ~ 21	3 : 12 ~ 13
3 : 16 ~ 19 ~ 33	3 : 17 ~ 18 ~ 20 ~ 36 ~ 38 ~ 43
3 : 22 ~ 23 ~ 24	3 : 25 ~ 28 ~ 31 ~ 40
3 : 26 ~ 27 ~ 29 ~ 35 ~ 39 ~ 41	3 : 30 ~ 32 ~ 34 ~ 37 ~ 42 ~ 44
4 : 2 ~ 17	4 : 7 ~ 27
4 : 8 ~ 37 ~ 39 ~ 41	4 : 10 ~ 11
4 : 12 ~ 18	4 : 13 ~ 20
4 : 19 ~ 52	4 : 21 ~ 24 ~ 57 ~ 60 ~ 61
4 : 25 ~ 62	4 : 26 ~ 29 ~ 31 ~ 34
4 : 28 ~ 30 ~ 32 ~ 33 ~ 35 ~ 36	4 : 38 ~ 40 ~ 42 ~ 43 ~ 81
4 : 46 ~ 79	4 : 47 ~ 49
4 : 48 ~ 50	4 : 51 ~ 53 ~ 56
4 : 55 ~ 58 ~ 59	4 : 64 ~ 69
4 : 65 ~ 67 ~ 68 ~ 70	4 : 71 ~ 74
4 : 72 ~ 75 ~ 77	4 : 73 ~ 76
4 : 78 ~ 80 ~ 82 ~ 84 ~ 85 ~ 87	4 : 83 ~ 86
5 : 1 ~ 5	5 : 4 ~ 7 ~ 12 ~ 16 ~ 42 ~ 48
5 : 8 ~ 43	5 : 13 ~ 17
5 : 19 ~ 21	5 : 20 ~ 22 ~ 23
5 : 31 ~ 34	5 : 37 ~ 40
5 : 44 ~ 50	6 : 8 ~ 37
6 : 11 ~ 38	6 : 30 ~ 35

TABLE 1. Equivalence classes of root systems, under the relation of having isometric chambers. For example, the last entry means that $\Pi_{6,30}$ and $\Pi_{6,35}$ share a chamber.

$\mathbb{R}^{2,1}$ sending each α_i to a scalar multiple of α'_i . (In the language of [21], the root systems are twisted to each other and these scalars are the twisting coefficients, up to a common multiple.) For ease of cross-reference, the root systems that share their chamber (and with which other systems) are listed in table 1.

Coordinates compatible with Aut Π : In every case we took $-\rho/2\rho^2$ as the first basis vector, for reasons explained below. When Π doesn't admit a symmetry of order 3 then we chose two more basis vectors in $L \otimes \mathbb{Q}$ making Aut Π visible, as described below. When Π does then we describe vectors in ρ^\perp by three-component vectors summing to 0, so we represent elements of L by four-component vectors whose last three coordinates sum to zero. This is also to make Aut Π visible. The inner product matrix (see below) gives the inner products of our basis for $\mathbb{R}^{2,1}$ or $\mathbb{R}^{3,1}$, and the roots (also below) are specified as vectors in $\mathbb{R}^{2,1}$ or $\mathbb{R}^{3,1}$. We have already described Aut Π in terms of its action on the chamber, and now we make concrete its action on $\mathbb{R}^{2,1}$ or $\mathbb{R}^{3,1}$.

First we describe Aut Π when using only three coordinates. If it is D_2 then the only symmetry is negation of the last coordinate. If it is C_2 then the only symmetry is the negation of the last two coordinates. If it is D_4 then Aut Π is generated by these two transformations. If it is C_4 or D_8 then the last two basis vectors are orthogonal and have the same norm, and the obvious rotations of order 4

are symmetries. In the D_8 case, $\text{Aut } \Pi$ is generated by these and the exchange of the last two coordinates.

Now we describe $\text{Aut } \Pi$ when using four coordinates. In every case it contains the cyclic permutations of the last three coordinates. If it is D_6 then one enlarges this to all permutations of these coordinates. If it is C_6 then instead one adjoins the simultaneous negation of the last three coordinates. If it is D_{12} then one makes both these enlargements.

Inner product matrix and Weyl vector: The first displayed matrix is the inner product matrix of our basis for $\mathbb{R}^{2,1}$ or $\mathbb{R}^{3,1}$. It is invariant under the symmetries just described. Recall that our first basis vector was $-\rho/2\rho^2$. The top left entry (call it E) of the inner product matrix is the norm of this vector, namely $1/4\rho^2$. It follows that $\rho^2 = 1/4E$ and $\rho = (-1/2E, 0, \dots, 0)$.

The simple roots and their norms: The second displayed matrix gives the components of some of the simple roots. Their norms are the top row of the matrix. This pleasant circumstance arises from the defining property $\rho \cdot \alpha = -\alpha^2/2$ of ρ , and is the reason for our curious choice of $-\rho/2\rho^2$ as our first basis vector.

In the absence of symmetry, we have printed all of $\alpha_1, \dots, \alpha_n$. If $\text{Aut } \Pi = C_m$ then we have printed $\alpha_1, \dots, \alpha_{n/m}$. So suppose $\text{Aut } \Pi = D_m$. We explained above how we used |'s to indicate how the mirrors of $\text{Aut } \Pi$ meet the boundary of the chamber. We printed only the roots corresponding to the edges involved in the portion of the boundary of the chamber lying strictly between the first two displayed |'s. For example, for $\sharp 44\sharp 44 \times D_2$ we would display $\alpha_2, \alpha_3, \alpha_4$ because the first | represents the vertex $\alpha_1^\perp \cap \alpha_2^\perp$ and the second represents $\alpha_4^\perp \cap \alpha_5^\perp$. And for $24|424|4 \times D_2$ we would display $\alpha_3, \dots, \alpha_6$ since the space between the first two 4's corresponds to α_3 and the space between the second two to α_6 . (It would have been nice to be able to print the table so that the roots displayed were always some initial segment of $\alpha_1, \dots, \alpha_n$. But this is incompatible with our convention that if two root systems share a chamber then their roots correspond in the obvious way.)

In all cases the full root system is got from the displayed roots by applying the automorphisms described above.

Dynkin diagram: This is essentially the Coxeter diagram for the Weyl group, with edges decorated to show the relationship between the norms of pairs of roots. The Coxeter diagram can be read from the chamber angles. For example, $\Pi_{3,21}$ has chamber with angles $\pi/4, \pi/4$ and π/∞ . The norms of the roots are the top row of the second matrix: 1, 2 and 4. The arrow on each edge of the Dynkin diagram points from the larger-norm root to the smaller-norm root. It follows that the diagram is number 54 in [7, table 2].

We close this section with a remark on recognizing Π 's root lattice. In all except 14 cases it was enough to work out L 's genus and observe that only one lattice in [3] has that genus. In the remaining cases, two lattices $L_{k,l}$ from [3] have that genus and we had to determine which one was L . In 12 of the cases ($\Pi_{4,16}, \Pi_{5,43}, \Pi_{6,9}, \Pi_{6,25}, \Pi_{6,42}, \Pi_{7,20}, \Pi_{8,35}, \Pi_{8,41}, \Pi_{8,52}, \Pi_{9,12}, \Pi_{10,5}$ and $\Pi_{12,13}$) one of the possibilities could be excluded because it lacked any roots of some norm appearing among the norms of the roots in Π . (Because Π 's roots are not assumed primitive, while roots were assumed primitive in [3], we had to divide some of Π 's roots by 2 before making this comparison.) We remark that $L_{10,1}$ occurs seven times, accounting for more than half of these 12 cases.

Two cases remained. For $\Pi = \Pi_{4,86}$ it turned out that Π 's inner product matrix coincides with that of the simple roots for $L_{10,9}$. For $\Pi = \Pi_{5,32}$ it turned out that Π admits a reflective symmetry, with $W(\Pi)$ having index 2 in a larger Coxeter group with the same root lattice. And a set of simple roots for this larger group also admits a reflective symmetry, so again we can enlarge the Coxeter group. It happens that the Gram matrix for the simple roots of this supergroup of index 4 coincides with that of the simple roots for $L_{10,4}$. Curiously, the root lattices in these two hard cases have the same genus.

3. RELATION TO WORK OF GRITSENKO-NIKULIN

Gritsenko and Nikulin have published three lists of root systems that we make explicit reference to in the table. The first enumeration appears in theorem 4.1 of [11] and consists of all twelve rank 3 hyperbolic simple root systems that have equal root norms and admit a timelike Weyl vector, and whose chamber is noncompact of finite area. (They also consider the case of a lightlike Weyl vector, which yields a thirteenth root system.) See theorem 1.3.1 of [14] for the proof of this classification. Their and our names for these root systems correspond as follows:

$$\begin{aligned} A_{1,0} &\leftrightarrow \Pi_{3,23} & A_{1,I} &\leftrightarrow \Pi_{3,2} & A_{1,II} &\leftrightarrow \Pi_{3,1} & A_{1,III} &\leftrightarrow \Pi_{5,31} \\ A_{2,0} &\leftrightarrow \Pi_{3,18} & A_{2,I} &\leftrightarrow \Pi_{4,8} & A_{2,II} &\leftrightarrow \Pi_{4,2} & A_{2,III} &\leftrightarrow \Pi_{8,4} \\ A_{3,0} &\leftrightarrow \Pi_{3,7} & A_{3,I} &\leftrightarrow \Pi_{4,3} & A_{3,II} &\leftrightarrow \Pi_{6,3} & A_{3,III} &\leftrightarrow \Pi_{12,4} \end{aligned}$$

The first detailed study of any hyperbolic KMA was by Feingold-Frenkel [9], whose A is $A_{1,0}$. A detailed treatment of the automorphic corrections to this KMA and the one for $A_{1,I}$ appears in [11]. These two cases are also called G_1 and G_2 in [12]. Examples 1 and 2 in [13] are the automorphic corrections of the KMAs associated to $A_{1,II}$ and $A_{2,II}$.

Their second enumeration appears in table 1 of [14] and consists of the sixty rank 3 hyperbolic root systems of elliptic type that admit a Weyl vector and are twisted to a hyperbolic root system all of whose roots have norm 2. (This means that one can rescale all the roots to have norm 2 and still have a root system.) The completeness of their listing is conjecture 1.2.2 of [14]. Our enumeration proves this conjecture. For each $k = 1, \dots, 60$, we call the k th member of their list “GN $_k$ ”. In order of increasing k , these root systems are

$$\begin{array}{cccccccccc} \Pi_{3,24} & \Pi_{3,22} & \Pi_{3,38} & \Pi_{3,23} & \Pi_{3,43} & \Pi_{3,17} & \Pi_{3,20} & \Pi_{4,27} & \Pi_{4,79} & \Pi_{3,36} \\ \Pi_{3,5} & \Pi_{3,2} & \Pi_{3,9} & \Pi_{3,15} & \Pi_{4,39} & \Pi_{4,59} & \Pi_{3,18} & \Pi_{3,14} & \Pi_{3,7} & \Pi_{4,37} \\ \Pi_{4,7} & \Pi_{4,46} & \Pi_{4,41} & \Pi_{4,58} & \Pi_{3,1} & \Pi_{5,34} & \Pi_{4,8} & \Pi_{4,55} & \Pi_{4,1} & \Pi_{4,6} \\ \Pi_{4,17} & \Pi_{8,11} & \Pi_{4,3} & \Pi_{5,38} & \Pi_{4,2} & \Pi_{5,30} & \Pi_{6,1} & \Pi_{6,45} & \Pi_{6,22} & \Pi_{7,13} \\ \Pi_{6,17} & \Pi_{5,31} & \Pi_{6,2} & \Pi_{6,16} & \Pi_{8,31} & \Pi_{6,3} & \Pi_{7,12} & \Pi_{8,45} & \Pi_{8,46} & \Pi_{8,25} \\ \Pi_{9,7} & \Pi_{9,16} & \Pi_{9,1} & \Pi_{10,21} & \Pi_{10,22} & \Pi_{10,11} & \Pi_{11,8} & \Pi_{12,8} & \Pi_{8,4} & \Pi_{12,4} \end{array}$$

Of the sixty, sixteen have all root norms equal, and twelve of these are the cases $A_{1,0}, \dots, A_{3,III}$ referred to above. The remaining four have compact Weyl chambers, with Gram matrices B_1, \dots, B_4 listed in theorem 1.5.6 of [16]. The correspondence with our root systems is

$$B_1 \leftrightarrow \Pi_{4,7} \quad B_2 \leftrightarrow \Pi_{4,1} \quad B_3 \leftrightarrow \Pi_{6,1} \quad B_4 \leftrightarrow \Pi_{6,2}$$

The third enumeration [16] is really an enumeration of automorphic corrections to Kac-Moody algebras rather than root systems. That is, they classified the pairs (Π, ξ) where Π is a simple root system satisfying (i)–(iii) of theorem 1 and consists of roots of some lattice of the form $L_t = \langle 2t \rangle \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, and ξ is a suitable automorphic

form for a certain subgroup of $O(L_t \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})$. They showed that there are exactly 29 pairs (Π, ξ) . Of their 29 root systems, 23 have timelike Weyl vectors and therefore must appear in our tables. The root systems $A_{1,II}$, $A_{2,II}$ and $A_{3,II}$ mentioned above appear twice each and $A_{1,0}$, $A_{2,0}$ and $A_{3,0}$ appear once each. The remaining 14 are the following.

$$\begin{array}{llll}
A_{1,I,\bar{0}} \leftrightarrow \Pi_{3,24} & L_{2.3} \cong \langle 2 \rangle \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & A_{2,I,\bar{0}} \leftrightarrow \Pi_{3,17} & L_{1.9} \cong \langle 4 \rangle \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
A_{2,II,\bar{1}} \leftrightarrow \Pi_{4,38} & L_{1.9} & A_{2,I,\bar{1}} \leftrightarrow \Pi_{3,29} & L_{1.9} \\
A_{2,0,\bar{1}} \leftrightarrow \Pi_{3,27} & L_{1.4} \cong \langle 1 \rangle \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & A_{3,I,\bar{0}} \leftrightarrow \Pi_{3,5} & L_{7.7} \cong \langle -3 \rangle \oplus A_2[2] \\
A_{3,II,\bar{1}} \leftrightarrow \Pi_{4,36} & L_{7.7} & A_{3,I,\bar{1}} \leftrightarrow \Pi_{3,34} & L_{7.7} \\
A_{3,0,\bar{1}} \leftrightarrow \Pi_{3,32} & L_{7.6} \cong \langle -6 \rangle \oplus A_2 & A_{4,I,\bar{0}} \leftrightarrow \Pi_{3,15} & L_{140.4} \cong \langle 1 \rangle \oplus \begin{pmatrix} 0 & 8 \\ 8 & 0 \end{pmatrix} \\
A_{4,II,\bar{1}} \leftrightarrow \Pi_{4,39} & L_{140.4} & A_{4,I,\bar{1}} \leftrightarrow \Pi_{3,38} & L_{140.4} \\
A_{4,0,\bar{1}} \leftrightarrow \Pi_{3,36} & L_{2.3} & \xi_{0,9}^{(2)} \leftrightarrow \Pi_{5,34} & L_{148.9} \cong \langle 1 \rangle \oplus \begin{pmatrix} 0 & 18 \\ 18 & 0 \end{pmatrix}
\end{array}$$

(Gritsenko and Nikulin had found all of these automorphic corrections previously, in [11], [12] and [15]. Also, they used signature $(1, 2)$ where we use $(2, 1)$, and in the last case they did not name the generalized Cartan matrix. We used their name for the automorphic form.)

The use of “root lattice” in [16] differs slightly from ours. There, it was the lattice L_t fixed beforehand, from which one chooses a root system that doesn’t necessarily span the whole lattice. So we have also described the root lattice in our sense, meaning the span of the roots, rescaled to be unscaled. For completeness we also do this for $A_{1,0}, \dots, A_{3,0}$ and $A_{1,II}, \dots, A_{3,II}$:

$$\begin{array}{llll}
A_{1,0} \leftrightarrow \Pi_{3,23} & L_{2.2} \cong \langle 2 \rangle \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & A_{1,II} \leftrightarrow \Pi_{3,1} & L_{2.3} \cong \langle 1 \rangle \oplus \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \\
A_{2,0} \leftrightarrow \Pi_{3,18} & L_{1.4} \cong \langle 1 \rangle \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & A_{2,II} \leftrightarrow \Pi_{4,2} & L_{1.9} \cong \langle 1 \rangle \oplus \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix} \\
A_{3,0} \leftrightarrow \Pi_{3,7} & L_{7.6} \cong \langle -6 \rangle \oplus A_2 & A_{3,II} \leftrightarrow \Pi_{6,3} & L_{7.7} \cong \langle -3 \rangle \oplus A_2[2]
\end{array}$$

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Table of Root Systems (see theorem 1)

$\Pi_{3,1} = A_{1,II} = \text{GN}_{25}$ spans $L_{2,3}$ $ \circ \circ \circ \circ \circ \circ \times D_6$ (shared) $\begin{bmatrix} -1/3 & & & \\ & 2/9 & & \\ & & 2/9 & \\ & & & 2/9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix}$	$\Pi_{3,2} = A_{1,I} = \text{GN}_{12}$ spans $L_{2,1}$ $3 3 \circ \circ \times D_2$ $\begin{bmatrix} -1/16 & & \\ & 1/4 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & -1 \\ 0 & -1 \end{bmatrix}$
$\Pi_{3,3}$ spans $L_{3,2}$ $\mathfrak{3}4 4 \times D_2$ (shared) $\begin{bmatrix} -1/17 & & \\ & 1/34 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & -6 \\ 1 & 0 \end{bmatrix}$	$\Pi_{3,4}$ spans $L_{155,1}$ $\mathfrak{3}6 6 \times D_2$ (shared) $\begin{bmatrix} -1/25 & & \\ & 3/50 & \\ & & 9/2 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 7 & -6 \\ 1 & 0 \end{bmatrix}$
$\Pi_{3,5} = A_{3,I,0} = \text{GN}_{11}$ spans $L_{7,7}$ $\mathfrak{3}\infty \infty \times D_2$ (shared) $\begin{bmatrix} -1/11 & & \\ & 3/11 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 3 & -2 \\ 1 & 0 \end{bmatrix}$	$\Pi_{3,6}$ spans $L_{3,1}$ $\mathfrak{3}4 4 \times D_2$ (shared) $\begin{bmatrix} -1/28 & & \\ & 1/14 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -3 & 8 \\ -1 & 0 \end{bmatrix}$
$\Pi_{3,7} = A_{3,0} = \text{GN}_{19}$ spans $L_{7,6}$ $\mathfrak{3}\infty \infty \times D_2$ (shared) $\begin{bmatrix} -3/26 & & \\ & 1/26 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -5 & 8 \\ -1 & 0 \end{bmatrix}$	$\Pi_{3,8}$ spans $L_{3,4}$ $\mathfrak{3}6 6 \times D_2$ (shared) $\begin{bmatrix} -1/19 & & \\ & 3/38 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 3 & -10 \\ 1 & 0 \end{bmatrix}$
$\Pi_{3,9} = \text{GN}_{13}$ spans $L_{7,9}$ $\mathfrak{3}\infty \infty \times D_2$ (shared) $\begin{bmatrix} -1/16 & & \\ & 3/4 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 8 \\ -1 & 4 \\ 1 & 0 \end{bmatrix}$	$\Pi_{3,10}$ spans $L_{1,6}$ $4 4 \circ \circ \times D_2$ (shared) $\begin{bmatrix} -1/6 & & \\ & 1/6 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 0 & -1 \end{bmatrix}$
$\Pi_{3,11}$ spans $L_{1,3}$ $4 4 \circ \circ \times D_2$ (shared) $\begin{bmatrix} -1/8 & & \\ & 1/8 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ 0 & 1 \end{bmatrix}$	$\Pi_{3,12}$ spans $L_{7,9}$ $6 6 \circ \circ \times D_2$ (shared) $\begin{bmatrix} -3/32 & & \\ & 3/8 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 5 & -1 \\ 0 & 1 \end{bmatrix}$

$\Pi_{3,13}$ spans $L_{7,13}$

$6|6\infty \times D_2$ (shared)

$$\begin{bmatrix} -1/16 & & \\ & 9/4 & \\ & & 6 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$\Pi_{3,15} = A_{4,I,\bar{0}} = \text{GN}_{14}$ spans $L_{140,4}$

$|\infty\infty\infty \times D_2$ (shared)

$$\begin{bmatrix} -1/8 & & \\ & 1/8 & \\ & & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -3 & 4 \\ 0 & 1 \end{bmatrix}$$

$\Pi_{3,17} = A_{2,I,\bar{0}} = \text{GN}_6$ spans $L_{1,9}$

$|\infty\mathbb{2}\infty \times D_2$ (shared)

$$\begin{bmatrix} -1/17 & & \\ & 2/17 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -3 & 5 \\ 0 & 1 \end{bmatrix}$$

$\Pi_{3,19}$ spans $L_{3,1}$

$6\mathbb{2}6 \times D_2$ (shared)

$$\begin{bmatrix} -1/44 & & \\ & 3/11 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ -5 & 2 \\ 0 & -1 \end{bmatrix}$$

$\Pi_{3,21}$ spans $L_{145,1}$

44∞ (shared)

$$\begin{bmatrix} -4/41 & & \\ & 5/41 & -2/41 \\ & -2/41 & 9/41 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & -6 \\ 0 & 3 & -4 \end{bmatrix}$$

$\Pi_{3,23} = A_{1,0} = \text{GN}_4$ spans $L_{2,2}$

32∞ (shared)

$$\begin{bmatrix} -1/46 & & \\ & 4/23 & -1/23 \\ & -1/23 & 6/23 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 3 & -3 & -2 \\ -1 & -2 & 2 \end{bmatrix}$$

$\Pi_{3,25}$ spans $L_{3,1}$

426 (shared)

$$\begin{bmatrix} -1/88 & & \\ & 5/22 & -1/22 \\ & -1/22 & 53/22 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 3 & -3 & -4 \\ 0 & -1 & 1 \end{bmatrix}$$

$\Pi_{3,27} = A_{2,0,\bar{1}}$ spans $L_{1,4}$

42∞ (shared)

$$\begin{bmatrix} -1/15 & & \\ & 4/15 & -1/15 \\ & -1/15 & 4/15 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 2 \\ -1 & 3 & 0 \end{bmatrix}$$

$\Pi_{3,29} = A_{2,I,\bar{1}}$ spans $L_{1,9}$

42∞ (shared)

$$\begin{bmatrix} -1/41 & & \\ & 10/41 & -4/41 \\ & -4/41 & 18/41 \end{bmatrix} \begin{bmatrix} 4 & 8 & 1 \\ -3 & -2 & 2 \\ -3 & 4 & 1 \end{bmatrix}$$

$\Pi_{3,31}$ spans $L_{3,2}$

426 (shared)

$$\begin{bmatrix} -1/65 & & \\ & 14/65 & -1/65 \\ & -1/65 & 14/65 \end{bmatrix} \begin{bmatrix} 2 & 1 & 6 \\ -3 & 1 & 5 \\ -1 & 2 & -2 \end{bmatrix}$$

$\Pi_{3,33}$ spans $L_{155,1}$

626 (shared)

$$\begin{bmatrix} -1/88 & & \\ & 45/22 & -9/11 \\ & -9/11 & 30/11 \end{bmatrix} \begin{bmatrix} 6 & 18 & 2 \\ -1 & -1 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$

$\Pi_{3,35}$ spans $L_{1,3}$

42∞ (shared)

$$\begin{bmatrix} -1/40 & & \\ & 9/40 & -1/40 \\ & -1/40 & 9/40 \end{bmatrix} \begin{bmatrix} 2 & 1 & 8 \\ 3 & -1 & -6 \\ -1 & -2 & 2 \end{bmatrix}$$

$\Pi_{3,14} = \text{GN}_{18}$ spans $L_{1,9}$

$\infty|\infty\infty \times D_2$ (shared)

$$\begin{bmatrix} -1/5 & & \\ & 1/5 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 6 & -1 \\ 0 & 1 \end{bmatrix}$$

$\Pi_{3,16}$ spans $L_{3,4}$

$6\mathbb{2}6 \times D_2$ (shared)

$$\begin{bmatrix} -1/52 & & \\ & 3/13 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -3 & 4 \\ 0 & 1 \end{bmatrix}$$

$\Pi_{3,18} = A_{2,0} = \text{GN}_{17}$ spans $L_{1,4}$

$|\infty\mathbb{2}\infty \times D_2$ (shared)

$$\begin{bmatrix} -1/7 & & \\ & 1/14 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 4 & -3 \\ 0 & -1 \end{bmatrix}$$

$\Pi_{3,20} = \text{GN}_7$ spans $L_{1,6}$

$|\infty\mathbb{2}\infty \times D_2$ (shared)

$$\begin{bmatrix} -1/14 & & \\ & 4/7 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 3 & -1 \\ 0 & -1 \end{bmatrix}$$

