

M. Finkelberg - Uhlenbeck Spaces  
 $C$  curve  $K = \text{function on } C$   $A = \text{adibos}$

10/2 1/07

$T$  split tors

$$F(G_G \backslash G_A / G_K) = F(\text{Bun}_G(C))$$

Inducter from tors:

$$F_C(T_G \backslash T_A / T_K) \xrightarrow{\text{Eis}} F(G_G \backslash G_A / G_K)$$

Defined by  $G_G \backslash G_A / G_K \xrightarrow{\pi} B_G \backslash B_A / B_K \xrightarrow{\rho} T_G \backslash T_A / T_K$

$$E_D = \pi, \rho^*$$

$$\text{Bun}_G C \xleftarrow{\pi} \text{Bun}_B C \xrightarrow{\rho} \text{Bun}_T C$$

$\pi$  not proper  $\Rightarrow$  Drinfeld proposed a relative compactification

$$\pi: \text{Bun}_B \rightarrow \text{Bun}_G$$

Modified Eisenstein series, defined using  $\pi$ , satisfy simple functional equation, w/o L-funct-ns

$$\text{Plicker}: G/B \hookrightarrow \prod_{\lambda \in \Lambda_G^+} \mathbb{P}(V_\lambda^*)$$

Point of  $\text{Bun}_G \iff G$ -bundle + line in associated  $V_\lambda^*$  bundle, for all  $\lambda \quad L_\lambda = FV_\lambda^*$

Points of  $\overline{\text{Bun}}_G(C) : F G$ -bundle on  $C \triangleleft L_\lambda \subset FV_\lambda^*$  invertible subsheaf -- i.e. allow zeros.

$$G/P \hookrightarrow \prod_{\lambda \in \Lambda_G^+} \mathbb{P}(V_\lambda^*) \quad \lambda \in \text{well associated to } P$$

$$\Rightarrow \overline{\text{Bun}}_P(C) \xrightarrow{\pi} \overline{\text{Bun}}_G(C)$$

If  $C = \mathbb{P}^1 \Rightarrow \text{Bun}_G(C)$  contains an open part formed by trivial  $G$ -bundles. Fiber of  $\pi$  over this is

$$\text{maps } C \rightarrow G/P$$

Fiber of  $\pi$  over trivial bundle is quasi-singular space  
 $\text{Maps}(C, G/P) \subset \text{Maps}(C, G/P)$

Components are numbered by degree  $[C] \in H_2(G/P, \mathbb{Z}) = \check{N}_{G,P}^v$   
 cocharacters of  $M/[M, M]$   $M = \text{Lie}$   
 ... only get  $\check{N}_{G,P}^+$  positive cochars.

Stratification:  $Q\text{Maps}^\alpha(C, G/P) = \coprod_{\beta \leq \alpha} \text{Maps}^\beta(C, G/P) \times C^{\alpha-\beta}$   
 ... introduce zeros along  $\check{N}_{G,P}^+$ -colored divisors

Ex =  $\pi, q^*$  ... need to study geometry of  $\overline{Bun}_P$  or  $Q\text{Maps}(C, G/P)$   
 $\Rightarrow$  consider "reduced version", based questions

$Q\text{Maps}(C, \infty; G/P, [P] \in G/P)$  locally base  $\times G/P$ ...

1. Factorization property:  $Q\text{Maps}(C, \infty; G/P, P) \supset \text{Maps}^\alpha(C, \infty; G/P, P)$

Look at torus Bruhat cells  $B_- \mid G/P$   $\eta^* \rightarrow (C, \infty) \xleftarrow{q^*}$

$\Rightarrow$  - Bruhat divisors,

$\eta^*$  takes a map to the inverse image of the Bruhat divisor.  
 [depends on choice of  $B_-$ ]

Factorization describes fibers of  $\eta$ :

if  $D^\alpha \in (C, \infty)^\alpha$  is a disjoint union  $\mathbb{L}^B \sqcup \mathbb{L}^\alpha$

$$\Rightarrow (\eta^\alpha)^{-1}(D^\alpha) = (\eta^B)^{-1}(\mathbb{L}^B) \times (\eta^\alpha)^{-1}(D^\alpha)$$

