MATH 380, PROBLEM SET 1

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1. PROBLEMS

(1) From the text: 1.6.18, 1.7.23, 2.1.7, 2.2.12, 2.4.19, 2.5.13, 3.1.14, 3.1.15.
(2) Describe the Cayley graphs associated to the standard generating set for D_n.
(3) Prove the "orbit-stabilizer" theorem, which says that given a finite group G acting on a finite set X, the equation

$$|G_x| = \frac{|G|}{\mathrm{Stab}_G(x)}$$

holds for all $x \in X$, where G_x is the orbit of x and $\mathrm{Stab}_G(x)$ is the set of elements of $g \in G$ that fix x.
(4) A group action is transitive if there is only a single orbit. Prove that if a finite group G acts transitively on a finite set S such that $|S| \geq 2$, then there exists $g \in G$ such that there are no elements of S fixed by g.