$\Pi_{3,22} = \text{GN}_2$ spans $L_{2,1}$

32∞ (shared)

$$\begin{bmatrix} -1/88 & & \\ & 5/22 & -1/11 \\ & -1/11 & 18/11 \end{bmatrix} \begin{bmatrix} 2 & 2 & 8 \\ 3 & -1 & -6 \\ 0 & 1 & -1 \end{bmatrix}$$

$\Pi_{3,24} = A_{1,I,\bar{0}} = \text{GN}_1$ spans $L_{2,3}$

32∞ (shared)

$$\begin{bmatrix} -1/59 & & \\ & 12/59 & -2/59 \\ & -2/59 & 20/59 \end{bmatrix} \begin{bmatrix} 4 & 4 & 1 \\ 3 & 2 & -2 \\ 3 & -3 & -1 \end{bmatrix}$$

$\Pi_{3,26}$ spans $L_{1,6}$

42∞ (shared)

$$\begin{bmatrix} -1/26 & & \\ & 3/26 & -1/26 \\ & -1/26 & 35/26 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ -3 & 3 & 5 \\ 0 & 1 & -1 \end{bmatrix}$$

$\Pi_{3,28}$ spans $L_{3,4}$

426 (shared)

$$\begin{bmatrix} -1/136 & & \\ & 21/34 & -3/17 \\ & -3/17 & 30/17 \end{bmatrix} \begin{bmatrix} 6 & 12 & 2 \\ -3 & 4 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$\Pi_{3,30}$ spans $L_{7,9}$

62∞ (shared)

$$\begin{bmatrix} -1/40 & & \\ & 21/10 & -3/5 \\ & -3/5 & 18/5 \end{bmatrix} \begin{bmatrix} 2 & 6 & 8 \\ -1 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

$\Pi_{3,32} = A_{3,0,\bar{1}}$ spans $L_{7,6}$

62∞ (shared)

$$\begin{bmatrix} -3/74 & & \\ & 8/37 & -1/37 \\ & -1/37 & 14/37 \end{bmatrix} \begin{bmatrix} 2 & 6 & 2 \\ -3 & 5 & 2 \\ -1 & -2 & 2 \end{bmatrix}$$

$\Pi_{3,34} = A_{3,I,\bar{1}}$ spans $L_{7,7}$

62∞ (shared)

$$\begin{bmatrix} -1/35 & & \\ & 36/35 & -6/35 \\ & -6/35 & 36/35 \end{bmatrix} \begin{bmatrix} 4 & 12 & 1 \\ 2 & -1 & -1 \\ 1 & -4 & 0 \end{bmatrix}$$

$\Pi_{3,36} = A_{4,0,\bar{1}} = \text{GN}_{10}$ spans $L_{2,3}$

$\infty\mathbb{2}\infty$ (shared)

$$\begin{bmatrix} -1/11 & & \\ & 4/11 & -2/11 \\ & -2/11 & 12/11 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 \\ -1 & 4 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$\Pi_{3,37}$ spans $L_{7.13}$
 62∞ (shared)

$$\begin{bmatrix} -1/72 & & & \\ & 13/18 & -1/3 & \\ & -1/3 & 2 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 2 & 24 \\ 3 & -1 & -6 \\ 1 & -1 & 1 \end{bmatrix}$$

 $\Pi_{3,39}$ spans $L_{1.5}$
 42∞ (shared)

$$\begin{bmatrix} -1/22 & & & \\ & 3/22 & -1/22 & \\ & -1/22 & 15/22 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 4 & -2 & -3 \\ 0 & -1 & 1 \end{bmatrix}$$

 $\Pi_{3,41}$ spans $L_{1.7}$
 42∞ (shared)

$$\begin{bmatrix} -1/29 & & & \\ & 6/29 & -2/29 & \\ & -2/29 & 20/29 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 4 & -3 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

 $\Pi_{3,43} = \text{GN}_5$ spans $L_{140.3}$
 $\infty 2\infty$ (shared)

$$\begin{bmatrix} -1/20 & & & \\ & 1/4 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 4 & 1 \\ -4 & 4 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

 $\Pi_{3,38} = A_{4,I,\bar{1}} = \text{GN}_3$ spans $L_{140.4}$
 $\infty 2\infty$ (shared)

$$\begin{bmatrix} -1/32 & & & \\ & 9/32 & -1/8 & \\ & -1/8 & 1/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 16 & 1 \\ 0 & -8 & 1 \\ 3 & -6 & -1 \end{bmatrix}$$

 $\Pi_{3,40}$ spans $L_{3.3}$
 426 (shared)

$$\begin{bmatrix} -1/91 & & & \\ & 30/91 & -9/91 & \\ & -9/91 & 30/91 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 3 & 2 \\ -1 & -2 & 2 \\ 4 & -3 & -1 \end{bmatrix}$$

 $\Pi_{3,42}$ spans $L_{7.8}$
 62∞ (shared)

$$\begin{bmatrix} -1/42 & & & \\ & 8/21 & -1/21 & \\ & -1/21 & 8/21 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 2 & 6 \\ -4 & 1 & 4 \\ 1 & -2 & 2 \end{bmatrix}$$

 $\Pi_{3,44}$ spans $L_{7.11}$
 62∞ (shared)

$$\begin{bmatrix} -1/57 & & & \\ & 28/57 & -10/57 & \\ & -10/57 & 28/57 & \\ & & & \end{bmatrix} \begin{bmatrix} 12 & 4 & 3 \\ 5 & -3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$

 $\Pi_{4,1} = B_2 = \text{GN}_{29}$ spans $L_{3.4}$
 $\mathbb{3}\mathbb{3}\mathbb{3}\mathbb{3}\mathbb{3} \times D_8$

$$\begin{bmatrix} -1/4 & & & \\ & 3 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

 $\Pi_{4,3} = A_{3,I} = \text{GN}_{33}$ spans $L_{7.9}$
 $\mathbb{3}\mathbb{3}\mathbb{3}\mathbb{3} \times D_4$

$$\begin{bmatrix} -3/8 & & & \\ & 2 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

 $\Pi_{4,5}$ spans $L_{16.8}$
 $6\mathbb{6}\mathbb{6}\mathbb{6} \times D_4$

$$\begin{bmatrix} -1/4 & & & \\ & 3 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

 $\Pi_{4,7} = B_1 = \text{GN}_{21}$ spans $L_{3.1}$
 $\mathbb{3}\mathbb{2}\mathbb{3}\mathbb{2} \times D_4$ (shared)

$$\begin{bmatrix} -1/8 & & & \\ & 1 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

 $\Pi_{4,9}$ spans $L_{6.2}$
 $3\mathbb{3}\mathbb{2}\mathbb{2} \times D_2$

$$\begin{bmatrix} -1/7 & & & \\ & 1/14 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ -6 & 1 & 4 \\ 0 & 1 & 0 \end{bmatrix}$$

 $\Pi_{4,11}$ spans $L_{7.13}$
 $\mathbb{3}\mathbb{6}\mathbb{6} \times D_2$ (shared)

$$\begin{bmatrix} -2/9 & & & \\ & 1/18 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -5 & 12 \\ 1 & 2 \end{bmatrix}$$

 $\Pi_{4,13}$ spans $L_{1.3}$
 $4\mathbb{4}\mathbb{4}\mathbb{2} \times D_2$ (shared)

$$\begin{bmatrix} -2/5 & & & \\ & 1/10 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & -3 \\ -2 & -1 \end{bmatrix}$$

 $\Pi_{4,15}$ spans $L_{228.1}$
 $6\mathbb{6}\mathbb{6}\mathbb{6} \times D_2$

$$\begin{bmatrix} -1/7 & & & \\ & 9/14 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 18 & 6 & 2 \\ 10 & 1 & -2 \\ 0 & -1 & 0 \end{bmatrix}$$

 $\Pi_{4,2} = A_{2,II} = \text{GN}_{35}$ spans $L_{1.9}$
 $\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{4} \times D_8$ (shared)

$$\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

 $\Pi_{4,4}$ spans $L_{123.4}$
 $4\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{4} \times D_4$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

 $\Pi_{4,6} = \text{GN}_{30}$ spans $L_{4.17}$
 $\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{4} \times D_4$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

 $\Pi_{4,8} = A_{2,I} = \text{GN}_{27}$ spans $L_{1.6}$
 $\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{4}\mathbb{2} \times D_4$ (shared)

$$\begin{bmatrix} -1/2 & & & \\ & 1/2 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

 $\Pi_{4,10}$ spans $L_{7.9}$
 $\mathbb{3}\mathbb{6}\mathbb{6} \times D_2$ (shared)

$$\begin{bmatrix} -6/25 & & & \\ & 3/50 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 13 & -4 \\ 3 & 2 \end{bmatrix}$$

 $\Pi_{4,12}$ spans $L_{19.5}$
 $4\mathbb{4}\mathbb{2}\mathbb{2} \times D_2$ (shared)

$$\begin{bmatrix} -1/20 & & & \\ & 3/10 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 12 & 6 & 4 \\ 8 & -1 & -4 \\ 0 & 1 & 0 \end{bmatrix}$$

 $\Pi_{4,14}$ spans $L_{31.3}$
 $6\mathbb{6}\mathbb{2}\mathbb{2} \times D_2$

$$\begin{bmatrix} -1/13 & & & \\ & 3/13 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 12 & 4 & 3 \\ 10 & -1 & -4 \\ 0 & 1 & 0 \end{bmatrix}$$

 $\Pi_{4,16}$ spans $L_{10.1}$
 $\mathbb{4}\mathbb{4}\mathbb{2}\mathbb{2} \times D_2$

$$\begin{bmatrix} -1/32 & & & \\ & 5/8 & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 40 & 10 & 8 \\ -12 & 1 & 4 \\ 0 & -1 & 0 \end{bmatrix}$$

$\Pi_{4,17} = \text{GN}_{31}$ spans $L_{140.4}$

$\infty \circ \circ \infty \circ \circ \times D_2$ (shared)

$$\begin{bmatrix} -1/2 & & 4 & 1 \\ & 1/2 & 4 & -1 \\ & & 1 & 2 & 1 \end{bmatrix}$$

$\Pi_{4,19}$ spans $L_{123.5}$

$4|42|2 \times D_2$ (shared)

$$\begin{bmatrix} -1/7 & & 2 & 4 & 1 \\ & 2/7 & -3 & 1 & 2 \\ & & 6 & 0 & 1 & 0 \end{bmatrix}$$

$\Pi_{4,21}$ spans $L_{4.10}$

$\infty|\infty 2|2 \times D_2$ (shared)

$$\begin{bmatrix} -1/5 & & 3 & 3 & 1 \\ & 3/10 & 4 & -1 & -2 \\ & & 9/2 & 0 & 1 & 0 \end{bmatrix}$$

$\Pi_{4,23}$ spans $L_{22.10}$

$6|62|2 \times D_2$

$$\begin{bmatrix} -1/40 & & 14 & 42 & 2 \\ & 21/10 & -3 & 1 & 1 \\ & & 84 & 0 & 1 & 0 \end{bmatrix}$$

$\Pi_{4,25}$ spans $L_{141.11}$

$\infty|\infty 2|2 \times D_2$ (shared)

$$\begin{bmatrix} -1/7 & & 2 & 8 & 1 \\ & 2/7 & -3 & 2 & 2 \\ & & 16 & 0 & 1 & 0 \end{bmatrix}$$

$\Pi_{4,27} = \text{GN}_8$ spans $L_{155.1}$

$\mathfrak{3}2\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/28 & & 2 & 18 \\ & 9/14 & 1 & -5 \\ & & 3/2 & 1 & 3 \end{bmatrix}$$

$\Pi_{4,29}$ spans $L_{3.3}$

$\mathfrak{3}2\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/19 & & 2 & 3 \\ & 3/38 & 3 & -5 \\ & & 3/2 & -1 & -1 \end{bmatrix}$$

$\Pi_{4,31}$ spans $L_{3.1}$

$\mathfrak{3}2\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/40 & & 6 & 4 \\ & 12/5 & -1 & 1 \\ & & 1/2 & 3 & 2 \end{bmatrix}$$

$\Pi_{4,33}$ spans $L_{7.11}$

$\mathfrak{3}2\circ 2 \times D_2$ (shared)

$$\begin{bmatrix} -1/9 & & 4 & 3 \\ & 1/9 & -5 & 3 \\ & & 3 & -1 & -1 \end{bmatrix}$$

$\Pi_{4,35}$ spans $L_{7.6}$

$\mathfrak{3}2\circ 2 \times D_2$ (shared)

$$\begin{bmatrix} -3/26 & & 6 & 2 \\ & 3/26 & -7 & 2 \\ & & 1/2 & 3 & 2 \end{bmatrix}$$

$\Pi_{4,37} = \text{GN}_{20}$ spans $L_{145.1}$

$\infty\mathfrak{2}\infty\mathfrak{2} \times D_2$ (shared)

$$\begin{bmatrix} -4/17 & & 4 & 1 \\ & 1/34 & 14 & -5 \\ & & 1/2 & -2 & -1 \end{bmatrix}$$

$\Pi_{4,39} = A_{4,II,\bar{1}} = \text{GN}_{15}$ spans $L_{140.4}$

$\circ\mathfrak{2}\circ\mathfrak{2} \times D_2$ (shared)

$$\begin{bmatrix} -1/8 & & 1 & 16 \\ & 1/8 & -1 & 16 \\ & & 1 & 1 & 4 \end{bmatrix}$$

$\Pi_{4,18}$ spans $L_{19.2}$

$4|42|2 \times D_2$ (shared)

$$\begin{bmatrix} -1/5 & & 1 & 2 & 3 \\ & 3/10 & -2 & 1 & 4 \\ & & 5/2 & 0 & -1 & 0 \end{bmatrix}$$

$\Pi_{4,20}$ spans $L_{145.1}$

$4\circ\mathfrak{4}2 \times D_2$ (shared)

$$\begin{bmatrix} -4/9 & & 1 & 2 \\ & 1/9 & 2 & -5 \\ & & 1 & 1 & 1 \end{bmatrix}$$

$\Pi_{4,22}$ spans $L_{22.2}$

$6|62|2 \times D_2$

$$\begin{bmatrix} -1/7 & & 2 & 6 & 1 \\ & 1/14 & 6 & -3 & -4 \\ & & 21/2 & 0 & 1 & 0 \end{bmatrix}$$

$\Pi_{4,24}$ spans $L_{4.18}$

$\infty|\infty 2|2 \times D_2$ (shared)

$$\begin{bmatrix} -1/5 & & 1 & 4 & 3 \\ & 6/5 & -1 & 1 & 2 \\ & & 6 & 0 & 1 & 0 \end{bmatrix}$$

$\Pi_{4,26}$ spans $L_{3.4}$

$\mathfrak{3}2\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/40 & & 2 & 12 \\ & 3/5 & -1 & 4 \\ & & 3/2 & 1 & 2 \end{bmatrix}$$

$\Pi_{4,28}$ spans $L_{7.13}$

$\mathfrak{3}2\circ 2 \times D_2$ (shared)

$$\begin{bmatrix} -1/24 & & 2 & 24 \\ & 2/3 & -1 & 6 \\ & & 3/2 & 1 & 4 \end{bmatrix}$$

$\Pi_{4,30}$ spans $L_{7.8}$

$\mathfrak{3}2\circ 2 \times D_2$ (shared)

$$\begin{bmatrix} -1/10 & & 2 & 6 \\ & 9/10 & 1 & -2 \\ & & 3/2 & 1 & 2 \end{bmatrix}$$

$\Pi_{4,32}$ spans $L_{7.9}$

$\mathfrak{3}2\circ 2 \times D_2$ (shared)

$$\begin{bmatrix} -3/56 & & 6 & 8 \\ & 24/7 & -1 & 1 \\ & & 1/2 & -3 & -4 \end{bmatrix}$$

$\Pi_{4,34}$ spans $L_{3.2}$

$\mathfrak{3}2\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/25 & & 6 & 1 \\ & 3/50 & 7 & -3 \\ & & 1/2 & 3 & 1 \end{bmatrix}$$

$\Pi_{4,36} = A_{3,II,\bar{1}}$ spans $L_{7.7}$

$\mathfrak{3}2\circ 2 \times D_2$ (shared)

$$\begin{bmatrix} -3/25 & & 12 & 1 \\ & 3/25 & 13 & -1 \\ & & 1 & -3 & -1 \end{bmatrix}$$

$\Pi_{4,38} = A_{2,II,\bar{1}}$ spans $L_{1.9}$

$\circ\mathfrak{2}\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/9 & & 1 & 8 \\ & 1/9 & -1 & 10 \\ & & 1 & 1 & 2 \end{bmatrix}$$

$\Pi_{4,40}$ spans $L_{1.7}$

$\circ\mathfrak{2}\mathfrak{3}2 \times D_2$ (shared)

$$\begin{bmatrix} -1/5 & & 1 & 2 \\ & 1/5 & 1 & -3 \\ & & 1 & -1 & -1 \end{bmatrix}$$

$\Pi_{4,41} = \text{GN}_{23}$ spans $L_{140.3}$
 $\circlearrowleft 2 \circlearrowleft 2 \rtimes D_2$ (shared)

$$\begin{bmatrix} -1/4 & & & \\ & 1/4 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}$$

 $\Pi_{4,43}$ spans $L_{1.3}$
 $\circlearrowleft 2 \circlearrowleft 2 \rtimes D_2$ (shared)

$$\begin{bmatrix} -1/16 & & & \\ & 1/16 & & \\ & & 1/2 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 8 & 1 \\ -8 & 3 \\ 4 & 1 \end{bmatrix}$$

 $\Pi_{4,45}$ spans $L_{168.2}$
 $6262 \rtimes C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1 & \\ & -1 & 10 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$$

 $\Pi_{4,47}$ spans $L_{155.1}$
 3622 (shared)

$$\begin{bmatrix} -1/28 & & & \\ & 15/7 & -6/7 & \\ & -6/7 & 15/7 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 18 & 18 & 6 & 2 \\ -1 & -4 & 1 & 1 \\ -4 & -1 & 2 & 0 \end{bmatrix}$$

 $\Pi_{4,49}$ spans $L_{3.1}$
 3622 (shared)

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1/2 & \\ & -1/2 & 5/2 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 6 & 2 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & -2 & 0 \end{bmatrix}$$

 $\Pi_{4,51}$ spans $L_{1.7}$
 $4\infty 22$ (shared)

$$\begin{bmatrix} -1/5 & & & \\ & 4/5 & -2/5 & \\ & -2/5 & 6/5 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 & 2 \\ -1 & 3 & 0 & -2 \\ 1 & 2 & -1 & -1 \end{bmatrix}$$

 $\Pi_{4,53}$ spans $L_{142.3}$
 $4\infty 22$ (shared)

$$\begin{bmatrix} -1/8 & & & \\ & 9/8 & -3/8 & \\ & -3/8 & 17/8 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 8 & 9 \\ -1 & 1 & 2 & -2 \\ 0 & 1 & -2 & -3 \end{bmatrix}$$

 $\Pi_{4,55} = \text{GN}_{28}$ spans $L_{1.6}$
 $\infty\infty 2\infty$ (shared)

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & -1/2 & \\ & -1/2 & 3/2 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 3 & 0 & -1 \end{bmatrix}$$

 $\Pi_{4,57}$ spans $L_{144.8}$
 $\infty\infty 22$ (shared)

$$\begin{bmatrix} -1/12 & & & \\ & 13/12 & -5/12 & \\ & -5/12 & 13/12 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 3 & 12 & 12 & 1 \\ -2 & 1 & 5 & 0 \\ -1 & 5 & 1 & -1 \end{bmatrix}$$

 $\Pi_{4,59} = \text{GN}_{16}$ spans $L_{140.4}$
 $\infty\infty 2\infty$ (shared)

$$\begin{bmatrix} -1/8 & & & \\ & 1/2 & -1/4 & \\ & -1/4 & 9/8 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 16 & 4 & 1 & 16 \\ 6 & -3 & 0 & 10 \\ -4 & -2 & 1 & 4 \end{bmatrix}$$

 $\Pi_{4,61}$ spans $L_{7.11}$
 $\infty\infty 22$ (shared)

$$\begin{bmatrix} -1/9 & & & \\ & 4/9 & -2/9 & \\ & -2/9 & 28/9 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 12 & 4 \\ 1 & 2 & -7 & -3 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

 $\Pi_{4,63}$ spans $L_{6.5}$
 3222

$$\begin{bmatrix} -1/68 & & & \\ & 15/17 & -5/17 & \\ & -5/17 & 30/17 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 10 & 10 & 20 & 2 \\ 3 & -3 & -4 & 1 \\ -1 & -2 & 2 & 1 \end{bmatrix}$$

 $\Pi_{4,42}$ spans $L_{1.5}$
 $\circlearrowleft 2 \circlearrowleft 2 \rtimes D_2$ (shared)

$$\begin{bmatrix} -1/6 & & & \\ & 2/3 & & \\ & & 1/2 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 2 & 1 \end{bmatrix}$$

 $\Pi_{4,44}$ spans $L_{8.1}$
 $4242 \rtimes C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1 & \\ & -1 & 6 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

 $\Pi_{4,46} = \text{GN}_{22}$ spans $L_{148.2}$
 $\infty 2\infty 2 \rtimes C_2$ (shared)

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1/2 & \\ & -1/2 & 29/2 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 8 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$$

 $\Pi_{4,48}$ spans $L_{221.3}$
 $36\infty 2$ (shared)

$$\begin{bmatrix} -3/25 & & & \\ & 12/25 & -6/25 & \\ & -6/25 & 28/25 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 12 & 12 & 4 & 1 \\ 5 & 8 & -1 & -1 \\ -3 & 3 & 2 & -1 \end{bmatrix}$$

 $\Pi_{4,50}$ spans $L_{7.11}$
 $36\infty 2$ (shared)

$$\begin{bmatrix} -1/9 & & & \\ & 4/9 & -2/9 & \\ & -2/9 & 28/9 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 12 & 3 \\ -3 & -2 & 7 & 1 \\ -1 & 1 & 2 & -1 \end{bmatrix}$$

 $\Pi_{4,52}$ spans $L_{123.9}$
 4422 (shared)

$$\begin{bmatrix} -3/49 & & & \\ & 20/49 & -4/49 & \\ & -4/49 & 40/49 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 4 & 8 & 16 & 1 \\ 3 & -4 & -6 & 1 \\ -1 & -3 & 4 & 1 \end{bmatrix}$$

 $\Pi_{4,54}$ spans $L_{7.11}$
 $6\infty\infty 2$

$$\begin{bmatrix} -1/9 & & & \\ & 4/9 & -2/9 & \\ & -2/9 & 28/9 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 4 & 12 & 3 & 12 \\ -2 & 7 & 1 & -8 \\ 1 & 2 & -1 & -1 \end{bmatrix}$$

 $\Pi_{4,56}$ spans $L_{1.9}$
 $4\infty 22$ (shared)

$$\begin{bmatrix} -1/9 & & & \\ & 4/9 & -2/9 & \\ & -2/9 & 10/9 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 8 & 4 & 1 & 8 \\ 6 & -1 & -1 & 4 \\ 2 & 2 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,58} = \text{GN}_{24}$ spans $L_{140.3}$
 $\infty\infty 2\infty$ (shared)

$$\begin{bmatrix} -1/4 & & & \\ & 1 & -1/2 & \\ & -1/2 & 5/4 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 1 & 4 \\ 3 & -1 & -1 & 1 \\ 2 & 2 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,60}$ spans $L_{144.5}$
 $\infty\infty 22$ (shared)

$$\begin{bmatrix} -1/8 & & & \\ & 9/8 & -3/8 & \\ & -3/8 & 33/8 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 16 & 3 \\ 1 & -1 & -6 & 0 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

 $\Pi_{4,62}$ spans $L_{141.10}$
 $\infty\infty 22$ (shared)

$$\begin{bmatrix} -1/16 & & & \\ & 9/16 & -1/4 & \\ & -1/4 & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 8 & 8 & 32 & 1 \\ -4 & 4 & 8 & -1 \\ 1 & 3 & -6 & -1 \end{bmatrix}$$

 $\Pi_{4,64}$ spans $L_{19.1}$
 4222 (shared)

$$\begin{bmatrix} -1/40 & & & \\ & 21/10 & -3/10 & \\ & -3/10 & 29/10 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 10 & 48 \\ 1 & -1 & -2 & 2 \\ 0 & -1 & 1 & 6 \end{bmatrix}$$

$\Pi_{4,65}$ spans $L_{123.3}$

4222 (shared)

$$\begin{bmatrix} -1/14 & & & \\ & 4/7 & -1/7 & \\ & -1/7 & 11/14 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 2 & 8 & 3 \\ 1 & -2 & -2 & 2 \\ 1 & 0 & -4 & -1 \end{bmatrix}$$

 $\Pi_{4,67}$ spans $L_{123.6}$

4222 (shared)

$$\begin{bmatrix} -1/20 & & & \\ & 21/20 & -9/20 & \\ & -9/20 & 21/20 & \\ & & & \end{bmatrix} \begin{bmatrix} 3 & 6 & 6 & 1 \\ 1 & -3 & -1 & 1 \\ 2 & -1 & -3 & 0 \end{bmatrix}$$