2.  $(\eta^\alpha)^{-1}(\alpha \cdot c) \hookrightarrow \text{Gr}_c = G((z)) / G[[z]]$   $c \in C$

"  
 intersection of semi-infinite orbits

$$\text{Radical}(P^-)((z)) \cdot \bigcup_{\alpha} \{ \eta(P((z)) \cdot \alpha \}^0 \text{ compact}$$

$$\text{[M, M]}((z)) \cdot \text{Rad } P((z)) \cdot \alpha$$

If  $\mathfrak{g}$  is a symmetrizable  $K$ - $M$  Lie dial  $\Rightarrow$

Kashiwara constructed (for any parabolic  $\hat{p} \subset \mathfrak{g}$ )

a partial flag space  $\hat{G}/\hat{P}$ , scheme of infinite type

$X = \mathbb{P}^1$   
 $+ \infty$   
 $\mathbb{C} \cong \mathbb{P}^1 \setminus \infty$

Maps  $^d (C, \infty; \hat{G}/\hat{P}, \hat{P})$  is a finite-dim algebraic variety.

Take  $\hat{G}$  affine Lie algebra  $\mathfrak{g}((t^{-1})) \oplus \mathfrak{K} \oplus \mathbb{C}d$

$\hat{P} = G[t]$ ,  $\hat{G}/\hat{P} = G((t^{-1}))/G[t] = G$ -bundles on  $X$   
 $\mathbb{G}_x$  // Grassmannian slices + triv at  $\infty$

$\text{actN Maps}^a (C, c, \mathbb{G}_x, \hat{P}) = \left. \begin{array}{l} G = G_2 = \text{SU}(2) \\ G\text{-bundles on } (C \times X \text{ + trivialization}) \\ \text{at } \infty \times X \cup C \times \infty : \text{ for almost all } c \in C \\ \text{Bundle along fibers is canonically trivialized} \end{array} \right\} \text{Bun}_G^a(\mathbb{A}^2)$

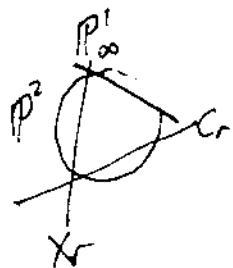
$\Rightarrow$  divisor  $\eta(F) \in \mathbb{C}^{\times}$  where  $F|_{C \times X}$  is normal.

See along  $C$ : get formal trivializations along both divisors but don't agree.

$\mathbb{Q} \text{ Maps}^a (C, \infty; \mathbb{G}_x, \hat{P}) = \bigsqcup_{b \leq a} \text{Bun}_G^b(\mathbb{A}^2) \times (C, \infty)^{a-b}$

( $\mathbb{P}^2$  rel  $\mathbb{P}^1$  picture)

Uhlenbeck completion  $U^a = \text{Bun}_G^a(\mathbb{A}^2)$  given by Drinfeld  
 compactification in the relative case



$\mathbb{P} =$  space of pairs of transversal lines  $(X, C)$   
 in  $\mathbb{P}^2$

$\Rightarrow$  ~~local~~ topological the families  $E \rightarrow \mathbb{P}^2$

Construct relative version  $\mathbb{Q} \text{ Maps}^a (C, \infty; \mathbb{G}_x, \hat{P}) \hookrightarrow \text{Bun}_G^a(\mathbb{A}^2)$

$U^a :=$  closure of  $\text{Bun}_G^a(\mathbb{A}^2)$  in the product

$\mathbb{Q} \text{ Maps}^a (C, \infty; \mathbb{G}_x, \hat{P}) \times \text{Set}(R, (C, \infty)^{(a)})$

$\downarrow \eta$   
 $\text{Set}(R, (C, \infty)^{(a)})$

$U^a = \bigsqcup_{b \leq a} \text{Bun}_G^b(\mathbb{A}^2) \times \text{Sym}^{a-b}(\mathbb{A}^2)$

finite type version