 $\Pi_{4,69}$ spans $L_{19.10}$

4222 (shared)

$$\begin{bmatrix} -1/184 & & & \\ & 165/46 & -75/46 & \\ & -75/46 & 285/46 & \\ & & & \end{bmatrix} \begin{bmatrix} 30 & 60 & 6 & 80 \\ 1 & -5 & 0 & 6 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

 $\Pi_{4,71}$ spans $L_{22.5}$

6222 (shared)

$$\begin{bmatrix} -1/28 & & & \\ & 15/7 & -3/7 & \\ & -3/7 & 30/7 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 6 & 14 & 12 \\ 1 & -1 & -3 & 0 \\ 0 & 1 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,73}$ spans $L_{31.13}$

6222 (shared)

$$\begin{bmatrix} -1/104 & & & \\ & 165/26 & -15/26 & \\ & -15/26 & 285/26 & \\ & & & \end{bmatrix} \begin{bmatrix} 10 & 30 & 160 & 6 \\ 0 & 2 & -2 & -1 \\ 1 & -1 & -6 & 0 \end{bmatrix}$$

 $\Pi_{4,75}$ spans $L_{16.20}$

6222 (shared)

$$\begin{bmatrix} -1/184 & & & \\ & 165/46 & -45/46 & \\ & -45/46 & 765/46 & \\ & & & \end{bmatrix} \begin{bmatrix} 90 & 30 & 36 & 20 \\ -6 & 2 & 3 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

 $\Pi_{4,77}$ spans $L_{16.13}$

6222 (shared)

$$\begin{bmatrix} -1/121 & & & \\ & 210/121 & -15/121 & \\ & -15/121 & 390/121 & \\ & & & \end{bmatrix} \begin{bmatrix} 90 & 30 & 9 & 5 \\ 7 & 2 & -2 & -1 \\ 5 & -3 & -1 & 1 \end{bmatrix}$$

 $\Pi_{4,79} = \text{GN}_9$ spans $L_{148.13}$ $\infty 2\infty 2$ (shared)

$$\begin{bmatrix} -1/24 & & & \\ & 13/6 & -1 & \\ & -1 & 6 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 8 & 18 & 72 \\ 1 & -2 & -3 & 6 \\ 0 & -1 & 1 & 7 \end{bmatrix}$$

 $\Pi_{4,81}$ spans $L_{141.7}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/13 & & & \\ & 10/13 & -4/13 & \\ & -4/13 & 12/13 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 8 & 4 & 1 \\ -1 & -2 & 2 & 1 \\ 1 & -4 & -1 & 1 \end{bmatrix}$$

 $\Pi_{4,83}$ spans $L_{10.3}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/56 & & & \\ & 3/2 & -1/2 & \\ & -1/2 & 29/14 & \\ & & & \end{bmatrix} \begin{bmatrix} 10 & 40 & 2 & 32 \\ -1 & 7 & 0 & -6 \\ 2 & 1 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,85}$ spans $L_{4.16}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/13 & & & \\ & 12/13 & -3/13 & \\ & -3/13 & 30/13 & \\ & & & \end{bmatrix} \begin{bmatrix} 3 & 3 & 2 & 9 \\ 1 & -2 & 0 & 4 \\ -1 & 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{4,87}$ spans $L_{4.5}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/18 & & & \\ & 7/18 & -1/9 & \\ & -1/9 & 8/9 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 6 & 1 & 8 \\ 4 & -4 & -1 & 4 \\ 2 & 1 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,66}$ spans $L_{5.6}$ $42\infty 2$

$$\begin{bmatrix} -1/10 & & & \\ & 11/10 & -3/10 & \\ & -3/10 & 19/10 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 2 & 20 & 5 \\ -1 & 1 & 7 & -1 \\ 0 & 1 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,68}$ spans $L_{3.3}$

4222 (shared)

$$\begin{bmatrix} -1/19 & & & \\ & 6/19 & -3/19 & \\ & -3/19 & 30/19 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 3 & 3 & 2 \\ 4 & -3 & -2 & 2 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

 $\Pi_{4,70}$ spans $L_{123.8}$

4222 (shared)

$$\begin{bmatrix} -1/49 & & & \\ & 30/49 & -6/49 & \\ & -6/49 & 60/49 & \\ & & & \end{bmatrix} \begin{bmatrix} 24 & 12 & 3 & 2 \\ 6 & 2 & -2 & -1 \\ 4 & -3 & -1 & 1 \end{bmatrix}$$

 $\Pi_{4,72}$ spans $L_{16.3}$

6222 (shared)

$$\begin{bmatrix} -1/24 & & & \\ & 13/6 & -1/3 & \\ & -1/3 & 14/3 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 6 & 20 & 4 \\ 1 & -1 & -4 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

 $\Pi_{4,74}$ spans $L_{22.8}$

6222 (shared)

$$\begin{bmatrix} -1/92 & & & \\ & 35/23 & -7/23 & \\ & -7/23 & 98/23 & \\ & & & \end{bmatrix} \begin{bmatrix} 42 & 14 & 6 & 28 \\ -5 & 3 & 1 & -4 \\ 2 & 1 & -1 & -2 \end{bmatrix}$$

 $\Pi_{4,76}$ spans $L_{31.5}$

6222 (shared)

$$\begin{bmatrix} -1/58 & & & \\ & 40/29 & -15/29 & \\ & -15/29 & 60/29 & \\ & & & \end{bmatrix} \begin{bmatrix} 30 & 10 & 30 & 2 \\ 6 & -1 & -6 & 0 \\ 1 & 2 & -2 & -1 \end{bmatrix}$$

 $\Pi_{4,78}$ spans $L_{4.17}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/14 & & & \\ & 15/14 & -3/7 & \\ & -3/7 & 18/7 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 & 12 \\ 1 & -2 & -2 & 2 \\ 0 & -1 & 1 & 3 \end{bmatrix}$$

 $\Pi_{4,80}$ spans $L_{4.15}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/16 & & & \\ & 9/16 & -3/16 & \\ & -3/16 & 33/16 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 8 & 3 & 6 \\ 1 & 2 & -2 & -3 \\ 1 & -2 & -1 & 1 \end{bmatrix}$$

 $\Pi_{4,82}$ spans $L_{4.6}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/18 & & & \\ & 7/18 & -1/18 & \\ & -1/18 & 31/18 & \\ & & & \end{bmatrix} \begin{bmatrix} 3 & 12 & 2 & 4 \\ -2 & 7 & 1 & -3 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

 $\Pi_{4,84}$ spans $L_{4.5}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/24 & & & \\ & 3/8 & & \\ & & 2/3 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 24 & 1 & 2 \\ -2 & -8 & 1 & 2 \\ -3 & 6 & 1 & -1 \end{bmatrix}$$

 $\Pi_{4,86}$ spans $L_{10.9}$ $\infty 222$ (shared)

$$\begin{bmatrix} -1/38 & & & \\ & 40/19 & -5/19 & \\ & -5/19 & 60/19 & \\ & & & \end{bmatrix} \begin{bmatrix} 10 & 10 & 2 & 50 \\ 2 & 0 & -1 & -2 \\ -1 & 2 & 0 & -6 \end{bmatrix}$$

$\Pi_{5,1}$ spans $L_{4,22}$ $22\circ\circ 22 \times D_2$ (shared)

$$\begin{bmatrix} -1/8 & & \\ & 9/8 & \\ & & 6 \end{bmatrix} \begin{bmatrix} 1 & 18 & 6 \\ 1 & 2 & -2 \\ 0 & 3 & 1 \end{bmatrix}$$

 $\Pi_{5,3}$ spans $L_{16,6}$ $22\mathfrak{z}22 \times D_2$

$$\begin{bmatrix} -1/9 & & \\ & 5/18 & \\ & & 15/2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 10 \\ 2 & 1 & -7 \\ 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{5,5}$ spans $L_{4,6}$ $22\circ\circ 22 \times D_2$ (shared)

$$\begin{bmatrix} -1/6 & & \\ & 1/6 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 2 & 16 & 3 \\ 4 & 8 & -3 \\ 0 & 4 & 1 \end{bmatrix}$$

 $\Pi_{5,7}$ spans $L_{4,14}$ $22\circ\circ 22 \times D_2$ (shared)

$$\begin{bmatrix} -1/3 & & \\ & 1/3 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -3 \\ 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{5,9}$ spans $L_{10,2}$ $22\circ\circ 22 \times D_2$

$$\begin{bmatrix} -1/6 & & \\ & 1/6 & \\ & & 5/2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 10 \\ 4 & 1 & -10 \\ 0 & -1 & -2 \end{bmatrix}$$

 $\Pi_{5,11}$ spans $L_{8,2}$ $22442 \times D_2$

$$\begin{bmatrix} -7/11 & & \\ & 1/22 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ -5 & 1 & 6 \\ -1 & -3 & 0 \end{bmatrix}$$

 $\Pi_{5,13}$ spans $L_{31,9}$ $22\mathfrak{z}22 \times D_2$ (shared)

$$\begin{bmatrix} -1/14 & & \\ & 15/14 & \\ & & 15/2 \end{bmatrix} \begin{bmatrix} 10 & 6 & 10 \\ -4 & -1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{5,15}$ spans $L_{3,4}$ $22332 \times D_2$

$$\begin{bmatrix} -1/4 & & \\ & 3 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 6 & 2 & 2 \\ 2 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$$

 $\Pi_{5,17}$ spans $L_{31,1}$ $22\mathfrak{z}22 \times D_2$ (shared)

$$\begin{bmatrix} -1/16 & & \\ & 15/4 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 24 & 10 & 6 \\ 4 & 1 & -1 \\ 0 & -5 & -3 \end{bmatrix}$$

 $\Pi_{5,19}$ spans $L_{123,4}$ $22|22\mathfrak{z} \times D_2$ (shared)

$$\begin{bmatrix} -1/10 & & \\ & 3/5 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 6 & 16 & 1 \\ 4 & 4 & -1 \\ 0 & -8 & -1 \end{bmatrix}$$

 $\Pi_{5,21}$ spans $L_{123,7}$ $22|22\mathfrak{z} \times D_2$ (shared)

$$\begin{bmatrix} -1/10 & & \\ & 12/5 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 12 & 3 \\ -1 & -1 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

 $\Pi_{5,23}$ spans $L_{123,3}$ $22\mathfrak{z}22 \times D_2$ (shared)

$$\begin{bmatrix} -1/4 & & \\ & 3/4 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & -1 \\ 0 & -3 & -1 \end{bmatrix}$$

 $\Pi_{5,2}$ spans $L_{22,7}$ $22\mathfrak{z}22 \times D_2$

$$\begin{bmatrix} -1/19 & & \\ & 21/38 & \\ & & 21/2 \end{bmatrix} \begin{bmatrix} 2 & 21 & 14 \\ -2 & -2 & 5 \\ 0 & 2 & 1 \end{bmatrix}$$

 $\Pi_{5,4}$ spans $L_{4,11}$ $22\circ\circ 22 \times D_2$ (shared)

$$\begin{bmatrix} -1/16 & & \\ & 9/16 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 2 & 9 & 24 \\ -2 & -1 & 8 \\ 0 & -3 & -4 \end{bmatrix}$$

 $\Pi_{5,6}$ spans $L_{141,7}$ $22\circ\circ 22 \times D_2$

$$\begin{bmatrix} -1/3 & & \\ & 4/3 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

 $\Pi_{5,8}$ spans $L_{5,4}$ $22\infty|\infty 2 \times D_2$ (shared)

$$\begin{bmatrix} -1/3 & & \\ & 5/6 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 5 \\ -1 & 1 & 4 \\ -1 & -5 & 0 \end{bmatrix}$$

 $\Pi_{5,10}$ spans $L_{16,5}$ $22\mathfrak{z}22 \times D_2$

$$\begin{bmatrix} -1/7 & & \\ & 15/14 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 3 & 15 & 2 \\ -2 & -3 & 1 \\ 0 & -5 & -1 \end{bmatrix}$$

 $\Pi_{5,12}$ spans $L_{4,11}$ $22\circ\circ 22 \times D_2$ (shared)

$$\begin{bmatrix} -1/10 & & \\ & 18/5 & \\ & & 3/2 \end{bmatrix} \begin{bmatrix} 8 & 9 & 6 \\ 2 & 1 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

 $\Pi_{5,14}$ spans $L_{41,10}$ $2|26\mathfrak{z}6 \times D_2$

$$\begin{bmatrix} -1/40 & & \\ & 39/10 & \\ & & 39 \end{bmatrix} \begin{bmatrix} 12 & 26 & 78 \\ -2 & -1 & 7 \\ 0 & -1 & -1 \end{bmatrix}$$

 $\Pi_{5,16}$ spans $L_{4,25}$ $22\circ\circ 22 \times D_2$ (shared)

$$\begin{bmatrix} -1/7 & & \\ & 9/7 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 9 & 8 & 3 \\ 4 & 2 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

 $\Pi_{5,18}$ spans $L_{19,8}$ $22\mathfrak{z}22 \times D_2$

$$\begin{bmatrix} -1/28 & & \\ & 15/7 & \\ & & 5 \end{bmatrix} \begin{bmatrix} 2 & 60 & 10 \\ -1 & -2 & 2 \\ 0 & -6 & -1 \end{bmatrix}$$

 $\Pi_{5,20}$ spans $L_{123,5}$ $22\mathfrak{z}22 \times D_2$ (shared)

$$\begin{bmatrix} -1/11 & & \\ & 3/11 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 8 \\ 2 & 1 & -6 \\ 0 & 3 & 2 \end{bmatrix}$$

 $\Pi_{5,22}$ spans $L_{123,8}$ $22\mathfrak{z}22 \times D_2$ (shared)

$$\begin{bmatrix} -1/13 & & \\ & 3/13 & \\ & & 3 \end{bmatrix} \begin{bmatrix} 3 & 8 & 6 \\ 4 & 2 & -5 \\ 0 & -2 & -1 \end{bmatrix}$$

 $\Pi_{5,24}$ spans $L_{19,6}$ $2\mathfrak{z}22|2 \times D_2$

$$\begin{bmatrix} -1/19 & & \\ & 15/38 & \\ & & 15/2 \end{bmatrix} \begin{bmatrix} 15 & 6 & 5 \\ 7 & -1 & -4 \\ -1 & -1 & 0 \end{bmatrix}$$

$\Pi_{5,49}$ spans $L_{9.5}$
 $22\infty 22$

$$\begin{bmatrix} -1/10 & & & & & \\ & 11/10 & -1/2 & & & \\ & -1/2 & 3/2 & & & \\ & & & 7 & 8 & 14 & 14 & 1 \\ & & & -2 & 2 & 6 & 1 & -1 \\ & & & -3 & -2 & 2 & 5 & 0 \end{bmatrix}$$

 $\Pi_{5,51}$ spans $L_{25.3}$
 22222

$$\begin{bmatrix} -1/24 & & & & & \\ & 13/6 & -2/3 & & & \\ & -2/3 & 20/3 & & & \\ & & & 2 & 28 & 12 & 14 & 16 \\ & & & 1 & 2 & -2 & -3 & 0 \\ & & & 0 & 3 & 1 & -1 & -2 \end{bmatrix}$$

 $\Pi_{5,53}$ spans $L_{1.6}$
 $\infty\infty 22\infty$

$$\begin{bmatrix} -1/2 & & & & & \\ & 3/2 & -1/2 & & & \\ & -1/2 & 3/2 & & & \\ & & & 1 & 4 & 4 & 2 & 4 \\ & & & 1 & -1 & -3 & -1 & 1 \\ & & & 0 & -3 & -1 & 1 & 3 \end{bmatrix}$$

 $\Pi_{5,50}$ spans $L_{127.6}$
 22422 (shared)

$$\begin{bmatrix} -1/10 & & & & & \\ & 12/5 & -3/5 & & & \\ & -3/5 & 39/10 & & & \\ & & & 15 & 24 & 6 & 3 & 2 \\ & & & -2 & 2 & 2 & 0 & -1 \\ & & & -3 & -4 & 0 & 1 & 0 \end{bmatrix}$$

 $\Pi_{5,52}$ spans $L_{17.7}$
 22222

$$\begin{bmatrix} -1/11 & & & & & \\ & 12/11 & -3/11 & & & \\ & -3/11 & 42/11 & & & \\ & & & 1 & 15 & 6 & 10 & 3 \\ & & & 1 & 2 & -2 & -4 & 0 \\ & & & 0 & 3 & 1 & -1 & -1 \end{bmatrix}$$

 $\Pi_{6,1} = B_3 = \text{GN}_{37}$ spans $L_{3.2}$
 $2|2|2|2|2|2 \times D_{12}$

$$\begin{bmatrix} -1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

 $\Pi_{6,3} = A_{3,II} = \text{GN}_{46}$ spans $L_{7.7}$
 $|\phi|\phi|\phi|\phi|\phi|\phi \times D_{12}$

$$\begin{bmatrix} -3 & & & & & \\ & 2 & & & & \\ & & 2 & & & \\ & & & 2 & & \\ & & & & 2 & \\ & & & & & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

 $\Pi_{6,5}$ spans $L_{19.5}$
 $2|222|22 \times D_4$

$$\begin{bmatrix} -1/8 & & & & & \\ & 3 & & & & \\ & & 15/2 & & & \\ & & & 16 & 6 & \\ & & & -4 & -1 & \\ & & & 0 & -1 & \end{bmatrix}$$

 $\Pi_{6,7}$ spans $L_{19.8}$
 $22|222|2 \times D_4$

$$\begin{bmatrix} -1/8 & & & & & \\ & 60 & & & & \\ & & 5/2 & & & \\ & & & 20 & 2 & \\ & & & -1 & 0 & \\ & & & -2 & -1 & \end{bmatrix}$$

 $\Pi_{6,9}$ spans $L_{10.1}$
 $2|2\phi 2|2\phi \times D_4$

$$\begin{bmatrix} -1/8 & & & & & \\ & 10 & & & & \\ & & 25/2 & & & \\ & & & 32 & 10 & \\ & & & 4 & 1 & \\ & & & 0 & 1 & \end{bmatrix}$$

 $\Pi_{6,11}$ spans $L_{4.12}$
 $2|2\phi 2|2\phi \times D_4$ (shared)

$$\begin{bmatrix} -1/2 & & & & & \\ & 3 & & & & \\ & & 9/2 & & & \\ & & & 4 & 3 & \\ & & & -2 & -1 & \\ & & & 0 & -1 & \end{bmatrix}$$

 $\Pi_{6,13}$ spans $L_{251.1}$
 $36|636|6 \times D_4$

$$\begin{bmatrix} -5/8 & & & & & \\ & 24 & & & & \\ & & 9/2 & & & \\ & & & 6 & 2 & \\ & & & 1 & 0 & \\ & & & 1 & 1 & \end{bmatrix}$$

 $\Pi_{6,15}$ spans $L_{123.1}$
 $4|424|42 \times D_4$

$$\begin{bmatrix} -3/2 & & & & & \\ & 1/2 & & & & \\ & & 2 & & & \\ & & & 2 & 1 & \\ & & & 4 & 1 & \\ & & & 0 & -1 & \end{bmatrix}$$

 $\Pi_{6,2} = B_4 = \text{GN}_{43}$ spans $L_{155.1}$
 $|\phi|\phi|\phi|\phi|\phi|\phi \times D_{12}$

$$\begin{bmatrix} -1 & & & & & \\ & 3 & & & & \\ & & 3 & & & \\ & & & 2 & & \\ & & & -1 & & \\ & & & 0 & & \\ & & & & 1 & \end{bmatrix}$$

 $\Pi_{6,4}$ spans $L_{6.3}$
 $2|2|2|2|2|2 \times D_6$

$$\begin{bmatrix} -1/3 & & & & & \\ & 5/9 & & & & \\ & & 5/9 & & & \\ & & & 5/9 & & \\ & & & & 5/9 & \\ & & & & & 5/9 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 2 & 2 \\ -1 & 2 \\ -1 & -4 \end{bmatrix}$$

 $\Pi_{6,6}$ spans $L_{123.6}$
 $22|222|2 \times D_4$

$$\begin{bmatrix} -1/2 & & & & & \\ & 6 & & & & \\ & & 3/2 & & & \\ & & & 3 & 1 & \\ & & & -1 & 0 & \\ & & & 1 & 1 & \end{bmatrix}$$

 $\Pi_{6,8}$ spans $L_{31.2}$
 $2|232|23 \times D_4$ (shared)

$$\begin{bmatrix} -1/2 & & & & & \\ & 3/2 & & & & \\ & & 5/2 & & & \\ & & & 6 & 2 & \\ & & & 4 & 1 & \\ & & & 0 & -1 & \end{bmatrix}$$

 $\Pi_{6,10}$ spans $L_{16.16}$
 $2|232|23 \times D_4$

$$\begin{bmatrix} -1/8 & & & & & \\ & 15/2 & & & & \\ & & 15 & & & \\ & & & 12 & 10 & \\ & & & -2 & -1 & \\ & & & 0 & -1 & \end{bmatrix}$$

 $\Pi_{6,12}$ spans $L_{22.1}$
 $2|232|23 \times D_4$

$$\begin{bmatrix} -1/4 & & & & & \\ & 1/2 & & & & \\ & & 21/2 & & & \\ & & & 4 & 6 & \\ & & & -4 & -3 & \\ & & & 0 & -1 & \end{bmatrix}$$

 $\Pi_{6,14}$ spans $L_{22.1}$
 $36|636|6 \times D_4$

$$\begin{bmatrix} -7/8 & & & & & \\ & 4 & & & & \\ & & 3/2 & & & \\ & & & 2 & 6 & \\ & & & -1 & 0 & \\ & & & -1 & -5 & \end{bmatrix}$$

 $\Pi_{6,16} = \text{GN}_{44}$ spans $L_{141.14}$
 $\infty|\infty\phi\infty|\infty\phi \times D_4$

$$\begin{bmatrix} -2 & & & & & \\ & 1 & & & & \\ & & 2 & & & \\ & & & 4 & 1 & \\ & & & 6 & 1 & \\ & & & 0 & -1 & \end{bmatrix}$$

$\Pi_{6,17} = \text{GN}_{41}$ spans $L_{10.8}$

$$|\infty\circ\circ\infty|\infty\circ\circ\infty \times D_4$$

$$\begin{bmatrix} -5/8 & & & \\ & 1/2 & & \\ & & 2 & 8 \\ & & 3 & 4 \\ & & 40 & \\ & & & 0 & -1 \end{bmatrix}$$

$\Pi_{6,19}$ spans $L_{22.8}$

$$\mathfrak{3}2|2\mathfrak{3}2|2 \times D_4$$

$$\begin{bmatrix} -1/8 & & & \\ & 28 & & \\ & & 14 & 6 \\ & & 1 & 0 \\ & & 21/2 & \\ & & & 1 & 1 \end{bmatrix}$$

$\Pi_{6,21}$ spans $L_{31.10}$

$$\mathfrak{3}2|2\mathfrak{3}2|2 \times D_4$$

$$\begin{bmatrix} -1/8 & & & \\ & 120 & & \\ & & 30 & 2 \\ & & -1 & 0 \\ & & 5/2 & \\ & & & -3 & -1 \end{bmatrix}$$

$\Pi_{6,23}$ spans $L_{141.3}$

$$\circ\circ 2|2\circ\circ 2|2 \times D_4$$

$$\begin{bmatrix} -1 & & & \\ & 4 & & \\ & & 2 & 1 \\ & & 1 & 0 \\ & & 2 & \\ & & & 1 & 1 \end{bmatrix}$$

$\Pi_{6,25}$ spans $L_{10.13}$

$$\circ\circ 2|2\circ\circ 2|2 \times D_4$$

$$\begin{bmatrix} -1/8 & & & \\ & 200 & & \\ & & 40 & 2 \\ & & -1 & 0 \\ & & 5/2 & \\ & & & -4 & -1 \end{bmatrix}$$

$\Pi_{6,27}$ spans $L_{172.4}$

$$|222|222 \times D_2$$

$$\begin{bmatrix} -1/4 & & & \\ & 3/4 & & \\ & & 2 & 18 & 3 & 2 \\ & & 2 & 6 & -1 & -2 \\ & & 9/2 & 0 & -4 & -1 & 0 \end{bmatrix}$$

$\Pi_{6,29}$ spans $L_{17.20}$

$$|222|222 \times D_2$$

$$\begin{bmatrix} -1/14 & & & \\ & 60/7 & & \\ & & 6 & 20 & 15 & 16 \\ & & 1 & 1 & -1 & -2 \\ & & 5/2 & 0 & 4 & 3 & 0 \end{bmatrix}$$

$\Pi_{6,31}$ spans $L_{6.5}$

$$22|222|2 \times D_2$$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & & \\ & & 2 & 80 & 10 & 2 \\ & & -1 & -8 & 1 & 1 \\ & & 20 & 0 & 6 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,33}$ spans $L_{123.6}$

$$22|224|4 \times D_2$$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & 1 & 3 & 3 & 6 \\ & & 1 & 1 & -1 & -4 \\ & & 6 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,35}$ spans $L_{151.6}$

$$22|222|2 \times D_2 \text{ (shared)}$$

$$\begin{bmatrix} -1/5 & & & \\ & 6/5 & & \\ & & 1 & 12 & 8 & 3 \\ & & 1 & 2 & -2 & -2 \\ & & 4 & 0 & 3 & 2 & 0 \end{bmatrix}$$

$\Pi_{6,37}$ spans $L_{251.1}$

$$22\mathfrak{3}22\mathfrak{3} \times D_2 \text{ (shared)}$$

$$\begin{bmatrix} -5/32 & & & \\ & 9/8 & & \\ & & 18 & 24 & 2 \\ & & 7 & 4 & -1 \\ & & 3/2 & 3 & 8 & 1 \end{bmatrix}$$

$\Pi_{6,39}$ spans $L_{31.10}$

$$36|632|2 \times D_2$$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & & \\ & & 10 & 30 & 30 & 2 \\ & & -3 & -3 & 3 & 1 \\ & & 120 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$\Pi_{6,18}$ spans $L_{19.1}$

$$4|4\mathfrak{2}4|4\mathfrak{2} \times D_4$$

$$\begin{bmatrix} -5/8 & & & \\ & 1/2 & & \\ & & 2 & 4 \\ & & -3 & -2 \\ & & 12 & \\ & & & 0 & -1 \end{bmatrix}$$

$\Pi_{6,20}$ spans $L_{16.12}$

$$\mathfrak{3}2|2\mathfrak{3}2|2 \times D_4$$

$$\begin{bmatrix} -1/8 & & & \\ & 45 & & \\ & & 18 & 4 \\ & & -1 & 0 \\ & & 3/2 & \\ & & & 3 & 2 \end{bmatrix}$$

$\Pi_{6,22} = \text{GN}_{39}$ spans $L_{142.10}$

$$|\infty\mathfrak{2}\infty|\infty\mathfrak{2}\infty \times D_4$$

$$\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 1 & 4 \\ & & -1 & -1 \\ & & 18 & \\ & & & 0 & -1 \end{bmatrix}$$

$\Pi_{6,24}$ spans $L_{4.22}$

$$\circ\circ 2|2\circ\circ 2|2 \times D_4$$

$$\begin{bmatrix} -1/2 & & & \\ & 18 & & \\ & & 6 & 1 \\ & & -1 & 0 \\ & & 3/2 & \\ & & & 2 & 1 \end{bmatrix}$$

$\Pi_{6,26}$ spans $L_{25.5}$

$$|222|222 \times D_2$$

$$\begin{bmatrix} -1/5 & & & \\ & 3/10 & & \\ & & 1 & 21 & 6 & 3 \\ & & -2 & -7 & 3 & 4 \\ & & 21/2 & 0 & -3 & -1 & 0 \end{bmatrix}$$

$\Pi_{6,28}$ spans $L_{6.1}$

$$22|222|2 \times D_2$$

$$\begin{bmatrix} -1/4 & & & \\ & 1/2 & & \\ & & 4 & 10 & 2 & 4 \\ & & 4 & 5 & -1 & -4 \\ & & 5/2 & 0 & 3 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,30}$ spans $L_{151.10}$

$$22|222|2 \times D_2 \text{ (shared)}$$

$$\begin{bmatrix} -1/5 & & & \\ & 6/5 & & \\ & & 3 & 16 & 6 & 1 \\ & & -2 & -4 & 1 & 1 \\ & & 12 & 0 & 2 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,32}$ spans $L_{187.2}$

$$22|223|3 \times D_2$$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & & \\ & & 2 & 80 & 10 & 10 \\ & & 1 & 8 & -1 & -3 \\ & & 20 & 0 & 6 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,34}$ spans $L_{213.2}$

$$22|222|2 \times D_2$$

$$\begin{bmatrix} -1/5 & & & \\ & 3/10 & & \\ & & 3 & 10 & 9 & 1 \\ & & 4 & 5 & -3 & -2 \\ & & 45/2 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,36}$ spans $L_{127.7}$

$$22|224|4 \times D_2$$

$$\begin{bmatrix} -1/4 & & & \\ & 5/4 & & \\ & & 1 & 10 & 10 & 5 \\ & & -1 & -2 & 2 & 3 \\ & & 30 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$\Pi_{6,38}$ spans $L_{144.5}$

$$22\circ\circ 22\circ\circ \times D_2 \text{ (shared)}$$

$$\begin{bmatrix} -3/4 & & & \\ & 3/4 & & \\ & & 4 & 3 & 1 \\ & & -4 & -1 & 1 \\ & & 1 & -2 & -3 & -1 \end{bmatrix}$$

$\Pi_{6,40}$ spans $L_{22.8}$

$$36|632|2 \times D_2$$

$$\begin{bmatrix} -1/8 & & & \\ & 21/2 & & \\ & & 42 & 14 & 14 & 6 \\ & & 5 & 1 & -1 & -1 \\ & & 28 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$\Pi_{6,41}$ spans $L_{141.3}$
 $\infty|\infty\infty 2|2\infty \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & \begin{bmatrix} 8 & 2 & 2 & 1 \\ 6 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,43}$ spans $L_{159.1}$
 $4\phi 422\ddot{2} \times D_2$

$$\begin{bmatrix} -8/9 & & & \\ & 1/18 & & \\ & & \begin{bmatrix} 2 & 4 & 1 \\ 8 & -2 & -5 \\ -2 & -6 & -1 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,45} = \text{GN}_{38}$ spans $L_{1.9}$
 $\phi\infty 2\phi 2\infty \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 1 & & \\ & & \begin{bmatrix} 1 & 4 & 4 \\ 1 & -2 & -4 \\ -1 & -4 & -2 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,47}$ spans $L_{124.8}$
 $222222 \times C_2$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & -1/2 & \\ & -1/2 & 7/2 & \\ & & & \begin{bmatrix} 2 & 5 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,49}$ spans $L_{45.3}$
 $222222 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1 & \\ & -1 & 34 & \\ & & & \begin{bmatrix} 14 & 32 & 2 \\ -1 & 4 & 1 \\ 1 & 2 & 0 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,51}$ spans $L_{48.4}$
 $224224 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 21/2 & -3 & \\ & -3 & 30 & \\ & & & \begin{bmatrix} 6 & 34 & 12 \\ 1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,53}$ spans $L_{7.9}$
 $22\infty 22\infty \times C_2$

$$\begin{bmatrix} -3/8 & & & \\ & 7/2 & -1/2 & \\ & -1/2 & 7/2 & \\ & & & \begin{bmatrix} 2 & 6 & 8 \\ 0 & -2 & -3 \\ 1 & 1 & -1 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,55}$ spans $L_{154.1}$
 $222222 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 6 & -3 & \\ & -3 & 21/2 & \\ & & & \begin{bmatrix} 4 & 36 & 6 \\ 1 & 1 & -1 \\ 0 & -4 & -1 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,57}$ spans $L_{26.1}$
 $\infty 22\infty 22 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1 & \\ & -1 & 18 & \\ & & & \begin{bmatrix} 22 & 88 & 2 \\ -3 & 6 & 1 \\ -2 & -7 & 0 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{7,1}$ spans $L_{123.6}$
 $22|222\ddot{2} \times D_2$

$$\begin{bmatrix} -1/2 & & & \\ & 6 & & \\ & & \begin{bmatrix} 6 & 24 & 1 & 3 \\ -2 & -6 & 0 & 1 \\ 0 & -8 & -1 & -1 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{7,3}$ spans $L_{127.6}$
 $|222\ddot{2}22 \times D_2$

$$\begin{bmatrix} -1/4 & & & \\ & 15/4 & & \\ & & \begin{bmatrix} 6 & 15 & 8 & 3 \\ 2 & 3 & 0 & -1 \\ 0 & 5 & 4 & 1 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{7,5}$ spans $L_{3.2}$
 $22|223\ddot{3} \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 1/2 & & \\ & & \begin{bmatrix} 1 & 6 & 2 & 2 \\ -2 & -6 & 0 & 3 \\ 0 & -4 & -2 & -1 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,42}$ spans $L_{149.21}$
 $|\infty\infty 2|2\infty\infty \times D_2$

$$\begin{bmatrix} -1/4 & & & \\ & 5/4 & & \\ & & \begin{bmatrix} 5 & 20 & 20 & 1 \\ -3 & -4 & 4 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,44}$ spans $L_{4.22}$
 $|\infty\infty 2|2\infty\infty \times D_2$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & \begin{bmatrix} 6 & 6 & 6 & 1 \\ -4 & -2 & 2 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,46}$ spans $L_{127.9}$
 $42|242|2 \times D_2$

$$\begin{bmatrix} -1/5 & & & \\ & 6/5 & & \\ & & \begin{bmatrix} 1 & 24 & 12 & 3 \\ 1 & 4 & -3 & -2 \\ 0 & 2 & 1 & 0 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,48}$ spans $L_{35.3}$
 $222222 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1/2 & \\ & -1/2 & 53/2 & \\ & & & \begin{bmatrix} 12 & 44 & 2 \\ 1 & -5 & -1 \\ -1 & -3 & 0 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,50}$ spans $L_{14.1}$
 $222222 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & -1/2 & \\ & -1/2 & 21/2 & \\ & & & \begin{bmatrix} 16 & 26 & 2 \\ -2 & 2 & 1 \\ -2 & -3 & 0 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,52}$ spans $L_{32.1}$
 $226226 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 21/2 & -9/2 & \\ & -9/2 & 117/2 & \\ & & & \begin{bmatrix} 6 & 22 & 18 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,54}$ spans $L_{6.5}$
 $222222 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 5/2 & & \\ & & \begin{bmatrix} 2 & 80 & 10 \\ -1 & -8 & 1 \\ 0 & -6 & -1 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{6,56}$ spans $L_{30.15}$
 $\infty 22\infty 22 \times C_2$

$$\begin{bmatrix} -1/8 & & & \\ & 21/2 & -3 & \\ & -3 & 18 & \\ & & & \begin{bmatrix} 10 & 40 & 6 \\ -1 & 2 & 1 \\ -1 & -3 & 0 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{6,58}$ spans $L_{17.14}$
 222222

$$\begin{bmatrix} -1/11 & & & \\ & 42/11 & -6/11 & \\ & -6/11 & 48/11 & \\ & & & \begin{bmatrix} 3 & 40 & 24 & 15 & 6 & 10 \\ 1 & 2 & -2 & -3 & -1 & 1 \\ 0 & -6 & -4 & -1 & 1 & 2 \end{bmatrix} \end{bmatrix}$$

 $\Pi_{7,2}$ spans $L_{3.2}$
 $22|222\ddot{2} \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 3/2 & & \\ & & \begin{bmatrix} 2 & 6 & 1 & 1 \\ 2 & 4 & 0 & -1 \\ 0 & 6 & 2 & 1 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{7,4}$ spans $L_{24.8}$
 $22\ddot{2}222|2 \times D_2$

$$\begin{bmatrix} -1/5 & & & \\ & 21/5 & & \\ & & \begin{bmatrix} 8 & 14 & 6 & 7 \\ 2 & 1 & -1 & -2 \\ -2 & -7 & -3 & 0 \end{bmatrix} & \end{bmatrix}$$

 $\Pi_{7,6}$ spans $L_{3.2}$
 $222|222\ddot{3} \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 1/2 & & \\ & & \begin{bmatrix} 1 & 1 & 6 & 2 \\ 2 & 1 & -3 & -3 \\ 0 & 1 & 5 & 1 \end{bmatrix} & \end{bmatrix}$$

$\Pi_{7,7}$ spans $L_{9,7}$ $22|222\circlearrowleft 2 \times D_2$

$$\begin{bmatrix} -1/3 & & & \\ & 1/3 & & \\ & & 7 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 14 & 8 & 7 \\ 2 & 7 & -2 & -7 \\ 0 & -3 & -2 & -1 \end{bmatrix}$$

 $\Pi_{7,9}$ spans $L_{150.13}$ $22\circlearrowleft 222|2 \times D_2$

$$\begin{bmatrix} -1/3 & & & \\ & 4/3 & & \\ & & 6 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 16 & 8 & 1 \\ 3 & 2 & -2 & -1 \\ -1 & -4 & -2 & 0 \end{bmatrix}$$

 $\Pi_{7,11}$ spans $L_{155.1}$ $33|332\circlearrowleft 2 \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 3/2 & & \\ & & 9/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 & 6 \\ -2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

 $\Pi_{7,13} = \text{GN}_{40}$ spans $L_{1.9}$ $|\infty 2\infty\circlearrowleft \infty 2\infty \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 2 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 & 4 \\ -1 & -1 & 1 & 3 \\ 0 & 3 & 3 & 1 \end{bmatrix}$$

 $\Pi_{7,15}$ spans $L_{31.2}$ 2222322

$$\begin{bmatrix} -1/2 & & & \\ & 4 & -1 & \\ & & -1 & 4 \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 6 & 10 & 6 & 2 & 2 & 6 \\ 0 & 2 & 4 & 2 & 0 & -1 & -2 \\ -1 & -1 & 1 & 2 & 1 & 0 & -2 \end{bmatrix}$$

 $\Pi_{7,17}$ spans $L_{141.3}$ $22\infty 22\infty 2$

$$\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 4 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 16 & 8 & 2 & 1 & 2 & 2 \\ 1 & 8 & 2 & -1 & -1 & -1 & 1 \\ 0 & -6 & -4 & -1 & 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{7,19}$ spans $L_{4.12}$ $22\infty 22\infty 2$

$$\begin{bmatrix} -1/2 & & & \\ & 15/2 & -3/2 & \\ & & -3/2 & 15/2 \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 18 & 12 & 3 & 4 & 3 & 3 \\ -1 & -1 & 1 & 1 & 1 & 0 & -1 \\ -1 & -5 & -3 & 0 & 1 & 1 & 0 \end{bmatrix}$$

 $\Pi_{7,21}$ spans $L_{22.8}$ 2226632

$$\begin{bmatrix} -1/8 & & & \\ & 21/2 & & \\ & & 28 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 112 & 42 & 14 & 42 & 14 & 14 \\ -1 & -8 & -1 & 1 & 5 & 1 & -1 \\ 0 & -6 & -3 & -1 & 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{8,1}$ spans $L_{19.10}$ $|2|2|2|2|2|2|2|2 \times D_8$

$$\begin{bmatrix} -1/4 & & & \\ & 15 & & \\ & & 15 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 20 \\ -1 & -2 \\ 0 & -2 \end{bmatrix}$$

 $\Pi_{8,3}$ spans $L_{8.1}$ $|4|4|4|4|4|4|4|4 \times D_8$

$$\begin{bmatrix} -7/4 & & & \\ & 1 & & \\ & & 1 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 4 \\ -3 & -4 \end{bmatrix}$$

 $\Pi_{8,5}$ spans $L_{69.1}$ $|22|22|22|22 \times D_4$

$$\begin{bmatrix} -1/4 & & & \\ & 3 & & \\ & & 35 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 84 & 10 \\ 1 & 14 & 0 \\ 0 & -6 & -1 \end{bmatrix}$$

 $\Pi_{8,7}$ spans $L_{25.1}$ $|22|22|22|22 \times D_4$

$$\begin{bmatrix} -1/4 & & & \\ & 3 & & \\ & & 63 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 36 & 14 \\ -1 & -6 & 0 \\ 0 & -2 & -1 \end{bmatrix}$$

 $\Pi_{7,8}$ spans $L_{3.2}$ $223|322\circlearrowleft \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 3/2 & & \\ & & 1/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 2 & 6 & 1 \\ -2 & -1 & 1 & 1 \\ 0 & -3 & -9 & -1 \end{bmatrix}$$

 $\Pi_{7,10}$ spans $L_{44.7}$ $2|262\circlearrowleft 26 \times D_2$

$$\begin{bmatrix} -3/7 & & & \\ & 3/7 & & \\ & & 7 & \\ & & & \end{bmatrix} \begin{bmatrix} 3 & 4 & 12 & 7 \\ -4 & -3 & 5 & 7 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

 $\Pi_{7,12} = \text{GN}_{47}$ spans $L_{7.7}$ $\circlearrowleft \infty \infty \infty | \infty \infty \infty \times D_2$

$$\begin{bmatrix} -3 & & & \\ & 1 & & \\ & & 3 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 & 1 \\ 7 & 1 & -1 & -2 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

 $\Pi_{7,14}$ spans $L_{22.8}$ 2222232

$$\begin{bmatrix} -1/8 & & & \\ & 21/2 & & \\ & & 28 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 112 & 42 & 14 & 6 & 14 & 14 \\ -1 & -8 & -1 & 1 & 1 & 1 & -1 \\ 0 & -6 & -3 & -1 & 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{7,16}$ spans $L_{16.16}$ 2222223

$$\begin{bmatrix} -1/8 & & & \\ & 45/2 & -15/2 & \\ & & -15/2 & 45/2 \\ & & & \end{bmatrix} \begin{bmatrix} 10 & 12 & 10 & 30 & 60 & 12 & 10 \\ 0 & -1 & -1 & -1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 2 & 5 & 1 & 0 \end{bmatrix}$$

 $\Pi_{7,18}$ spans $L_{141.3}$ $22\infty \infty \infty \infty 2$

$$\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 4 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 16 & 8 & 2 & 8 & 2 & 2 \\ 1 & 8 & 2 & -1 & -6 & -1 & 1 \\ 0 & -6 & -4 & -1 & 0 & 1 & 1 \end{bmatrix}$$

 $\Pi_{7,20}$ spans $L_{10.1}$ $22\infty 222\infty$

$$\begin{bmatrix} -1/8 & & & \\ & 45/2 & -5/2 & \\ & & -5/2 & 45/2 \\ & & & \end{bmatrix} \begin{bmatrix} 10 & 32 & 10 & 10 & 32 & 50 & 40 \\ 0 & -2 & -1 & 0 & 2 & 4 & 3 \\ -1 & -2 & 0 & 1 & 2 & 1 & -1 \end{bmatrix}$$

 $\Pi_{8,2}$ spans $L_{123.8}$ $|2|2|2|2|2|2|2|2 \times D_8$

$$\begin{bmatrix} -1 & & & \\ & 6 & & \\ & & 6 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}$$

 $\Pi_{8,4} = A_{2,III} = \text{GN}_{59}$ spans $L_{145.1}$ $\circlearrowleft 2\circlearrowleft 2\circlearrowleft 2\circlearrowleft 2\circlearrowleft 2 \times D_8$

$$\begin{bmatrix} -4 & & & \\ & 1 & & \\ & & 1 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

 $\Pi_{8,6}$ spans $L_{123.11}$ $|22|22|22|22 \times D_4$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & 24 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 24 & 6 \\ -1 & -8 & 0 \\ 0 & -3 & -1 \end{bmatrix}$$

 $\Pi_{8,8}$ spans $L_{17.5}$ $|22|22|22|22 \times D_4$

$$\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & 10 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 15 & 8 \\ 1 & 5 & 0 \\ 0 & -3 & -2 \end{bmatrix}$$

$$\begin{aligned}
 &\Pi_{8,9} \text{ spans } L_{17.12} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & 60 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 12 & 10 \\ -1 & -4 & 0 \\ 0 & -1 & -1 \end{bmatrix} \\
 &\Pi_{8,11} = \text{GN}_{32} \text{ spans } L_{172.5} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/2 & & & \\ & 3/2 & & \\ & & 36 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 9 & 16 \\ -1 & -3 & 0 \\ 0 & -1 & -2 \end{bmatrix} \\
 &\Pi_{8,13} \text{ spans } L_{17.6} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/2 & & & \\ & 4 & & \\ & & 15/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 20 & 3 \\ 1 & 5 & 0 \\ 0 & -4 & -1 \end{bmatrix} \\
 &\Pi_{8,15} \text{ spans } L_{151.6} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 12 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 4 & 3 \\ 1 & 2 & 0 \\ 0 & -1 & -1 \end{bmatrix} \\
 &\Pi_{8,17} \text{ spans } L_{69.11} \\
 &2|22|22|22|2 \times D_4 \\
 &\begin{bmatrix} -1/8 & & & \\ & 21 & & \\ & & 35/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 48 & 14 & 20 \\ -4 & -1 & 0 \\ 0 & -1 & -2 \end{bmatrix} \\
 &\Pi_{8,19} \text{ spans } L_{8.1} \\
 &2\bar{2}\bar{2}\bar{2}\bar{2}\bar{2}\bar{2} \times D_4 \\
 &\begin{bmatrix} -7/8 & & & \\ & 1 & & \\ & & 1/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 4 & 1 \\ 2 & 3 \end{bmatrix} \\
 &\Pi_{8,21} \text{ spans } L_{25.12} \\
 &2|22|22|22|2 \times D_4 \\
 &\begin{bmatrix} -1/8 & & & \\ & 63/2 & & \\ & & 3 & \\ & & & \end{bmatrix} \begin{bmatrix} 28 & 18 & 16 \\ 2 & 1 & 0 \\ 0 & 3 & 4 \end{bmatrix} \\
 &\Pi_{8,23} \text{ spans } L_{165.1} \\
 &2|22|22|22|2 \times D_4 \\
 &\begin{bmatrix} -1/4 & & & \\ & 45/2 & & \\ & & 1/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 36 & 10 & 4 \\ -4 & -1 & 0 \\ 0 & 5 & 4 \end{bmatrix} \\
 &\Pi_{8,25} = \text{GN}_{50} \text{ spans } L_{7.7} \\
 &\$ \infty \phi \infty \$ \infty \phi \infty \times D_4 \\
 &\begin{bmatrix} -3 & & & \\ & 1 & & \\ & & 3 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 7 & 1 \\ 1 & 1 \end{bmatrix} \\
 &\Pi_{8,27} \text{ spans } L_{168.1} \\
 &\$2\bar{2}\bar{2}\bar{2}\bar{2}\bar{2} \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 3/2 & & \\ & & 9/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 2 \\ -5 & -1 \\ -1 & -1 \end{bmatrix} \\
 &\Pi_{8,29} \text{ spans } L_{9.6} \\
 &|\infty 2|2\infty|\infty 2|2\infty \times D_4 \\
 &\begin{bmatrix} -1/2 & & & \\ & 7/2 & & \\ & & 4 & \\ & & & \end{bmatrix} \begin{bmatrix} 7 & 28 & 2 \\ -3 & -8 & 0 \\ 0 & -7 & -1 \end{bmatrix} \\
 &\Pi_{8,31} = \text{GN}_{45} \text{ spans } L_{148.2} \\
 &\phi 2 \phi 2 \phi 2 \phi 2 \times D_4 \\
 &\begin{bmatrix} -9/8 & & & \\ & 2 & & \\ & & 1/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 8 & 2 \\ 6 & 1 \\ -4 & -3 \end{bmatrix} \\
 &\Pi_{8,10} \text{ spans } L_{25.8} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/8 & & & \\ & 3/2 & & \\ & & 21 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 42 & 48 \\ 2 & 7 & 0 \\ 0 & -3 & -4 \end{bmatrix} \\
 &\Pi_{8,12} \text{ spans } L_{69.6} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/8 & & & \\ & 3/2 & & \\ & & 105 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 30 & 112 \\ 2 & 5 & 0 \\ 0 & -1 & -4 \end{bmatrix} \\
 &\Pi_{8,14} \text{ spans } L_{123.5} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 6 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 6 & 2 \\ 1 & 3 & 0 \\ 0 & -2 & -1 \end{bmatrix} \\
 &\Pi_{8,16} \text{ spans } L_{17.4} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 1/2 & & \\ & & 15/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \\
 &\Pi_{8,18} \text{ spans } L_{19.5} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/8 & & & \\ & 15/2 & & \\ & & 3 & \\ & & & \end{bmatrix} \begin{bmatrix} 12 & 30 & 16 \\ 2 & 3 & 0 \\ 0 & -5 & -4 \end{bmatrix} \\
 &\Pi_{8,20} \text{ spans } L_{17.11} \\
 &|22|22|22|22 \times D_4 \\
 &\begin{bmatrix} -1/2 & & & \\ & 15/2 & & \\ & & 10 & \\ & & & \end{bmatrix} \begin{bmatrix} 3 & 5 & 8 \\ -1 & -1 & 0 \\ 0 & -1 & -2 \end{bmatrix} \\
 &\Pi_{8,22} \text{ spans } L_{24.8} \\
 &2|24|42|24|4 \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 14 & & \\ & & 6 & \\ & & & \end{bmatrix} \begin{bmatrix} 7 & 4 & 2 \\ -2 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \\
 &\Pi_{8,24} \text{ spans } L_{37.7} \\
 &3|32|23|32|2 \times D_4 \\
 &\begin{bmatrix} -1/4 & & & \\ & 7 & & \\ & & 35 & \\ & & & \end{bmatrix} \begin{bmatrix} 14 & 14 & 10 \\ 3 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \\
 &\Pi_{8,26} \text{ spans } L_{9.4} \\
 &|\infty|\infty 2|2\infty|\infty 2|2 \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 7/2 & & \\ & & 1/2 & \\ & & & \end{bmatrix} \begin{bmatrix} 7 & 7 & 1 \\ 4 & 3 & 0 \\ 0 & -7 & -2 \end{bmatrix} \\
 &\Pi_{8,28} \text{ spans } L_{150.29} \\
 &|\infty 2|2\infty|\infty 2|2\infty \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 6 & & \\ & & 12 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 8 & 3 \\ -1 & -2 & 0 \\ 0 & -2 & -1 \end{bmatrix} \\
 &\Pi_{8,30} \text{ spans } L_{5.7} \\
 &\phi 2 \bar{2} \phi 2 \bar{2} \phi 2 \bar{2} \times D_4 \\
 &\begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 5 & \\ & & & \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 5 & 1 \\ 1 & 1 \end{bmatrix} \\
 &\Pi_{8,32} \text{ spans } L_{22.8} \\
 &|2222|2222 \times D_2 \\
 &\begin{bmatrix} -1/8 & & & \\ & 21/2 & & \\ & & 28 & \\ & & & \end{bmatrix} \begin{bmatrix} 6 & 112 & 42 & 14 & 6 \\ -1 & -8 & -1 & 1 & 1 \\ 0 & -6 & -3 & -1 & 0 \end{bmatrix}
 \end{aligned}$$

$\Pi_{9,1} = \text{GN}_{53}$ spans $L_{221.3}$

$3|\infty|\infty 3|\infty|\infty 3|\infty|\infty \times D_6$

$$\begin{bmatrix} -3 & & & \\ & 2/3 & & \\ & & 2/3 & \\ & & & 2/3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -7 & -1 \\ 2 & -1 \\ 5 & 2 \end{bmatrix}$$

$\Pi_{9,3}$ spans $L_{19.10}$

$22222222 | 22 \times D_2$

$$\begin{bmatrix} -1/4 & & & \\ & 15 & & \\ & & 15 & \\ & & & 15 \end{bmatrix} \begin{bmatrix} 30 & 20 & 6 & 20 & 6 \\ 4 & 2 & 0 & -2 & -1 \\ -1 & -2 & -1 & -2 & 0 \end{bmatrix}$$

$\Pi_{9,5}$ spans $L_{123.8}$

$2222 | 222222 \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 3 & & \\ & & 3 & \\ & & & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 3 & 2 & 24 \\ 2 & 1 & 0 & -1 & -14 \\ 0 & 1 & 2 & 1 & 2 \end{bmatrix}$$

$\Pi_{9,7} = \text{GN}_{51}$ spans $L_{7.7}$

$3|\infty|\infty|\infty|\infty|\infty|\infty|\infty \times D_2$

$$\begin{bmatrix} -3 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 & 4 & 4 \\ -2 & -1 & 2 & 5 & 7 \\ 0 & 1 & 4 & 3 & 1 \end{bmatrix}$$

$\Pi_{9,9}$ spans $L_{5.7}$

$\infty 2 | 2|\infty|\infty|\infty|\infty|\infty|\infty \times D_2$

$$\begin{bmatrix} -1 & & & \\ & 5 & & \\ & & 5 & \\ & & & 5 \end{bmatrix} \begin{bmatrix} 4 & 20 & 5 & 5 & 2 \\ -2 & -8 & -1 & 1 & 1 \\ 0 & -10 & -5 & -5 & -1 \end{bmatrix}$$

$\Pi_{9,11}$ spans $L_{17.11}$

2222222222

$$\begin{bmatrix} -1/2 & & & & & & & & & \\ & 15/2 & & & & & & & & \\ & & 10 & & & & & & & \\ & & & 10 & & & & & & \end{bmatrix} \begin{bmatrix} 3 & 5 & 8 & 5 & 3 & 5 & 8 & 30 & 40 \\ -1 & -1 & 0 & 1 & 1 & 10 & -4 & -8 \\ 0 & -1 & -2 & -1 & 0 & 1 & 2 & 6 & 6 \end{bmatrix}$$

$\Pi_{9,13}$ spans $L_{141.3}$

$22|\infty|\infty|\infty|\infty|\infty|\infty|\infty$

$$\begin{bmatrix} -1 & & & & & & \\ & 2 & & & & & \\ & & 4 & & & & \\ & & & 4 & & & \\ & & & & 4 & & \\ & & & & & 4 & \\ & & & & & & 4 \end{bmatrix} \begin{bmatrix} 1 & 16 & 8 & 2 & 1 & 16 & 8 & 8 & 16 \\ 1 & 8 & 2 & -1 & -1 & -8 & -2 & 2 & 8 \\ 0 & -6 & -4 & -1 & 0 & 6 & 4 & 4 & 6 \end{bmatrix}$$

$\Pi_{9,15}$ spans $L_{155.1}$

332362622

$$\begin{bmatrix} -1 & & & & & & & & \\ & 6 & -3 & & & & & & \\ & & & -3 & & & & & \\ & & & & 6 & & & & \\ & & & & & 6 & & & \\ & & & & & & 6 & & \\ & & & & & & & 6 & \\ & & & & & & & & 6 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 & 6 & 6 & 18 & 6 & 18 & 6 \\ 1 & 0 & -1 & -3 & -2 & -1 & 1 & 7 & 3 \\ 1 & 1 & 0 & -2 & -3 & -8 & -2 & -1 & 1 \end{bmatrix}$$

$\Pi_{9,17}$ spans $L_{168.1}$

323232226

$$\begin{bmatrix} -1 & & & & & & & & \\ & 6 & -3 & & & & & & \\ & & & -3 & & & & & \\ & & & & 6 & & & & \\ & & & & & 6 & & & \\ & & & & & & 6 & & \\ & & & & & & & 6 & \\ & & & & & & & & 6 \end{bmatrix} \begin{bmatrix} 6 & 6 & 2 & 2 & 6 & 6 & 2 & 6 & 18 \\ -3 & -2 & 0 & 1 & 3 & 2 & 0 & -2 & -8 \\ -2 & -3 & -1 & 0 & 2 & 3 & 1 & 1 & -1 \end{bmatrix}$$

$\Pi_{10,1}$ spans $L_{78.9}$

$22|22222|222 \times D_4$

$$\begin{bmatrix} -1/4 & & & & & & \\ & 15/2 & & & & & \\ & & 55/2 & & & & \\ & & & 30 & & & \end{bmatrix} \begin{bmatrix} 20 & 66 & 10 \\ -4 & -11 & -1 \\ 0 & 3 & 1 \end{bmatrix}$$

$\Pi_{10,3}$ spans $L_{24.7}$

$22|22222|222 \times D_4$

$$\begin{bmatrix} -1 & & & & & & \\ & 3/2 & & & & & \\ & & 21/2 & & & & \\ & & & 7 & & & \end{bmatrix} \begin{bmatrix} 2 & 21 & 3 \\ -2 & -14 & -1 \\ 0 & -4 & -1 \\ 4 & 4 & 1 \end{bmatrix}$$

$\Pi_{10,5}$ spans $L_{10.1}$

$22|22\curvearrowright 22|22\curvearrowright 22 \times D_4$

$$\begin{bmatrix} -1/8 & & & & & & \\ & 10 & & & & & \\ & & 25/2 & & & & \end{bmatrix} \begin{bmatrix} 32 & 50 & 40 \\ -4 & -5 & -2 \\ 0 & -3 & -4 \end{bmatrix}$$

$\Pi_{9,2}$ spans $L_{155.1}$

$3|2|23|2|23|2|2 \times D_6$

$$\begin{bmatrix} -1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \end{bmatrix} \begin{bmatrix} 6 & 2 \\ -5 & -1 \\ 1 & -1 \\ 4 & 2 \end{bmatrix}$$

$\Pi_{9,4}$ spans $L_{123.8}$

$22|22222222 \times D_2$

$$\begin{bmatrix} -1 & & & & & & \\ & 6 & & & & & \\ & & 6 & & & & \\ & & & 6 & & & \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 & 3 & 12 \\ 1 & 1 & 0 & -1 & -5 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\Pi_{9,6}$ spans $L_{155.1}$

$3|32263622 \times D_2$

$$\begin{bmatrix} -1 & & & & & & \\ & 3/2 & & & & & \\ & & 9/2 & & & & \\ & & & 2 & & & \end{bmatrix} \begin{bmatrix} 2 & 2 & 6 & 18 & 6 \\ -2 & -1 & 1 & 9 & 5 \\ 0 & -1 & -3 & -7 & -1 \end{bmatrix}$$

$\Pi_{9,8}$ spans $L_{145.1}$

$4|\infty|\infty|\infty|\infty|\infty|\infty|\infty \times D_2$

$$\begin{bmatrix} -4 & & & & & & \\ & 1/2 & & & & & \\ & & 1/2 & & & & \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 & 1 & 1 \\ 6 & 10 & 1 & -1 & -3 \\ 0 & -6 & -3 & -3 & -1 \end{bmatrix}$$

$\Pi_{9,10}$ spans $L_{69.11}$

2222222222

$$\begin{bmatrix} -1/8 & & & & & & & & & \\ & 77/2 & -7/2 & & & & & & & \\ & & & 77/2 & & & & & & \end{bmatrix} \begin{bmatrix} 20 & 84 & 70 & 48 & 14 & 20 & 14 & 48 & 14 \\ -1 & -1 & 1 & 2 & 1 & 1 & 0 & -2 & -1 \\ -1 & -5 & -4 & -2 & 0 & 1 & 1 & 2 & 0 \end{bmatrix}$$

$\Pi_{9,12}$ spans $L_{10.1}$

$2222\infty 2222\infty$

$$\begin{bmatrix} -1/8 & & & & & & & & & \\ & 45/2 & -5/2 & & & & & & & \\ & & & -5/2 & & & & & & \end{bmatrix} \begin{bmatrix} 40 & 50 & 32 & 50 & 40 & 10 & 32 & 50 & 40 \\ 1 & -1 & -2 & -4 & -3 & 0 & 2 & 4 & 3 \\ -3 & -4 & -2 & -1 & 1 & 1 & 2 & 1 & -1 \end{bmatrix}$$

$\Pi_{9,14}$ spans $L_{4.12}$

$22|\infty|\infty|\infty|\infty|\infty|\infty|\infty$

$$\begin{bmatrix} -1/2 & & & & & & & & & \\ & 15/2 & -3/2 & & & & & & & \\ & & & -3/2 & & & & & & \end{bmatrix} \begin{bmatrix} 4 & 18 & 12 & 3 & 4 & 18 & 12 & 12 & 18 \\ -1 & -1 & 1 & 1 & 1 & 1 & -1 & -3 & -5 \\ -1 & -5 & -3 & 0 & 1 & 5 & 3 & 1 & -1 \end{bmatrix}$$

$\Pi_{9,16} = \text{GN}_{52}$ spans $L_{7.7}$

$3|\infty|\infty|\infty|\infty|\infty|\infty|\infty$

$$\begin{bmatrix} -3 & & & & & & & & \\ & 4 & -2 & & & & & & \\ & & & -2 & & & & & \\ & & & & 4 & & & & \\ & & & & & 4 & & & \\ & & & & & & 4 & & \\ & & & & & & & 4 & \\ & & & & & & & & 4 \end{bmatrix} \begin{bmatrix} 4 & 4 & 1 & 1 & 4 & 4 & 1 & 4 & 4 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & 4 & 3 \\ -3 & -4 & -1 & 0 & 3 & 4 & 1 & 1 & -1 \end{bmatrix}$$

$\Pi_{9,18}$ spans $L_{155.1}$

323226322

$$\begin{bmatrix} -1 & & & & & & & & \\ & 6 & -3 & & & & & & \\ & & & -3 & & & & & \\ & & & & 6 & & & & \\ & & & & & 6 & & & \\ & & & & & & 6 & & \\ & & & & & & & 6 & \\ & & & & & & & & 6 \end{bmatrix} \begin{bmatrix} 6 & 6 & 2 & 2 & 6 & 18 & 6 & 6 & 2 \\ -1 & 1 & 1 & 1 & -1 & -2 & -3 & -1 \\ -3 & -2 & 0 & 1 & 3 & 7 & 1 & -1 & -1 \end{bmatrix}$$

$\Pi_{10,2}$ spans $L_{127.15}$

$222222|222222 \times D_4$

$$\begin{bmatrix} -1/2 & & & & & & \\ & 15/2 & & & & & \\ & & 30 & & & & \end{bmatrix} \begin{bmatrix} 3 & 10 & 15 \\ -1 & -2 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$\Pi_{10,4}$ spans $L_{9.6}$

$22|22\curvearrowright 22|22\curvearrowright 22 \times D_4$

$$\begin{bmatrix} -1/2 & & & & & & \\ & 7 & & & & & \\ & & 1/2 & & & & \end{bmatrix} \begin{bmatrix} 14 & 16 & 7 \\ 4 & 4 & 1 \\ 0 & -8 & -7 \end{bmatrix}$$

$\Pi_{10,6}$ spans $L_{186.2}$

$222322|222222 \times D_4$

$$\begin{bmatrix} -1 & & & & & & \\ & 1/2 & & & & & \\ & & 75/2 & & & & \end{bmatrix} \begin{bmatrix} 1 & 25 & 6 \\ -2 & -25 & -3 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\begin{aligned} &\Pi_{10,7} \text{ spans } L_{141.10} \\ &|22\phi 22|22\phi 22 \times D_4 \\ &\begin{bmatrix} -1 & & & \\ & 2 & & \\ & & 16 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 16 & 8 \\ -1 & -8 & -2 \\ 0 & -3 & -2 \end{bmatrix} \\ &\Pi_{10,9} \text{ spans } L_{123.8} \\ &2\phi 22|22\phi 22|2 \times D_4 \\ &\begin{bmatrix} -1 & & & \\ & 3 & & \\ & & 3 & \\ & & & \end{bmatrix} \begin{bmatrix} 24 & 2 & 3 \\ -14 & -1 & 0 \\ -2 & -1 & -2 \end{bmatrix} \\ &\Pi_{10,11} = \text{GN}_{56} \text{ spans } L_{7.7} \\ &3\infty|3\phi 3\phi 3\infty|3\phi 3 \times D_4 \\ &\begin{bmatrix} -3 & & & \\ & 1 & & \\ & & 3 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 \\ -2 & -5 & -2 \\ 0 & 3 & 4 \end{bmatrix} \\ &\Pi_{10,13} \text{ spans } L_{149.20} \\ &\infty 2|2\infty\phi\infty 2|2\infty\phi \times D_4 \\ &\begin{bmatrix} -1 & & & \\ & 5 & & \\ & & 25 & \\ & & & \end{bmatrix} \begin{bmatrix} 4 & 20 & 5 \\ -2 & -8 & -1 \\ 0 & 2 & 1 \end{bmatrix} \\ &\Pi_{10,15} \text{ spans } L_{19.10} \\ &2222|22222|2 \times D_2 \\ &\begin{bmatrix} -1/4 & & & & & & & \\ & 15/2 & & & & & & \\ & & 15/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 20 & 30 & 30 & 20 & 6 & 20 \\ 4 & 5 & 3 & 0 & -1 & -4 \\ 0 & -3 & -5 & -4 & -1 & 0 \end{bmatrix} \\ &\Pi_{10,17} \text{ spans } L_{17.11} \\ &|22222|22222 \times D_2 \\ &\begin{bmatrix} -1/2 & & & & & & & \\ & 15/2 & & & & & & \\ & & 10 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 3 & 5 & 8 & 30 & 40 & 3 \\ -1 & -1 & 0 & 4 & 8 & 1 \\ 0 & -1 & -2 & -6 & -6 & 0 \end{bmatrix} \\ &\Pi_{10,19} \text{ spans } L_{69.11} \\ &222|22222|22 \times D_2 \\ &\begin{bmatrix} -1/8 & & & & & & & \\ & 21 & & & & & & \\ & & 35/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 48 & 70 & 84 & 20 & 14 & 48 \\ 4 & 5 & 4 & 0 & -1 & -4 \\ 0 & -3 & -6 & -2 & -1 & 0 \end{bmatrix} \\ &\Pi_{10,21} = \text{GN}_{54} \text{ spans } L_{7.7} \\ &3\infty\phi\infty 3\infty 3\phi 3\infty \times D_2 \\ &\begin{bmatrix} -3 & & & & & & & \\ & 3 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & -1 & & \\ & & & & & & -1 & \\ & & & & & & & -2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 & 4 \\ 1 & 1 & -1 & -3 & -4 \\ -1 & -7 & -7 & -5 & -2 \end{bmatrix} \\ &\Pi_{10,23} \text{ spans } L_{145.1} \\ &4\infty 2\phi 2\infty 44\phi 4 \times D_2 \\ &\begin{bmatrix} -4 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 2 & & \\ & & & & & & 2 & \\ & & & & & & & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 & 2 & 4 \\ 2 & 1 & -2 & -3 & -8 \\ 1 & 2 & 8 & 3 & 2 \end{bmatrix} \\ &\Pi_{10,25} \text{ spans } L_{155.1} \\ &32|23622|226 \times D_2 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 3/2 & & & & & & \\ & & 9/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 18 & 6 & 2 \\ 2 & 4 & 1 & -6 & -4 & -2 \\ 0 & 2 & 3 & 8 & 2 & 0 \end{bmatrix} \\ &\Pi_{10,27} \text{ spans } L_{17.11} \\ &2222222222 \times C_2 \\ &\begin{bmatrix} -1/2 & & & & & & & \\ & 15/2 & & & & & & \\ & & 10 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 3 & 5 & 8 & 30 & 40 \\ -1 & -1 & 0 & 4 & 8 \\ 0 & -1 & -2 & -6 & -6 \end{bmatrix} \\ &\Pi_{10,29} \text{ spans } L_{123.8} \\ &2222222422 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 6 & & & & & & \\ & & 6 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 & 3 & 2 & 3 & 12 & 12 & 24 & 24 \\ 0 & 1 & 1 & 1 & 0 & -1 & -5 & -5 & -8 & -6 \\ 1 & 1 & 0 & -1 & -1 & -1 & 1 & 6 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\Pi_{10,8} \text{ spans } L_{150.14} \\ &|22\phi 22|22\phi 22 \times D_4 \\ &\begin{bmatrix} -1 & & & \\ & 6 & & \\ & & 36 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & 9 & 6 \\ -1 & -3 & -1 \\ 0 & -1 & -1 \end{bmatrix} \\ &\Pi_{10,10} \text{ spans } L_{123.8} \\ &2\phi 22|22\phi 22|2 \times D_4 \\ &\begin{bmatrix} -1 & & & \\ & 6 & & \\ & & 6 & \\ & & & \end{bmatrix} \begin{bmatrix} 12 & 3 & 2 \\ -5 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \\ &\Pi_{10,12} \text{ spans } L_{216.1} \\ &|4\infty 2\infty 4|4\infty 2\infty 4 \times D_4 \\ &\begin{bmatrix} -4 & & & & & & & \\ & 1/2 & & & & & & \\ & & 9/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 6 & 10 & 1 \\ 0 & 2 & 1 \end{bmatrix} \\ &\Pi_{10,14} \text{ spans } L_{136.10} \\ &|22222|22222 \times D_2 \\ &\begin{bmatrix} -1/4 & & & & & & & \\ & 35/4 & & & & & & \\ & & 21/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 10 & 42 & 35 & 24 & 7 & 10 \\ 2 & 6 & 3 & 0 & -1 & -2 \\ 0 & 4 & 5 & 4 & 1 & 0 \end{bmatrix} \\ &\Pi_{10,16} \text{ spans } L_{17.11} \\ &22|22222|222 \times D_2 \\ &\begin{bmatrix} -1/2 & & & & & & & \\ & 10 & & & & & & \\ & & 15/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 8 & 5 & 3 & 40 & 30 & 8 \\ 2 & 1 & 0 & -6 & -6 & -2 \\ 0 & -1 & -1 & -8 & -4 & 0 \end{bmatrix} \\ &\Pi_{10,18} \text{ spans } L_{123.8} \\ &22|22222|222 \times D_2 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 6 & & & & & & \\ & & 6 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 & 24 & 24 & 2 \\ 1 & 1 & 0 & -6 & -8 & -1 \\ 0 & 1 & 1 & 8 & 6 & 0 \end{bmatrix} \\ &\Pi_{10,20} \text{ spans } L_{123.8} \\ &|22222|22222 \times D_2 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 3 & & & & & & \\ & & 3 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 3 & 12 & 12 & 3 & 2 & 3 \\ -2 & -6 & -4 & 0 & 1 & 2 \\ 0 & 4 & 6 & 2 & 1 & 0 \end{bmatrix} \\ &\Pi_{10,22} = \text{GN}_{55} \text{ spans } L_{7.7} \\ &3\infty\infty 3\infty\infty 3\infty\infty \times D_2 \\ &\begin{bmatrix} -3 & & & & & & & \\ & 1 & & & & & & \\ & & 3 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 4 & 4 \\ -7 & -1 & 2 & 5 & 7 \\ 1 & 1 & 4 & 3 & 1 \end{bmatrix} \\ &\Pi_{10,24} \text{ spans } L_{168.1} \\ &3222636222 \times D_2 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 3/2 & & & & & & \\ & & 9/2 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 6 & 2 & 6 & 18 & 6 \\ -5 & -1 & 1 & 9 & 5 \\ -1 & -1 & -3 & -7 & -1 \end{bmatrix} \\ &\Pi_{10,26} \text{ spans } L_{69.11} \\ &2222222222 \times C_2 \\ &\begin{bmatrix} -1/8 & & & & & & & \\ & 77/2 & & -7/2 & & & & \\ & & -7/2 & 77/2 & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 20 & 84 & 70 & 48 & 14 \\ -1 & -1 & 1 & 2 & 1 \\ -1 & -5 & -4 & -2 & 0 \end{bmatrix} \\ &\Pi_{10,28} \text{ spans } L_{168.1} \\ &3222632226 \times C_2 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 6 & & -3 & & & & \\ & & -3 & 6 & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 6 & 6 & 2 & 6 & 18 \\ -3 & -2 & 0 & 2 & 8 \\ -2 & -3 & -1 & -1 & 1 \end{bmatrix} \\ &\Pi_{10,30} \text{ spans } L_{123.8} \\ &2222222222 \\ &\begin{bmatrix} -1 & & & & & & & \\ & 6 & & & & & & \\ & & 6 & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 2 & 3 & 2 & 3 & 12 & 12 & 3 \\ 0 & -6 & -8 & -1 & -1 & 0 & 1 & 5 & 5 & 1 \\ -1 & -8 & -6 & 0 & 1 & 1 & 1 & -1 & -1 \end{bmatrix} \end{aligned}$$

$\Pi_{10,31}$ spans $L_{155.1}$

$$3232262626 \begin{bmatrix} -1 \\ 6 & -3 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} 6 & 6 & 2 & 2 & 6 & 18 & 6 & 18 & 6 & 18 \\ -1 & 1 & 1 & 1 & -1 & -2 & -8 & -3 & -7 \\ 2 & 3 & 1 & 0 & -2 & -8 & -3 & -7 & -1 & 1 \end{bmatrix}$$

$\Pi_{10,33}$ spans $L_{155.1}$

$$6223226362 \begin{bmatrix} -1 \\ 6 & -3 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} 6 & 18 & 6 & 2 & 2 & 6 & 18 & 6 & 6 & 18 \\ -1 & -7 & -3 & -1 & 0 & 2 & 8 & 3 & 2 & 1 \\ -3 & -8 & -2 & 0 & 1 & 3 & 7 & 1 & -1 & -7 \end{bmatrix}$$

$\Pi_{10,32}$ spans $L_{155.1}$

$$3223222626 \begin{bmatrix} -1 \\ 6 & -3 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} 6 & 6 & 2 & 6 & 6 & 2 & 6 & 18 & 6 & 18 \\ -1 & 1 & 1 & 3 & 2 & 0 & -2 & -8 & -3 & -7 \\ 2 & 3 & 1 & 1 & -1 & -1 & -3 & -7 & -1 & 1 \end{bmatrix}$$

$\Pi_{11,1}$ spans $L_{19.10}$

$$222222222222 \times D_2 \begin{bmatrix} -1/4 \\ 15 \\ 15 \end{bmatrix} \begin{bmatrix} 6 & 20 & 30 & 30 & 20 & 30 \\ -1 & -2 & -1 & 1 & 2 & 4 \\ 0 & 2 & 4 & 4 & 2 & 1 \end{bmatrix}$$

$\Pi_{11,3}$ spans $L_{123.8}$

$$2242422222 \times D_2 \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 24 & 12 & 12 & 3 & 2 & 3 \\ 14 & 6 & 4 & 0 & -1 & -2 \\ -2 & -4 & -6 & -2 & -1 & 0 \end{bmatrix}$$

$\Pi_{11,5}$ spans $L_{123.8}$

$$222222222222 \times D_2 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 2 & 3 & 12 \\ -1 & -8 & -6 & 0 & 1 & 5 \\ 0 & 6 & 8 & 1 & 1 & 1 \end{bmatrix}$$

$\Pi_{11,7}$ spans $L_{123.8}$

$$222222222222 \times D_2 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 3 & 12 & 12 & 3 & 12 \\ 1 & 1 & 1 & -1 & -1 & -5 \\ 0 & 1 & 5 & 5 & 1 & 1 \end{bmatrix}$$

$\Pi_{11,9}$ spans $L_{145.1}$

$$4\infty 2\infty 44\infty 4 | 4\infty 4 \times D_2 \begin{bmatrix} -4 \\ 1/2 \\ 1/2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 & 4 & 4 & 2 \\ -3 & -6 & 0 & 6 & 10 & 6 \\ -1 & -10 & -6 & -10 & -6 & 0 \end{bmatrix}$$

$\Pi_{11,11}$ spans $L_{69.11}$

$$222222222222$$

$\Pi_{11,12}$ spans $L_{17.11}$

$$222222222222 \begin{bmatrix} -1/2 \\ 15/2 \\ 10 \end{bmatrix} \begin{bmatrix} 3 & 5 & 8 & 30 & 40 & 3 & 40 & 30 & 8 & 30 & 40 \\ -1 & -1 & 0 & 4 & 8 & 1 & 8 & 4 & 0 & -4 & -8 \\ 0 & -1 & -2 & -6 & -6 & 0 & 6 & 6 & 2 & 6 & 6 \end{bmatrix}$$

$\Pi_{11,14}$ spans $L_{123.8}$

$$224222222222 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 2 & 3 & 12 & 12 & 3 \\ 1 & 8 & 6 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 0 & 6 & 8 & 5 & 5 & 1 & 0 & -1 & -5 & -5 & -1 \end{bmatrix}$$

$\Pi_{11,16}$ spans $L_{123.8}$

$$224222222222 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 12 & 12 & 3 & 2 & 3 \\ 1 & 8 & 6 & 1 & -1 & -1 & -5 & -5 & -1 & 0 & 1 \\ 0 & 6 & 8 & 5 & 5 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

$\Pi_{11,18}$ spans $L_{155.1}$

$$62222636262 \begin{bmatrix} -1 \\ 6 & -3 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} 6 & 18 & 6 & 2 & 6 & 18 & 6 & 6 & 18 & 6 & 18 \\ -1 & -7 & -3 & -1 & -1 & 1 & 2 & 3 & 8 & 2 & 1 \\ -3 & -8 & -2 & 0 & 2 & 8 & 3 & 2 & 1 & -1 & -7 \end{bmatrix}$$

$\Pi_{11,2}$ spans $L_{123.8}$

$$22|222242422 \times D_2 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 & 24 & 24 & 12 \\ 1 & 1 & 0 & -6 & -8 & -5 \\ 0 & 1 & 1 & 8 & 6 & 1 \end{bmatrix}$$

$\Pi_{11,4}$ spans $L_{123.8}$

$$2222|222222 \times D_2 \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 24 & 24 & 2 & 24 \\ 2 & 1 & 2 & -2 & -1 & -14 \\ 0 & 1 & 14 & 14 & 1 & 2 \end{bmatrix}$$

$\Pi_{11,6}$ spans $L_{123.8}$

$$22222222|2222 \times D_2 \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 24 & 2 & 3 & 12 & 12 & 3 \\ -14 & -1 & 0 & 4 & 6 & 2 \\ -2 & -1 & -2 & -6 & -4 & 0 \end{bmatrix}$$

$\Pi_{11,8} = \text{GN}_{57}$ spans $L_{7.7}$

$$3\infty | \infty 3\infty 3\infty 3\infty | \infty 3\infty \times D_2 \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 & 4 & 4 & 4 \\ -2 & -5 & -2 & 2 & 5 & 7 \\ 0 & 3 & 4 & 4 & 3 & 1 \end{bmatrix}$$

$\Pi_{11,10}$ spans $L_{155.1}$

$$622|22626362 \times D_2 \begin{bmatrix} -1 \\ 3/2 \\ 9/2 \end{bmatrix} \begin{bmatrix} 2 & 6 & 18 & 6 & 18 & 6 \\ -2 & -4 & -6 & 1 & 9 & 5 \\ 0 & 2 & 8 & 3 & 7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1/8 \\ 77/2 & -7/2 \\ -7/2 & 77/2 \end{bmatrix} \begin{bmatrix} 20 & 84 & 70 & 48 & 14 & 20 & 84 & 70 & 48 & 70 & 84 \\ -1 & -1 & 1 & 2 & 1 & 1 & 1 & -1 & -2 & -4 & -5 \\ -1 & -5 & -4 & -2 & 0 & 1 & 5 & 4 & 2 & 1 & -1 \end{bmatrix}$$

$\Pi_{11,13}$ spans $L_{123.8}$

$$224222222222 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 2 & 3 & 2 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -1 & -1 & -1 & 0 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 1 & 0 & -1 & -1 & -8 & -6 \end{bmatrix}$$

$\Pi_{11,15}$ spans $L_{123.8}$

$$224222222222 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 2 & 24 & 24 & 2 & 3 \\ 1 & 8 & 6 & 1 & -1 & -1 & -1 & -8 & -6 & 0 & 1 \\ 0 & 6 & 8 & 5 & 5 & 1 & 0 & -6 & -8 & -1 & -1 \end{bmatrix}$$

$\Pi_{11,17}$ spans $L_{155.1}$

$$32226262626 \begin{bmatrix} -1 \\ 6 & -3 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} 6 & 6 & 2 & 6 & 18 & 6 & 18 & 6 & 18 & 6 & 18 \\ -1 & 1 & 1 & 3 & 7 & 1 & -1 & -2 & -8 & -3 & -7 \\ 2 & 3 & 1 & 1 & -1 & -2 & -8 & -3 & -7 & -1 & 1 \end{bmatrix}$$

$\Pi_{12,23}$ spans $L_{123.8}$

$224222|222422 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 2 \\ -1 & -8 & -6 & -1 & 1 & 1 & 1 \\ 0 & -6 & -8 & -5 & -5 & -1 & 0 \end{bmatrix}$$

$\Pi_{12,25}$ spans $L_{123.8}$

$2242422222 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 12 & 24 & 24 & 2 & 3 & 12 \\ 5 & 8 & 6 & 0 & -1 & -5 \\ -1 & -6 & -8 & -1 & -1 & -1 \end{bmatrix}$$

$\Pi_{12,27}$ spans $L_{123.8}$

$224222224222 \rtimes C_2$

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 \\ 1 & 8 & 6 & 1 & -1 & -1 \\ 0 & 6 & 8 & 5 & 5 & 1 \end{bmatrix}$$

$\Pi_{12,29}$ spans $L_{123.8}$

224222222222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 12 & 12 & 3 & 2 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -1 & -5 & -5 & -1 & 0 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 1 & 1 & -1 & -1 & -1 & -8 & -6 \end{bmatrix}$$

$\Pi_{12,31}$ spans $L_{123.8}$

224242222222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 24 & 24 & 2 & 3 & 2 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -6 & -8 & -1 & -1 & 0 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 8 & 6 & 0 & -1 & -1 & -8 & -6 \end{bmatrix}$$

$\Pi_{12,33}$ spans $L_{123.8}$

224242222222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 3 & 12 & 12 & 24 & 24 & 12 & 12 & 3 & 2 & 3 & 12 & 12 \\ -1 & -1 & 1 & 6 & 8 & 5 & 5 & 1 & 0 & -1 & -5 & -5 \\ -1 & -5 & -5 & -8 & -6 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

$\Pi_{12,24}$ spans $L_{123.8}$

$22422|224222|2 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 12 & 12 & 24 & 24 & 2 & 3 \\ -2 & -6 & -4 & -2 & 2 & 1 & 2 \\ 0 & 4 & 6 & 14 & 14 & 1 & 0 \end{bmatrix}$$

$\Pi_{12,26}$ spans $L_{123.8}$

$2242422222 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 24 & 12 & 12 & 3 & 2 & 24 \\ 14 & 6 & 4 & 0 & -1 & -14 \\ -2 & -4 & -6 & -2 & -1 & -2 \end{bmatrix}$$

$\Pi_{12,28}$ spans $L_{123.8}$

224222222222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 2 & 24 & 24 & 2 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -1 & -1 & -8 & -6 & 0 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 1 & 0 & -6 & -8 & -1 & -8 & -6 \end{bmatrix}$$

$\Pi_{12,30}$ spans $L_{123.8}$

224222222222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 12 & 12 & 3 & 12 & 12 & 3 \\ 1 & 8 & 6 & 1 & -1 & -1 & -5 & -5 & -1 & -1 & 1 & 1 \\ 0 & 6 & 8 & 5 & 5 & 1 & 1 & -1 & -1 & -5 & -5 & -1 \end{bmatrix}$$

$\Pi_{12,32}$ spans $L_{123.8}$

224242422222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 24 & 24 & 12 & 12 & 3 & 2 & 3 \\ 1 & 8 & 6 & 1 & -1 & -6 & -8 & -5 & -5 & -1 & 0 & 1 \\ 0 & 6 & 8 & 5 & 5 & 8 & 6 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

$\Pi_{13,1}$ spans $L_{31.7}$

$2222|222222322 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 5 \\ 15 \end{bmatrix} \begin{bmatrix} 4 & 15 & 4 & 15 & 4 & 15 & 20 \\ -2 & -6 & -1 & 0 & 1 & 6 & 9 \\ 0 & -2 & -1 & -4 & -1 & -2 & -1 \end{bmatrix}$$

$\Pi_{13,3}$ spans $L_{16.13}$

$2222|222222322 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 \\ -2 & -3 & -1 & 0 & 1 & 3 & 11 \\ 0 & 1 & 1 & 2 & 1 & 1 & 1 \end{bmatrix}$$

$\Pi_{13,5}$ spans $L_{123.8}$

$2242222222422 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 12 \\ -1 & -8 & -6 & -1 & 1 & 1 & 5 \\ 0 & -6 & -8 & -5 & -5 & -1 & -1 \end{bmatrix}$$

$\Pi_{13,7}$ spans $L_{123.11}$

$2242422222|222 \rtimes D_2$

$$\begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 6 & 12 & 12 & 1 & 12 & 12 & 1 \\ 5 & 8 & 6 & 0 & -6 & -8 & -1 \\ 1 & 6 & 8 & 1 & 8 & 6 & 0 \end{bmatrix}$$

$\Pi_{13,9}$ spans $L_{123.9}$

$2242422222|222 \rtimes D_2$

$$\begin{bmatrix} -3 \\ 4 \\ 4 \end{bmatrix} \begin{bmatrix} 8 & 4 & 4 & 1 & 4 & 4 & 1 \\ -7 & -3 & -2 & 0 & 2 & 3 & 1 \\ 1 & 2 & 3 & 1 & 3 & 2 & 0 \end{bmatrix}$$

$\Pi_{13,11}$ spans $L_{123.8}$

2242222242422

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 2 & 24 & 24 & 12 & 12 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -1 & -1 & -8 & -6 & -1 & 1 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 1 & 0 & -6 & -8 & -5 & -5 & -8 & -6 \end{bmatrix}$$

$\Pi_{13,2}$ spans $L_{16.13}$

$22222|22222232 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 9 & 5 & 9 & 5 & 90 \\ -2 & -1 & -1 & 0 & 1 & 1 & 19 \\ 0 & 1 & 3 & 2 & 3 & 1 & 3 \end{bmatrix}$$

$\Pi_{13,4}$ spans $L_{22.4}$

$2222|222222322 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 21/2 \\ 7/2 \end{bmatrix} \begin{bmatrix} 6 & 7 & 6 & 7 & 6 & 7 & 42 \\ -2 & -2 & -1 & 0 & 1 & 2 & 13 \\ 0 & -2 & -3 & -4 & -3 & -2 & -3 \end{bmatrix}$$

$\Pi_{13,6}$ spans $L_{123.8}$

$22422|2242222 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 12 & 12 & 24 & 24 & 2 & 24 \\ -2 & -6 & -4 & -2 & 2 & 1 & 14 \\ 0 & 4 & 6 & 14 & 14 & 1 & 2 \end{bmatrix}$$

$\Pi_{13,8}$ spans $L_{123.8}$

$224242424222|2 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 24 & 12 & 12 & 24 & 24 & 2 & 3 \\ -14 & -6 & -4 & -2 & 2 & 1 & 2 \\ 2 & 4 & 6 & 14 & 14 & 1 & 0 \end{bmatrix}$$

$\Pi_{13,10}$ spans $L_{123.8}$

$224242424222|2 \rtimes D_2$

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 12 & 24 & 24 & 12 & 12 & 3 & 2 \\ 5 & 8 & 6 & 1 & -1 & -1 & -1 \\ -1 & -6 & -8 & -5 & -5 & -1 & 0 \end{bmatrix}$$

$\Pi_{13,12}$ spans $L_{123.8}$

2242222424222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 12 & 12 & 24 & 24 & 12 & 12 & 3 \\ 1 & 8 & 6 & 1 & -1 & -1 & -5 & -5 & -8 & -6 & -1 & 1 & 1 \\ 0 & 6 & 8 & 5 & 5 & 1 & 1 & -1 & -6 & -8 & -5 & -5 & -1 \end{bmatrix}$$

$\Pi_{13,13}$ spans $L_{123.8}$

224242422222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 24 & 24 & 12 & 12 & 3 & 2 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -6 & -8 & -5 & -5 & -1 & 0 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 8 & 6 & 1 & -1 & -1 & -1 & -8 & -6 \end{bmatrix}$$

 $\Pi_{13,15}$ spans $L_{142.20}$ $\infty 4 2 2 \infty 2 2 \infty 2 2 \infty 2 2$

$$\begin{bmatrix} -1 \\ 18 \\ 18 \end{bmatrix} \begin{bmatrix} 9 & 36 & 72 & 8 & 9 & 9 & 8 & 9 & 9 & 8 & 9 & 9 & 8 \\ 1 & 7 & 16 & 2 & 2 & 1 & 0 & -1 & -2 & -2 & -2 & -1 & 0 \\ 2 & 5 & 6 & 0 & -1 & -2 & -2 & -2 & -1 & 0 & 1 & 2 & 2 \end{bmatrix}$$

 $\Pi_{14,1}$ spans $L_{16.13}$ 22|222 \sharp 222|222 \sharp 2 $\times D_4$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 90 \\ -2 & -3 & -1 & -3 \\ 0 & 1 & 1 & 19 \end{bmatrix}$$

 $\Pi_{14,3}$ spans $L_{123.9}$ 224 \sharp 422|224 \sharp 422 $\times D_4$

$$\begin{bmatrix} -3 \\ 4 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 & 8 \\ -1 & -3 & -2 & -1 \\ 0 & 2 & 3 & 7 \end{bmatrix}$$

 $\Pi_{14,5}$ spans $L_{22.4}$ 2|222 \sharp 222|222 \sharp 22 $\times D_4$

$$\begin{bmatrix} -1 \\ 7/2 \\ 21/2 \end{bmatrix} \begin{bmatrix} 7 & 6 & 7 & 42 \\ -4 & -3 & -2 & -3 \\ 0 & -1 & -2 & -13 \end{bmatrix}$$

 $\Pi_{14,7}$ spans $L_{16.13}$ $\sharp 2 2 2 | 2 2 2 \sharp 2 2 2 | 2 2 2 \times D_4$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 9 & 5 & 9 \\ 11 & 3 & 1 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

 $\Pi_{14,9}$ spans $L_{31.7}$ 222|2222322|2322 $\times D_2$

$$\begin{bmatrix} -1 \\ 15 \\ 5 \end{bmatrix} \begin{bmatrix} 15 & 4 & 15 & 4 & 15 & 20 & 20 & 15 \\ -4 & -1 & -2 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & 6 & 2 & 6 & 6 & 3 & 0 \end{bmatrix}$$

 $\Pi_{14,11}$ spans $L_{16.13}$ 2222|2222322|232 $\times D_2$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 90 & 90 & 5 \\ -2 & -3 & -1 & 0 & 1 & 27 & 30 & 2 \\ 0 & 1 & 1 & 2 & 1 & 11 & 8 & 0 \end{bmatrix}$$

 $\Pi_{14,13}$ spans $L_{16.13}$ 222|2222322|2232 $\times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 9 & 5 & 90 & 5 & 9 \\ 2 & 1 & 1 & 0 & -8 & -11 & -1 & -2 \\ 0 & 1 & 3 & 2 & 30 & 27 & 1 & 0 \end{bmatrix}$$

 $\Pi_{14,15}$ spans $L_{22.4}$ 222|2222322|2322 $\times D_2$

$$\begin{bmatrix} -1 \\ 7/2 \\ 21/2 \end{bmatrix} \begin{bmatrix} 7 & 6 & 7 & 6 & 7 & 42 & 42 & 7 \\ 4 & 3 & 2 & 0 & -2 & -18 & -21 & -4 \\ 0 & 1 & 2 & 2 & 2 & 8 & 5 & 0 \end{bmatrix}$$

 $\Pi_{14,17}$ spans $L_{123.11}$ 22424 \sharp 424222 \sharp 2 $\times D_2$

$$\begin{bmatrix} -2 \\ 3/2 \\ 3/2 \end{bmatrix} \begin{bmatrix} 12 & 6 & 6 & 12 & 12 & 1 & 12 \\ -14 & -6 & -4 & -2 & 2 & 1 & 14 \\ 2 & 4 & 6 & 14 & 14 & 1 & 2 \end{bmatrix}$$

 $\Pi_{14,19}$ spans $L_{142.20}$ $\infty 4 2 2 \infty 2 | 2 \infty 2 2 4 \infty 2 | 2 \times D_2$

$$\begin{bmatrix} -1 \\ 18 \\ 18 \end{bmatrix} \begin{bmatrix} 8 & 9 & 8 & 72 & 36 & 9 & 8 \\ 2 & 2 & 1 & 0 & -6 & -5 & -2 & -2 \\ 0 & 1 & 2 & 2 & 16 & 7 & 1 & 0 \end{bmatrix}$$

 $\Pi_{13,14}$ spans $L_{123.8}$

224242422222

$$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 24 & 24 & 12 & 12 & 3 & 12 & 12 & 3 \\ 1 & 8 & 6 & 1 & -1 & -6 & -8 & -5 & -5 & -1 & -1 & 1 & 1 \\ 0 & 6 & 8 & 5 & 5 & 8 & 6 & 1 & -1 & -1 & -5 & -5 & -1 \end{bmatrix}$$

 $\Pi_{14,2}$ spans $L_{123.11}$ 224 \sharp 422|224 \sharp 422 $\times D_4$

$$\begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 12 & 12 & 6 \\ 1 & 8 & 6 & 1 \\ 0 & -6 & -8 & -5 \end{bmatrix}$$

 $\Pi_{14,4}$ spans $L_{31.7}$ 22 \sharp 222|222 \sharp 222|2 $\times D_4$

$$\begin{bmatrix} -1 \\ 5 \\ 15 \end{bmatrix} \begin{bmatrix} 20 & 15 & 4 & 15 \\ 9 & 6 & 1 & 0 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

 $\Pi_{14,6}$ spans $L_{30.27}$ $\infty | \infty 2 2 \wp 2 2 \infty | \infty 2 2 \wp 2 2 \times D_4$

$$\begin{bmatrix} -3/2 \\ 5/2 \\ 75/2 \end{bmatrix} \begin{bmatrix} 10 & 10 & 6 & 10 \\ 8 & 7 & 3 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

 $\Pi_{14,8}$ spans $L_{161.6}$ $| \infty 2 2 \wp 2 2 \infty | \infty 2 2 \wp 2 2 \infty \times D_4$

$$\begin{bmatrix} -8 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 & 1 \\ -3 & -10 & -4 & -1 \\ 0 & -4 & -3 & -2 \end{bmatrix}$$

 $\Pi_{14,10}$ spans $L_{31.7}$ 22|2222322|22322 $\times D_2$

$$\begin{bmatrix} -1 \\ 5 \\ 15 \end{bmatrix} \begin{bmatrix} 4 & 15 & 4 & 15 & 20 & 20 & 15 & 4 \\ 2 & 6 & 1 & 0 & -3 & -6 & -6 & -2 \\ 0 & 2 & 1 & 4 & 5 & 4 & 2 & 0 \end{bmatrix}$$

 $\Pi_{14,12}$ spans $L_{16.13}$ 222|2222322|2322 $\times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 \\ 2 & 1 & 1 & 0 & -1 & -5 & -6 & -2 \\ 0 & 1 & 3 & 2 & 3 & 7 & 4 & 0 \end{bmatrix}$$

 $\Pi_{14,14}$ spans $L_{16.13}$ 22|2222322|22322 $\times D_2$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 30 & 30 & 9 & 5 \\ -2 & -3 & -1 & 0 & 4 & 7 & 3 & 2 \\ 0 & 1 & 1 & 2 & 6 & 5 & 1 & 0 \end{bmatrix}$$

 $\Pi_{14,16}$ spans $L_{22.4}$ 22|2222322|22322 $\times D_2$

$$\begin{bmatrix} -1 \\ 21/2 \\ 7/2 \end{bmatrix} \begin{bmatrix} 6 & 7 & 6 & 7 & 42 & 42 & 7 & 6 \\ 2 & 2 & 1 & 0 & -5 & -8 & -2 & -2 \\ 0 & 2 & 3 & 4 & 21 & 18 & 2 & 0 \end{bmatrix}$$

 $\Pi_{14,18}$ spans $L_{123.9}$ 22424 \sharp 424222 \sharp 2 $\times D_2$

$$\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 4 & 8 & 8 & 4 & 4 & 1 & 4 \\ 5 & 8 & 6 & 1 & -1 & -1 & -5 \\ -1 & -6 & -8 & -5 & -5 & -1 & -1 \end{bmatrix}$$

 $\Pi_{14,20}$ spans $L_{142.20}$ $\infty 4 2 2 \wp 2 2 4 \infty 2 \wp 2 2 \times D_2$

$$\begin{bmatrix} -1 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 9 & 8 & 72 & 36 & 9 & 8 & 9 \\ 3 & 2 & 10 & 2 & -1 & -2 & -3 \\ -1 & -2 & -22 & -12 & -3 & -2 & -1 \end{bmatrix}$$

$\Pi_{14,21}$ spans $L_{142.20}$
 $\infty 42|24\infty 22\infty 2|2\infty 22 \times D_2$
 $\begin{bmatrix} -1 \\ 18 \\ 18 \end{bmatrix} \begin{bmatrix} 8 & 72 & 36 & 9 & 8 & 9 & 9 & 8 \\ 2 & 16 & 7 & 1 & 0 & -1 & -2 & -2 \\ 0 & -6 & -5 & -2 & -2 & -2 & -1 & 0 \end{bmatrix}$

$\Pi_{14,23}$ spans $L_{142.20}$
 $\infty 422\infty 22\infty 422\infty 22 \times C_2$
 $\begin{bmatrix} -1 \\ 18 \\ 18 \end{bmatrix} \begin{bmatrix} 9 & 36 & 72 & 8 & 9 & 9 & 8 \\ 1 & 7 & 16 & 2 & 2 & 1 & 0 \\ 2 & 5 & 6 & 0 & -1 & -2 & -2 \end{bmatrix}$

$\Pi_{14,25}$ spans $L_{16.13}$
 22222222223632

$\Pi_{14,26}$ spans $L_{16.13}$
 22222222322232

$\Pi_{14,27}$ spans $L_{16.13}$
 22222232222232

$\Pi_{14,28}$ spans $L_{123.8}$
 22422224242422

$\Pi_{14,29}$ spans $L_{123.8}$
 22424242424222

$\Pi_{14,22}$ spans $L_{142.20}$
 $422\infty 22\infty 22\infty 224\infty \times D_2$
 $\begin{bmatrix} -1 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 9 & 8 & 9 & 9 & 8 & 72 & 36 \\ -3 & -2 & -1 & 1 & 2 & 22 & 12 \\ -1 & -2 & -3 & -3 & -2 & -10 & -2 \end{bmatrix}$

$\Pi_{14,24}$ spans $L_{142.20}$
 $\infty 422\infty 22\infty 22\infty 422$
 $\begin{bmatrix} -1 \\ 18 \\ 18 \end{bmatrix} \begin{bmatrix} 9 & 36 & 72 & 8 & 9 & 9 & 8 & 9 & 9 & 8 & 9 & 36 & 72 & 8 \\ 1 & 7 & 16 & 2 & 2 & 1 & 0 & -1 & -2 & -2 & -2 & -5 & -6 & 0 \\ 2 & 5 & 6 & 0 & -1 & -2 & -2 & -2 & -1 & 0 & 1 & 7 & 16 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & 0 & 1 & 1 & 2 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -1 & -1 & 0 & 1 & 5 & 6 & 19 & 19 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 & 5 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & 0 & 1 & 5 & 6 & 2 & 1 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 19 & 19 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 & 5 & 9 & 5 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 1 & 1 & 2 & 1 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -1 & 0 & 1 & 1 & 19 & 19 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 3 & 12 & 12 & 24 & 24 & 12 & 12 & 24 & 24 \\ 1 & 8 & 6 & 1 & -1 & -1 & -5 & -5 & -8 & -6 & -1 & 1 & 6 & 8 \\ 0 & 6 & 8 & 5 & 5 & 1 & 1 & -1 & -6 & -8 & -5 & -5 & -8 & -6 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 24 & 24 & 12 & 12 & 24 & 24 & 12 & 12 & 24 & 24 & 12 & 12 & 3 \\ 1 & 8 & 6 & 1 & -1 & -6 & -8 & -5 & -5 & -8 & -6 & -1 & 1 & 1 \\ 0 & 6 & 8 & 5 & 5 & 8 & 6 & 1 & -1 & -6 & -8 & -5 & -5 & -1 \end{bmatrix}$

$\Pi_{15,1}$ spans $L_{31.7}$
 $|22\}22|22\}22|22\}22 \times D_6$
 $\begin{bmatrix} -1 \\ 10/3 \\ 10/3 \\ 10/3 \end{bmatrix} \begin{bmatrix} 4 & 15 & 20 \\ -2 & -6 & -6 \\ 1 & 0 & -3 \\ 1 & 6 & 9 \end{bmatrix}$

$\Pi_{15,3}$ spans $L_{22.4}$
 $\}22|22\}22|22\}22 \times D_6$
 $\begin{bmatrix} -1 \\ 7 \\ 7 \\ 7 \end{bmatrix} \begin{bmatrix} 42 & 7 & 6 \\ 13 & 2 & 1 \\ -5 & 0 & 1 \\ -8 & -2 & -2 \end{bmatrix}$

$\Pi_{15,5}$ spans $L_{31.7}$
 $22|2222322\}22322 \times D_2$
 $\begin{bmatrix} -1 \\ 5 \\ 15 \end{bmatrix} \begin{bmatrix} 4 & 15 & 4 & 15 & 20 & 20 & 15 & 20 \\ 2 & 6 & 1 & 0 & -3 & -6 & -6 & -9 \\ 0 & 2 & 1 & 4 & 5 & 4 & 2 & 1 \end{bmatrix}$

$\Pi_{15,7}$ spans $L_{16.13}$
 $222|222236\}6322 \times D_2$
 $\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 9 & 5 & 9 & 30 & 30 & 90 \\ 2 & 1 & 1 & 0 & -1 & -5 & -6 & -19 \\ 0 & 1 & 3 & 2 & 3 & 7 & 4 & 3 \end{bmatrix}$

$\Pi_{15,9}$ spans $L_{16.13}$
 $22|2222322\}22322 \times D_2$
 $\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 30 & 30 & 9 & 30 \\ -2 & -3 & -1 & 0 & 4 & 7 & 3 & 11 \\ 0 & 1 & 1 & 2 & 6 & 5 & 1 & 1 \end{bmatrix}$

$\Pi_{15,11}$ spans $L_{22.4}$
 $22|2222322\}22322 \times D_2$
 $\begin{bmatrix} -1 \\ 21/2 \\ 7/2 \end{bmatrix} \begin{bmatrix} 6 & 7 & 6 & 7 & 42 & 42 & 7 & 42 \\ 2 & 2 & 1 & 0 & -5 & -8 & -2 & -13 \\ 0 & 2 & 3 & 4 & 21 & 18 & 2 & 3 \end{bmatrix}$

$\Pi_{15,2}$ spans $L_{16.13}$
 $\}22|22\}22|22\}22|22 \times D_6$
 $\begin{bmatrix} -1 \\ 5 \\ 5 \\ 5 \end{bmatrix} \begin{bmatrix} 30 & 9 & 5 \\ 11 & 3 & 1 \\ -4 & 0 & 1 \\ -7 & -3 & -2 \end{bmatrix}$

$\Pi_{15,4}$ spans $L_{16.13}$
 $\}22|22\}22|22\}22|22 \times D_6$
 $\begin{bmatrix} -1 \\ 15 \\ 15 \\ 15 \end{bmatrix} \begin{bmatrix} 90 & 5 & 9 \\ -19 & -1 & -1 \\ 8 & 0 & -1 \\ 11 & 1 & 2 \end{bmatrix}$

$\Pi_{15,6}$ spans $L_{16.13}$
 $2222|222236\}632 \times D_2$
 $\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 90 & 90 & 30 \\ -2 & -3 & -1 & 0 & 1 & 27 & 30 & 11 \\ 0 & 1 & 1 & 2 & 1 & 11 & 8 & 1 \end{bmatrix}$

$\Pi_{15,8}$ spans $L_{16.13}$
 $222|2222322\}2232 \times D_2$
 $\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 9 & 5 & 90 & 90 & 5 & 90 \\ 2 & 1 & 1 & 0 & -8 & -11 & -1 & -19 \\ 0 & 1 & 3 & 2 & 30 & 27 & 1 & 3 \end{bmatrix}$

$\Pi_{15,10}$ spans $L_{16.13}$
 $22|2222322\}2232 \times D_2$
 $\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 90 & 90 & 5 & 9 & 30 \\ -2 & -3 & -1 & -3 & 3 & 1 & 3 & 11 \\ 0 & 1 & 1 & 19 & 19 & 1 & 1 & 1 \end{bmatrix}$

$\Pi_{15,12}$ spans $L_{123.11}$
 $|2242424\}4242422 \times D_2$
 $\begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 12 & 12 & 6 & 6 & 12 & 12 & 6 \\ 1 & 8 & 6 & 1 & -1 & -6 & -8 & -5 \\ 0 & -6 & -8 & -5 & -5 & -8 & -6 & -1 \end{bmatrix}$

$\Pi_{16,15}$ spans $L_{31.7}$

$22\{2222322\}22322 \times D_2$

$$\begin{bmatrix} -1 \\ 5 \\ 15 \end{bmatrix} \begin{bmatrix} 20 & 15 & 4 & 15 & 20 & 20 & 15 & 20 \\ 9 & 6 & 1 & 0 & -3 & -6 & -6 & -9 \\ 1 & 2 & 1 & 4 & 5 & 4 & 2 & 1 \end{bmatrix}$$

$\Pi_{16,17}$ spans $L_{22.4}$

$2|22232232|2322322 \times D_2$

$$\begin{bmatrix} -1 \\ 7/2 \\ 21/2 \end{bmatrix} \begin{bmatrix} 7 & 6 & 7 & 42 & 42 & 7 & 42 & 42 & 7 \\ -4 & -3 & -2 & -3 & 3 & 2 & 18 & 21 & 4 \\ 0 & -1 & -2 & -13 & -13 & -2 & -8 & -5 & 0 \end{bmatrix}$$

$\Pi_{16,19}$ spans $L_{16.13}$

$\{3632222\}2222236 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 30 & 30 & 9 & 5 & 9 & 5 & 90 \\ 19 & 6 & 5 & 1 & 0 & -1 & -1 & -19 \\ 3 & 4 & 7 & 3 & 2 & 3 & 1 & 3 \end{bmatrix}$$

$\Pi_{16,21}$ spans $L_{142.20}$

$\infty 422 \circ 224 \infty 224 \circ 422 \times D_2$

$$\begin{bmatrix} -1 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 9 & 8 & 72 & 36 & 9 & 8 & 72 & 36 \\ 3 & 2 & 10 & 2 & -1 & -2 & -22 & -12 \\ -1 & -2 & -22 & -12 & -3 & -2 & -10 & -2 \end{bmatrix}$$

$\Pi_{16,23}$ spans $L_{142.20}$

$\infty 4224 \circ 4224 \infty 22 \circ 22 \times D_2$

$$\begin{bmatrix} -1 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 36 & 72 & 8 & 72 & 36 & 9 & 8 & 9 \\ 12 & 22 & 2 & 10 & 2 & -1 & -2 & -3 \\ -2 & -10 & -2 & -22 & -12 & -3 & -2 & -1 \end{bmatrix}$$

$\Pi_{16,25}$ spans $L_{16.13}$

$32222\{2222322\}22 \times D_2$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 9 & 5 & 9 & 30 & 30 & 9 & 30 \\ 11 & 3 & 1 & 0 & -4 & -7 & -3 & -11 \\ 1 & 1 & 1 & 2 & 6 & 5 & 1 & 1 \end{bmatrix}$$

$\Pi_{16,27}$ spans $L_{16.13}$

$3222322|22322232|2 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 90 & 90 & 5 & 9 & 30 & 30 & 9 \\ 2 & 1 & 11 & 8 & 0 & -1 & -5 & -6 & -2 \\ 0 & 1 & 27 & 30 & 2 & 3 & 7 & 4 & 0 \end{bmatrix}$$

$\Pi_{16,29}$ spans $L_{16.13}$

$32222\{2222322\}22 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 5 & 9 & 5 & 90 & 90 & 5 & 90 \\ -19 & -1 & -1 & 0 & 8 & 11 & 1 & 19 \\ 3 & 1 & 3 & 2 & 30 & 27 & 1 & 3 \end{bmatrix}$$

$\Pi_{16,31}$ spans $L_{16.13}$

$3632222236322222 \times C_2$

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 90 & 90 & 30 & 30 & 9 & 5 & 9 & 5 \\ -8 & -11 & -5 & -6 & -2 & -1 & -1 & 0 \\ -19 & -19 & -6 & -5 & -1 & 0 & 1 & 1 \end{bmatrix}$$

$\Pi_{16,32}$ spans $L_{16.13}$

2222222236363632

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & 0 & 1 & 5 & 6 & 19 & 19 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 8 & 11 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{16,33}$ spans $L_{16.13}$

2222222322363632

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 5 & 90 & 90 & 5 & 90 & 90 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & 0 & 8 & 11 & 1 & 19 & 19 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -1 & -11 & -8 & 0 & 8 & 11 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{16,34}$ spans $L_{16.13}$

2222223222363632

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 & 5 & 90 & 90 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 1 & 1 & 19 & 19 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -1 & 0 & 8 & 11 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{16,35}$ spans $L_{16.13}$

22222232223632

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 & 30 & 30 & 9 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 1 & 5 & 6 & 2 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -1 & -1 & 1 & 1 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{16,36}$ spans $L_{16.13}$

22222232223632232

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 & 30 & 30 & 90 & 90 & 5 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 1 & 5 & 6 & 19 & 19 & 1 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -1 & -1 & 1 & 8 & 11 & 1 & 19 & 19 \end{bmatrix}$$

$\Pi_{17,5}$ spans $L_{31.7}$

$$|22322322\}22322322 \times D_2$$

$$\begin{bmatrix} -1 \\ 5 \\ 15 \end{bmatrix} \begin{bmatrix} 4 & 15 & 20 & 15 & 20 & 20 & 15 & 20 \\ -2 & -6 & -6 & -3 & 0 & 3 & 6 & 6 & 9 \\ 0 & -2 & -4 & -5 & -4 & -5 & -4 & -2 & -1 \end{bmatrix}$$

$\Pi_{17,7}$ spans $L_{16.13}$

$$3632222\}222223632|2 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 9 & 5 & 9 & 30 & 30 & 90 & 90 & 5 \\ 11 & 3 & 1 & 0 & -4 & -7 & -27 & -30 & -2 \\ 1 & 1 & 1 & 2 & 6 & 5 & 11 & 8 & 0 \end{bmatrix}$$

$\Pi_{17,9}$ spans $L_{16.13}$

$$36322|22363222\}222 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 30 & 30 & 90 & 90 & 5 & 9 & 30 \\ -2 & -3 & -7 & -4 & -3 & 3 & 1 & 3 & 11 \\ 0 & -1 & -5 & -6 & -19 & -19 & -1 & -1 & -1 \end{bmatrix}$$

$\Pi_{17,11}$ spans $L_{16.13}$

$$363222\}22236322|22 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 5 & 9 & 30 & 30 & 90 & 90 & 5 & 9 \\ 19 & 1 & 1 & 1 & -1 & -8 & -11 & -1 & -2 \\ 3 & 1 & 3 & 11 & 11 & 30 & 27 & 1 & 0 \end{bmatrix}$$

$\Pi_{17,13}$ spans $L_{16.13}$

$$3632|23632222\}2222 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 30 & 30 & 90 & 90 & 5 & 9 & 5 & 90 \\ -2 & -6 & -5 & -11 & -8 & 0 & 1 & 1 & 19 \\ 0 & 4 & 7 & 27 & 30 & 2 & 3 & 1 & 3 \end{bmatrix}$$

$\Pi_{17,15}$ spans $L_{16.13}$

$$36\}63222322|223222 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 90 & 90 & 5 & 9 & 30 & 30 & 9 & 5 \\ -11 & -30 & -27 & -1 & 0 & 4 & 7 & 3 & 2 \\ -1 & -8 & -11 & -1 & -2 & -6 & -5 & -1 & 0 \end{bmatrix}$$

$\Pi_{17,17}$ spans $L_{16.13}$

$$3222\}22232232|2322 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 5 & 9 & 30 & 30 & 9 & 30 & 30 & 9 \\ 19 & 1 & 1 & 1 & -1 & -1 & -5 & -6 & -2 \\ -3 & -1 & -3 & -11 & -11 & -3 & -7 & -4 & 0 \end{bmatrix}$$

$\Pi_{17,19}$ spans $L_{16.13}$

$$3222322\}22322232|2 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 5 & 90 & 90 & 5 & 9 & 30 & 30 & 9 \\ 19 & 1 & 11 & 8 & 0 & -1 & -5 & -6 & -2 \\ 3 & 1 & 27 & 30 & 2 & 3 & 7 & 4 & 0 \end{bmatrix}$$

$\Pi_{17,21}$ spans $L_{22.4}$

$$322|223223222\}22322 \times D_2$$

$$\begin{bmatrix} -1 \\ 21/2 \\ 7/2 \end{bmatrix} \begin{bmatrix} 6 & 7 & 42 & 42 & 7 & 42 & 42 & 7 & 42 \\ -2 & -2 & -8 & -5 & 0 & 5 & 8 & 2 & 13 \\ 0 & 2 & 18 & 21 & 4 & 21 & 18 & 2 & 3 \end{bmatrix}$$

$\Pi_{17,23}$ spans $L_{16.13}$

$$222222322236363632 \begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 1 & 5 & 6 & 19 & 19 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -1 & -1 & 1 & 8 & 11 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{17,24}$ spans $L_{16.13}$

$$22222236322363632 \begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 90 & 90 & 5 & 90 & 90 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 8 & 11 & 1 & 19 & 19 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -11 & -8 & 0 & 8 & 11 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{17,25}$ spans $L_{16.13}$

$$22222236363223632 \begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 9 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 8 & 11 & 5 & 6 & 2 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -11 & -8 & -1 & 1 & 1 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{17,26}$ spans $L_{16.13}$

$$22222236363632232 \begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 & 90 & 5 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -1 & -1 & 1 & 8 & 11 & 5 & 6 & 19 & 19 & 1 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -2 & -6 & -5 & -11 & -8 & -1 & 1 & 8 & 11 & 1 & 19 & 19 \end{bmatrix}$$

$\Pi_{17,27}$ spans $L_{16.13}$

$$22222322236363632 \begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 5 & 9 & 5 & 9 & 5 & 90 & 90 & 5 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 & 90 \\ 0 & -1 & -1 & -2 & -1 & -11 & -8 & 0 & 1 & 5 & 6 & 19 & 19 & 6 & 5 & 11 & 8 \\ 1 & 1 & 0 & -1 & -1 & -19 & -19 & -1 & -1 & 1 & 8 & 11 & 5 & 6 & 19 & 19 \end{bmatrix}$$

$\Pi_{17,6}$ spans $L_{16.13}$

$$363222|22236322\}22 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 9 & 30 & 30 & 90 & 90 & 5 & 90 \\ 2 & 1 & 1 & 1 & -1 & -8 & -11 & -1 & -19 \\ 0 & 1 & 3 & 11 & 11 & 30 & 27 & 1 & 3 \end{bmatrix}$$

$\Pi_{17,8}$ spans $L_{16.13}$

$$\}63222232|23222236 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 30 & 30 & 9 & 5 & 9 & 30 & 30 & 9 \\ 19 & 6 & 5 & 1 & 0 & -1 & -5 & -6 & -2 \\ 3 & 4 & 7 & 3 & 2 & 3 & 7 & 4 & 0 \end{bmatrix}$$

$\Pi_{17,10}$ spans $L_{16.13}$

$$\}63222322|22322236 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 30 & 30 & 9 & 5 & 90 & 90 & 5 & 9 \\ 19 & 6 & 5 & 1 & 0 & -8 & -11 & -1 & -2 \\ 3 & 4 & 7 & 3 & 2 & 30 & 27 & 1 & 0 \end{bmatrix}$$

$\Pi_{17,12}$ spans $L_{16.13}$

$$\}63223222|22232236 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 30 & 30 & 9 & 30 & 30 & 9 & 5 & 9 \\ 19 & 6 & 5 & 1 & 1 & -1 & -1 & -1 & -2 \\ 3 & 4 & 7 & 3 & 11 & 11 & 3 & 1 & 0 \end{bmatrix}$$

$\Pi_{17,14}$ spans $L_{16.13}$

$$36\}63222232|232222 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 90 & 90 & 5 & 9 & 5 & 90 & 90 & 5 \\ -11 & -30 & -27 & -1 & 0 & 1 & 27 & 30 & 2 \\ -1 & -8 & -11 & -1 & -2 & -1 & -11 & -8 & 0 \end{bmatrix}$$

$\Pi_{17,16}$ spans $L_{16.13}$

$$322|223222322\}22322 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 30 & 30 & 9 & 30 & 30 & 9 & 30 \\ 2 & 3 & 7 & 4 & 0 & -4 & -7 & -3 & -11 \\ 0 & 1 & 5 & 6 & 2 & 6 & 5 & 1 & 1 \end{bmatrix}$$

$\Pi_{17,18}$ spans $L_{16.13}$

$$322232|23222322\}22 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 90 & 90 & 5 & 9 & 30 & 30 & 9 & 30 \\ 2 & 30 & 27 & 1 & 0 & -4 & -7 & -3 & -11 \\ 0 & 8 & 11 & 1 & 2 & 6 & 5 & 1 & 1 \end{bmatrix}$$

$\Pi_{17,20}$ spans $L_{16.13}$

$$\}22232232|23222222 \times D_2$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 9 & 5 & 90 & 90 & 5 & 90 & 90 & 5 \\ 11 & 3 & 1 & 3 & -3 & -1 & -27 & -30 & -2 \\ 1 & 1 & 1 & 19 & 19 & 1 & 11 & 8 & 0 \end{bmatrix}$$

$\Pi_{17,22}$ spans $L_{16.13}$

$$322|223222322\}22322 \times D_2$$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 5 & 90 & 90 & 5 & 90 & 90 & 5 & 90 \\ 2 & 1 & 11 & 8 & 0 & -8 & -11 & -1 & -19 \\ 0 & -1 & -27 & -30 & -2 & -30 & -27 & -1 & -3 \end{bmatrix}$$

$\Pi_{20,1}$ spans $L_{216.2}$ $42|24\circ\circ 42|24\circ\circ 42|24\circ\circ 42|24\circ\circ \times D_8$

$$\begin{bmatrix} -4 \\ 9 \\ 9 \end{bmatrix} \begin{bmatrix} 2 & 18 & 9 \\ 1 & 11 & 6 \\ -1 & -5 & -1 \end{bmatrix}$$

 $\Pi_{20,3}$ spans $L_{16.13}$ $3632|23632|23632|23632|2 \times D_4$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 30 & 30 & 90 & 90 & 5 \\ -2 & -6 & -5 & -11 & -8 & 0 \\ 0 & 4 & 7 & 27 & 30 & 2 \end{bmatrix}$$

 $\Pi_{20,5}$ spans $L_{16.13}$ $363222\}2223636\}3636 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 5 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 \\ 19 & 1 & 1 & 1 & -1 & -8 & -11 & -5 & -6 & -19 \\ 3 & 1 & 3 & 11 & 11 & 30 & 27 & 7 & 4 & 3 \end{bmatrix}$$

 $\Pi_{20,7}$ spans $L_{16.13}$ $\}3632223636\}3636\}22236 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 30 & 30 & 9 & 5 & 90 & 90 & 30 & 30 & 90 \\ 19 & 6 & 5 & 1 & 0 & -8 & -11 & -5 & -6 & -19 \\ 3 & 4 & 7 & 3 & 2 & 30 & 27 & 7 & 4 & 3 \end{bmatrix}$$

 $\Pi_{20,9}$ spans $L_{16.13}$ $36322\}22236322\}36322 \times D_2$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 9 & 30 & 30 & 90 & 90 & 5 & 90 & 90 & 30 \\ -11 & -3 & -7 & -4 & -3 & 3 & 1 & 27 & 30 & 11 \\ -1 & -1 & -5 & -6 & -19 & -19 & -1 & -11 & -8 & -1 \end{bmatrix}$$

 $\Pi_{20,11}$ spans $L_{16.13}$ $3632|2363636322|223636 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 & 90 & 5 & 9 \\ -2 & -6 & -5 & -11 & -8 & -1 & 1 & 8 & 11 & 1 & 2 \\ 0 & 4 & 7 & 27 & 30 & 11 & 11 & 30 & 27 & 1 & 0 \end{bmatrix}$$

 $\Pi_{20,13}$ spans $L_{16.7}$ $363632232|2322363632|2 \times D_2$

$$\begin{bmatrix} -5 \\ 3/2 \\ 9/2 \end{bmatrix} \begin{bmatrix} 1 & 18 & 18 & 1 & 18 & 18 & 6 & 6 & 18 & 18 & 1 \\ -2 & -30 & -27 & -1 & -3 & 3 & 4 & 7 & 27 & 30 & 2 \\ 0 & -8 & -11 & -1 & -19 & -19 & -6 & -5 & -11 & -8 & 0 \end{bmatrix}$$

 $\Pi_{20,15}$ spans $L_{251.3}$ $32232|2322322622|22622 \times D_2$

$$\begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} \begin{bmatrix} 3 & 4 & 4 & 3 & 4 & 4 & 3 & 36 & 12 & 4 & 3 \\ -4 & -5 & -4 & -2 & -1 & 1 & 2 & 33 & 13 & 5 & 4 \\ 0 & 1 & 2 & 2 & 3 & 3 & 2 & 19 & 5 & 1 & 0 \end{bmatrix}$$

 $\Pi_{20,17}$ spans $L_{251.3}$ $322\}223226222\}222622 \times D_2$

$$\begin{bmatrix} -5 \\ 9 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 3 & 4 & 4 & 3 & 36 & 12 & 4 & 3 & 4 \\ -3 & -2 & -2 & -1 & 0 & 7 & 4 & 2 & 2 & 3 \\ 1 & 2 & 4 & 5 & 4 & 45 & 14 & 4 & 2 & 1 \end{bmatrix}$$

 $\Pi_{20,19}$ spans $L_{16.13}$ $36322|2236363632|23636 \times D_2$ $\Pi_{20,20}$ spans $L_{16.9}$ $3632232|2322363632|236 \times D_2$ $\Pi_{20,21}$ spans $L_{16.13}$ $\}36363222\}2223636\}36 \times D_2$ $\Pi_{20,22}$ spans $L_{16.13}$ $36322236363632223636 \times C_2$

$$\begin{bmatrix} -1 \\ 30 & -15 \\ -15 & 30 \end{bmatrix} \begin{bmatrix} 90 & 90 & 30 & 30 & 9 & 5 & 90 & 90 & 30 & 30 \\ 8 & 11 & 5 & 6 & 2 & 1 & 11 & 8 & 1 & -1 \\ 19 & 19 & 6 & 5 & 1 & 0 & -8 & -11 & -5 & -6 \end{bmatrix}$$

 $\Pi_{20,2}$ spans $L_{16.9}$ $\}6322\}2236\}6322\}2236 \times D_4$

$$\begin{bmatrix} -3 \\ 15/2 \\ 5/2 \end{bmatrix} \begin{bmatrix} 30 & 10 & 10 & 3 & 10 \\ -19 & -6 & -5 & -1 & -1 \\ 3 & 4 & 7 & 3 & 11 \end{bmatrix}$$

 $\Pi_{20,4}$ spans $L_{16.7}$ $\}36\}6322\}2236\}6322\}22 \times D_4$

$$\begin{bmatrix} -5 \\ 3/2 \\ 9/2 \end{bmatrix} \begin{bmatrix} 6 & 18 & 18 & 1 & 18 \\ 11 & 30 & 27 & 1 & 3 \\ -1 & -8 & -11 & -1 & -19 \end{bmatrix}$$

 $\Pi_{20,6}$ spans $L_{16.13}$ $36322236\}632223636\}6 \times D_2$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 90 & 90 & 5 & 9 & 30 & 30 & 90 & 90 & 30 \\ 11 & 30 & 27 & 1 & 0 & -4 & -7 & -27 & -30 & -11 \\ -1 & -8 & -11 & -1 & -2 & -6 & -5 & -11 & -8 & -1 \end{bmatrix}$$

 $\Pi_{20,8}$ spans $L_{16.9}$ $\}36322322\}223223636\}6 \times D_2$

$$\begin{bmatrix} -3 \\ 5/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 10 & 3 & 10 & 10 & 3 & 10 & 10 & 30 & 30 & 10 \\ -11 & -3 & -7 & -4 & 0 & 4 & 7 & 27 & 30 & 11 \\ 1 & 1 & 5 & 6 & 2 & 6 & 5 & 11 & 8 & 1 \end{bmatrix}$$

 $\Pi_{20,10}$ spans $L_{16.13}$ $\}632236322\}223632236 \times D_2$

$$\begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 90 & 30 & 30 & 9 & 30 & 30 & 90 & 90 & 5 & 90 \\ 19 & 6 & 5 & 1 & 1 & -1 & -8 & -11 & -1 & -19 \\ 3 & 4 & 7 & 3 & 11 & 11 & 30 & 27 & 1 & 3 \end{bmatrix}$$

 $\Pi_{20,12}$ spans $L_{16.7}$ $\}636322322\}223223636 \times D_2$

$$\begin{bmatrix} -5 \\ 9/2 \\ 3/2 \end{bmatrix} \begin{bmatrix} 18 & 6 & 6 & 18 & 18 & 1 & 18 & 1 & 18 \\ 19 & 6 & 5 & 11 & 8 & 0 & -8 & -11 & -1 & -19 \\ 3 & 4 & 7 & 27 & 30 & 2 & 30 & 27 & 1 & 3 \end{bmatrix}$$

 $\Pi_{20,14}$ spans $L_{251.3}$ $32232|2322322622|26222 \times D_2$

$$\begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} \begin{bmatrix} 3 & 4 & 4 & 3 & 4 & 4 & 3 & 4 & 12 & 36 & 3 \\ -4 & -5 & -4 & -2 & -1 & 1 & 2 & 4 & 14 & 45 & 4 \\ 0 & 1 & 2 & 2 & 3 & 3 & 2 & 2 & 4 & 7 & 0 \end{bmatrix}$$

 $\Pi_{20,16}$ spans $L_{251.3}$ $\}322\}223226222\}226222 \times D_2$

$$\begin{bmatrix} -5 \\ 9 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 3 & 4 & 4 & 3 & 4 & 12 & 36 & 3 & 4 \\ -3 & -2 & -2 & -1 & 0 & 1 & 5 & 19 & 2 & 3 \\ 1 & 2 & 4 & 5 & 4 & 5 & 13 & 33 & 2 & 1 \end{bmatrix}$$

 $\Pi_{20,18}$ spans $L_{251.3}$ $32|2322262232|23226222 \times D_2$

$$\begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} \begin{bmatrix} 3 & 4 & 4 & 3 & 4 & 12 & 36 & 3 & 4 & 4 & 3 \\ -4 & -5 & -4 & -2 & -1 & 1 & 12 & 2 & 4 & 5 & 4 \\ 0 & -1 & -2 & -2 & -3 & -9 & -26 & -2 & -2 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 & 9 & 30 & 30 & 90 & 90 & 30 & 30 & 90 & 90 & 5 \\ 2 & 3 & 7 & 4 & 3 & -3 & -4 & -7 & -27 & -30 & -2 \\ 0 & -1 & -5 & -6 & -19 & -19 & -6 & -5 & -11 & -8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 15/2 \\ 5/2 \end{bmatrix} \begin{bmatrix} 3 & 10 & 10 & 3 & 10 & 10 & 30 & 30 & 10 & 10 & 3 \\ -2 & -6 & -5 & -1 & -1 & 1 & 8 & 11 & 5 & 6 & 2 \\ 0 & -4 & -7 & -3 & -11 & -11 & -30 & -27 & -7 & -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 & 9 & 5 & 90 & 90 & 30 & 30 & 90 & 90 & 30 \\ 11 & 3 & 1 & 3 & -3 & -4 & -7 & -27 & -30 & -11 \\ -1 & -1 & -1 & -19 & -19 & -6 & -5 & -11 & -8 & -1 \end{bmatrix}$$

 $\Pi_{20,23}$ spans $L_{251.3}$ $32232226223223222622 \times C_2$

$$\begin{bmatrix} -5 \\ 12 & -6 \\ -6 & 12 \end{bmatrix} \begin{bmatrix} 4 & 4 & 3 & 4 & 4 & 3 & 4 & 12 & 36 & 3 \\ -1 & -2 & -2 & -3 & -3 & -2 & -2 & -4 & -7 & 0 \\ -3 & -3 & -2 & -2 & -1 & 0 & 1 & 5 & 19 & 2 \end{bmatrix}$$

$$\begin{array}{l}
 \Pi_{20,56} \text{ spans } L_{16.13} \qquad \qquad \qquad 36363222232236363636 \\
 \begin{bmatrix} -1 \\ 30 \ -15 \\ -15 \ 30 \end{bmatrix} \begin{bmatrix} 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 5 \ 9 \ 5 \ 90 \ 90 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \\ 8 \ 11 \ 5 \ 6 \ 19 \ 19 \ 1 \ 1 \ 0 \ -8 \ -11 \ -1 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \\ 19 \ 19 \ 6 \ 5 \ 11 \ 8 \ 0 \ -1 \ -1 \ -19 \ -19 \ -1 \ -11 \ -8 \ -1 \ 1 \ 8 \ 11 \ 5 \ 6 \end{bmatrix} \\
 \Pi_{20,57} \text{ spans } L_{16.13} \qquad \qquad \qquad 36363222236363223636 \\
 \begin{bmatrix} -1 \\ 30 \ -15 \\ -15 \ 30 \end{bmatrix} \begin{bmatrix} 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 5 \ 9 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 5 \ 90 \ 90 \ 30 \ 30 \\ 8 \ 11 \ 5 \ 6 \ 19 \ 19 \ 1 \ 1 \ 0 \ -8 \ -11 \ -5 \ -6 \ -19 \ -19 \ -1 \ -11 \ -8 \ -1 \ 1 \\ 19 \ 19 \ 6 \ 5 \ 11 \ 8 \ 0 \ -1 \ -1 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ 0 \ 8 \ 11 \ 5 \ 6 \end{bmatrix} \\
 \Pi_{20,58} \text{ spans } L_{251.3} \qquad \qquad \qquad 322322322226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 4 \ 3 \ 4 \ 4 \ 3 \ 4 \ 4 \ 3 \ 4 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -3 \ -2 \ -2 \ -1 \ 0 \ 1 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -1 \ 0 \ 1 \ 2 \ 2 \ 3 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{20,59} \text{ spans } L_{251.3} \qquad \qquad \qquad 32232232226223222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 4 \ 3 \ 4 \ 4 \ 3 \ 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -3 \ -2 \ -2 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -1 \ 0 \ 1 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \hline
 \Pi_{21,1} \text{ spans } L_{16.9} \qquad \qquad \qquad \Pi_{21,2} \text{ spans } L_{16.7} \\
 \mathfrak{3632|236\mathfrak{3632|236\mathfrak{3632|236} \rtimes D_6} \qquad \qquad \qquad 36\mathfrak{3632|236\mathfrak{3632|236\mathfrak{3632|2} \rtimes D_6} \\
 \begin{bmatrix} -3 \\ 5 \\ 5 \\ 5 \end{bmatrix} \begin{bmatrix} 30 \ 10 \ 10 \ 3 \\ 19 \ 6 \ 5 \ 1 \\ -8 \ -1 \ 1 \ 1 \\ -11 \ -5 \ -6 \ -2 \end{bmatrix} \qquad \qquad \qquad \begin{bmatrix} -5 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 6 \ 18 \ 18 \ 1 \\ -11 \ -30 \ -27 \ -1 \\ 4 \ 3 \ -3 \ -1 \\ 7 \ 27 \ 30 \ 2 \end{bmatrix} \\
 \Pi_{21,3} \text{ spans } L_{251.3} \\
 322262232226223222622 \rtimes C_3 \\
 \begin{bmatrix} -5 \\ 6 \\ 6 \\ 6 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ 3 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -2 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \end{bmatrix} \\
 \Pi_{21,4} \text{ spans } L_{16.13} \qquad \qquad \qquad 36322|2236363636\mathfrak{3636} \rtimes D_2 \\
 \begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 \ 9 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \\ 2 \ 3 \ 7 \ 4 \ 3 \ -3 \ -4 \ -7 \ -27 \ -30 \ -11 \\ 0 \ -1 \ -5 \ -6 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \end{bmatrix} \\
 \Pi_{21,5} \text{ spans } L_{16.9} \qquad \qquad \qquad 3632232|2322363636\mathfrak{3636} \rtimes D_2 \\
 \begin{bmatrix} -3 \\ 15/2 \\ 5/2 \end{bmatrix} \begin{bmatrix} 3 \ 10 \ 10 \ 3 \ 10 \ 10 \ 30 \ 30 \ 10 \ 10 \ 30 \\ -2 \ -6 \ -5 \ -1 \ -1 \ 1 \ 8 \ 11 \ 5 \ 6 \ 19 \\ 0 \ -4 \ -7 \ -3 \ -11 \ -11 \ -30 \ -27 \ -7 \ -4 \ -3 \end{bmatrix} \\
 \Pi_{21,6} \text{ spans } L_{16.13} \qquad \qquad \qquad 36322\mathfrak{32236363632|23636} \rtimes D_2 \\
 \begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 30 \ 9 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 5 \\ 11 \ 3 \ 7 \ 4 \ 3 \ -3 \ -4 \ -7 \ -27 \ -30 \ -2 \\ -1 \ -1 \ -5 \ -6 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ 0 \end{bmatrix} \\
 \Pi_{21,7} \text{ spans } L_{16.13} \qquad \qquad \qquad 363223632|2363223636\mathfrak{36} \rtimes D_2 \\
 \begin{bmatrix} -1 \\ 15/2 \\ 45/2 \end{bmatrix} \begin{bmatrix} 5 \ 90 \ 90 \ 30 \ 30 \ 9 \ 30 \ 30 \ 90 \ 90 \ 30 \\ 2 \ 30 \ 27 \ 7 \ 4 \ 0 \ -4 \ -7 \ -27 \ -30 \ -11 \\ 0 \ -8 \ -11 \ -5 \ -6 \ -2 \ -6 \ -5 \ -11 \ -8 \ -1 \end{bmatrix} \\
 \Pi_{21,8} \text{ spans } L_{16.13} \qquad \qquad \qquad \Pi_{21,9} \text{ spans } L_{16.13} \\
 3632|2363223636\mathfrak{3636322} \rtimes D_2 \qquad \qquad \qquad 3632|2363636322\mathfrak{3223636} \rtimes D_2 \\
 \begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 \ 30 \ 30 \ 90 \ 90 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \\ -2 \ -6 \ -5 \ -11 \ -8 \ 0 \ 8 \ 11 \ 5 \ 6 \ 19 \\ 0 \ 4 \ 7 \ 27 \ 30 \ 2 \ 30 \ 27 \ 7 \ 4 \ 3 \end{bmatrix} \qquad \qquad \qquad \begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 5 \ 90 \\ -2 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \ 8 \ 11 \ 1 \ 19 \\ 0 \ 4 \ 7 \ 27 \ 30 \ 11 \ 11 \ 30 \ 27 \ 1 \ 3 \end{bmatrix} \\
 \Pi_{21,10} \text{ spans } L_{16.13} \qquad \qquad \qquad \Pi_{21,11} \text{ spans } L_{16.7} \\
 3636322|2236363636\mathfrak{3636} \rtimes D_2 \qquad \qquad \qquad 363632232|2322363636\mathfrak{36} \rtimes D_2 \\
 \begin{bmatrix} -1 \\ 45/2 \\ 15/2 \end{bmatrix} \begin{bmatrix} 9 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \ 90 \\ 2 \ 1 \ 11 \ 8 \ 1 \ -1 \ -8 \ -11 \ -5 \ -6 \ -19 \\ 0 \ 1 \ 27 \ 30 \ 11 \ 11 \ 30 \ 27 \ 7 \ 4 \ 3 \end{bmatrix} \qquad \qquad \qquad \begin{bmatrix} -5 \\ 3/2 \\ 9/2 \end{bmatrix} \begin{bmatrix} 1 \ 18 \ 18 \ 1 \ 18 \ 18 \ 6 \ 6 \ 18 \ 18 \ 6 \\ 2 \ 30 \ 27 \ 1 \ 3 \ -3 \ -4 \ -7 \ -27 \ -30 \ -11 \\ 0 \ 8 \ 11 \ 1 \ 19 \ 19 \ 6 \ 5 \ 11 \ 8 \ 1 \end{bmatrix} \\
 \Pi_{21,12} \text{ spans } L_{16.13} \qquad \qquad \qquad 363222322363636363636 \\
 \begin{bmatrix} -1 \\ 30 \ -15 \\ -15 \ 30 \end{bmatrix} \begin{bmatrix} 90 \ 90 \ 30 \ 30 \ 9 \ 5 \ 90 \ 90 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \\ 8 \ 11 \ 5 \ 6 \ 2 \ 1 \ 11 \ 8 \ 0 \ -8 \ -11 \ -5 \ -6 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \\ 19 \ 19 \ 6 \ 5 \ 1 \ 0 \ -8 \ -11 \ -1 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \ 8 \ 11 \ 5 \ 6 \end{bmatrix} \\
 \Pi_{21,13} \text{ spans } L_{16.13} \qquad \qquad \qquad 36322363223636363636 \\
 \begin{bmatrix} -1 \\ 30 \ -15 \\ -15 \ 30 \end{bmatrix} \begin{bmatrix} 90 \ 90 \ 30 \ 30 \ 9 \ 5 \ 90 \ 90 \ 30 \ 30 \ 9 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \\ 8 \ 11 \ 5 \ 6 \ 2 \ 1 \ 11 \ 8 \ 1 \ -1 \ -1 \ -5 \ -6 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \\ 19 \ 19 \ 6 \ 5 \ 1 \ 0 \ -8 \ -11 \ -5 \ -6 \ -2 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \ 8 \ 11 \ 5 \ 6 \end{bmatrix} \\
 \Pi_{21,14} \text{ spans } L_{16.13} \qquad \qquad \qquad 363222363632236363636 \\
 \begin{bmatrix} -1 \\ 30 \ -15 \\ -15 \ 30 \end{bmatrix} \begin{bmatrix} 90 \ 90 \ 30 \ 30 \ 9 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 5 \ 90 \ 90 \ 30 \ 30 \ 90 \ 90 \ 30 \ 30 \\ 8 \ 11 \ 5 \ 6 \ 2 \ 1 \ 11 \ 8 \ 1 \ -1 \ -8 \ -11 \ -1 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \ 1 \\ 19 \ 19 \ 6 \ 5 \ 1 \ 0 \ -8 \ -11 \ -5 \ -6 \ -19 \ -19 \ -1 \ -11 \ -8 \ -1 \ 1 \ 8 \ 11 \ 5 \ 6 \end{bmatrix}
 \end{array}$$

$$\begin{array}{l}
 \Pi_{22,30} \text{ spans } L_{251.3} \quad 3222622322622226226222 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -1 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix} \\
 \Pi_{22,31} \text{ spans } L_{251.3} \quad 3222622322622226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -1 \ 0 \ 7 \ 4 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{22,32} \text{ spans } L_{251.3} \quad 3222622322622226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -1 \ 0 \ 7 \ 4 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix} \\
 \Pi_{22,33} \text{ spans } L_{251.3} \quad 3222622262232226226222 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix} \\
 \Pi_{22,34} \text{ spans } L_{251.3} \quad 3222622262232262222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \hline
 \Pi_{23,1} \text{ spans } L_{16.9} \quad 3632|236363636363636363636 \rtimes D_2 \quad \begin{bmatrix} -3 \\ 15/2 \\ 5/2 \end{bmatrix} \begin{bmatrix} 3 \ 10 \ 10 \ 30 \ 30 \ 10 \ 10 \ 30 \ 30 \ 10 \ 10 \ 30 \\ 2 \ 6 \ 5 \ 11 \ 8 \ 1 \ -1 \ -8 \ -11 \ -5 \ -6 \ -19 \\ 0 \ -4 \ -7 \ -27 \ -30 \ -11 \ -11 \ -30 \ -27 \ -7 \ -4 \ -3 \end{bmatrix} \\
 \Pi_{23,2} \text{ spans } L_{16.7} \quad 363632|23636363636363636 \rtimes D_2 \quad \begin{bmatrix} -5 \\ 3/2 \\ 9/2 \end{bmatrix} \begin{bmatrix} 1 \ 18 \ 18 \ 6 \ 6 \ 18 \ 18 \ 6 \ 6 \ 18 \ 18 \ 6 \\ -2 \ -30 \ -27 \ -7 \ -4 \ -3 \ 3 \ 4 \ 7 \ 27 \ 30 \ 11 \\ 0 \ -8 \ -11 \ -5 \ -6 \ -19 \ -19 \ -6 \ -5 \ -11 \ -8 \ -1 \end{bmatrix} \\
 \Pi_{23,3} \text{ spans } L_{251.3} \quad 32226222622262226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{23,4} \text{ spans } L_{251.3} \quad 32226222622262226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix} \\
 \Pi_{23,5} \text{ spans } L_{251.3} \quad 32226222622262226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{23,6} \text{ spans } L_{251.3} \quad 32226222622262226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix} \\
 \Pi_{23,7} \text{ spans } L_{251.3} \quad 32226222622622226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{23,8} \text{ spans } L_{251.3} \quad 32226222622622226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix} \\
 \Pi_{23,9} \text{ spans } L_{251.3} \quad 32226222622622226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{23,10} \text{ spans } L_{251.3} \quad 32226226222262226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\
 \Pi_{23,11} \text{ spans } L_{251.3} \quad 32226226222262226222622 \\
 \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -7 \ -4 \ -2 \ -2 \end{bmatrix}
 \end{array}$$

$$\begin{array}{ll} \Pi_{23,12} \text{ spans } L_{251.3} & 322262262222622262222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \Pi_{23,13} \text{ spans } L_{251.3} & 322262262222622226222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -19 \ -5 \ -1 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \Pi_{23,14} \text{ spans } L_{251.3} & 322262262222622226222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -3 \ -9 \ -26 \ -2 \ -19 \ -5 \ -1 \ 0 \ 7 \ 4 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 26 \ 9 \ 3 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \Pi_{23,15} \text{ spans } L_{251.3} & 322622226222622262226222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -26 \ -9 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \Pi_{23,16} \text{ spans } L_{251.3} & 322622226222622262226222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -26 \ -9 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \Pi_{23,17} \text{ spans } L_{251.3} & 322622226222622262226222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -26 \ -9 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \Pi_{23,18} \text{ spans } L_{251.3} & 322622226222622262226222622 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 4 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -1 \ -2 \ -2 \ -26 \ -9 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \\ -3 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \ 7 \ 4 \ 2 \ 2 \ 3 \ 9 \ 26 \ 2 \ 2 \ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \end{bmatrix} \\ \hline \Pi_{24,1} \text{ spans } L_{16.8} & \Pi_{24,2} \text{ spans } L_{251.3} \\ 36\ 36\ 36\ 36\ 36\ 36\ 36\ 36\ 36\ 36 \times D_{12} & 62|2622|2262|2622|2262|2622|22 \times D_6 \\ \begin{bmatrix} -15 \\ 1 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 6 \ 2 \\ 19 \ 6 \\ -8 \ -1 \\ -11 \ -5 \end{bmatrix} \\ \begin{bmatrix} 6 \ 2 \\ 19 \ 6 \\ -8 \ -1 \\ -11 \ -5 \end{bmatrix} & \begin{bmatrix} -5 \\ 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \ 36 \ 12 \ 4 \ 3 \\ -4 \ -45 \ -14 \ -4 \ -2 \\ 2 \ 12 \ 1 \ -1 \ -2 \\ 2 \ 33 \ 13 \ 5 \ 4 \end{bmatrix} \\ \Pi_{24,3} \text{ spans } L_{251.3} & \Pi_{24,4} \text{ spans } L_{251.3} \\ 6222622262226222622262222 \times C_6 & 6222622262|262226222622|22 \times D_2 \\ \begin{bmatrix} -5 \\ 6 \\ 6 \\ 6 \end{bmatrix} & \begin{bmatrix} 12 \ 36 \ 3 \ 4 \\ -5 \ -19 \ -2 \ -3 \\ 9 \ 26 \ 2 \ 2 \\ -4 \ -7 \ 0 \ 1 \end{bmatrix} \\ \begin{bmatrix} -5 \\ 6 \\ 6 \\ 6 \end{bmatrix} & \begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} \begin{bmatrix} 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \\ 4 \ 45 \ 14 \ 4 \ 2 \ 12 \ 1 \ -1 \ -2 \ -33 \ -13 \ -5 \ -4 \\ 0 \ -7 \ -4 \ -2 \ -2 \ -26 \ -9 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \end{bmatrix} \\ \Pi_{24,5} \text{ spans } L_{251.3} & \Pi_{24,6} \text{ spans } L_{251.3} \\ 6222622622|226226222622|22 \times D_2 & 622262|262226222262|262222 \times D_2 \\ \begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} & \begin{bmatrix} 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \\ 4 \ 5 \ 13 \ 33 \ 2 \ 12 \ 1 \ -1 \ -2 \ -33 \ -13 \ -5 \ -4 \\ 0 \ -1 \ -5 \ -19 \ -2 \ -26 \ -9 \ -3 \ -2 \ -19 \ -5 \ -1 \ 0 \end{bmatrix} \\ \begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} & \begin{bmatrix} -5 \\ 3 \\ 9 \end{bmatrix} \begin{bmatrix} 3 \ 36 \ 12 \ 4 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \ 12 \ 36 \ 3 \\ -4 \ -45 \ -14 \ -4 \ -2 \ -12 \ -1 \ 1 \ 2 \ 4 \ 14 \ 45 \ 4 \\ 0 \ -7 \ -4 \ -2 \ -2 \ -26 \ -9 \ -3 \ -2 \ -2 \ -4 \ -7 \ 0 \end{bmatrix} \\ \Pi_{24,7} \text{ spans } L_{251.3} & \\ 6222622622226222622222 \times C_2 & \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \\ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \ -26 \ -9 \ -3 \ -2 \ -2 \\ -5 \ -19 \ -2 \ -3 \ -9 \ -26 \ -2 \ -19 \ -5 \ -1 \ 0 \ 1 \end{bmatrix} \\ \Pi_{24,8} \text{ spans } L_{251.3} & 6222622262226222622262222 \\ \begin{bmatrix} -5 \\ 12 \ -6 \\ -6 \ 12 \end{bmatrix} & \begin{bmatrix} 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 4 \ 12 \ 36 \ 3 \ 36 \ 12 \ 4 \ 3 \ 4 \\ 4 \ 7 \ 0 \ -1 \ -5 \ -19 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 26 \ 9 \ 3 \ 2 \ 2 \\ -5 \ -19 \ -2 \ -3 \ -9 \ -26 \ -2 \ -2 \ -4 \ -7 \ 0 \ 1 \ 5 \ 19 \ 2 \ 3 \ 9 \ 26 \ 2 \ 19 \ 5 \ 1 \ 0 \ -1 \end{bmatrix} \end{array}$$

$$\begin{array}{l}
 \Pi_{24,9} \text{ spans } L_{251.3} \qquad \qquad \qquad 6222622262226222622222 \\
 \begin{bmatrix} -5 & & \\ 12 & -6 & \\ -6 & 12 & \end{bmatrix} \begin{bmatrix} 12 & 36 & 3 & 4 & 12 & 36 & 3 & 4 & 12 & 36 & 3 & 4 & 12 & 36 & 3 & 36 & 12 & 4 & 3 & 36 & 12 & 4 & 3 & 4 \\ 4 & 7 & 0 & -1 & -5 & -19 & -2 & -3 & -9 & -26 & -2 & -2 & -4 & -7 & 0 & 7 & 4 & 2 & 2 & 26 & 9 & 3 & 2 & 2 \\ -5 & -19 & -2 & -3 & -9 & -26 & -2 & -2 & -4 & -7 & 0 & 1 & 5 & 19 & 2 & 26 & 9 & 3 & 2 & 19 & 5 & 1 & 0 & -1 \end{bmatrix} \\
 \Pi_{24,10} \text{ spans } L_{251.3} \qquad \qquad \qquad 6222622262226222622222 \\
 \begin{bmatrix} -5 & & \\ 12 & -6 & \\ -6 & 12 & \end{bmatrix} \begin{bmatrix} 12 & 36 & 3 & 4 & 12 & 36 & 3 & 4 & 12 & 36 & 3 & 36 & 12 & 4 & 3 & 4 & 12 & 36 & 3 & 36 & 12 & 4 & 3 & 4 \\ 4 & 7 & 0 & -1 & -5 & -19 & -2 & -3 & -9 & -26 & -2 & -19 & -5 & -1 & 0 & 1 & 5 & 19 & 2 & 26 & 9 & 3 & 2 & 2 \\ -5 & -19 & -2 & -3 & -9 & -26 & -2 & -2 & -4 & -7 & 0 & 7 & 4 & 2 & 2 & 3 & 9 & 26 & 2 & 19 & 5 & 1 & 0 & -1 \end{bmatrix}
 \end{array}$$

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