MATH 380, PROBLEM SET 4

ANDREW J. BLUMBERG

1. Problems

You are in a maze of twisty little passages, all like. If you go north, please do the first set of problems below and the first item from the second list. If you go east, please do the second set of problems and problems 15, 16, and 17 from Atiyah-Macdonald.

(But really, you should do all the problems.)

The beginnings of algebraic geometry.

(1) Obtain the a copy of Atiyah-Macdonald. From the first chapter, do problems 15, 16, 17, 18, 27, and 28.

Profinite phenomena.

(1) From the text, 7.6.10 and 7.6.11.
(2) Prove that (if we give Z/p the discrete topology), the ring Z_p can be topologized via the inverse limit and is compact and totally disconnected.
(3) Prove that $\mathbb{Z}$ is dense in $\mathbb{Z}_p$.
(4) Consider the system of rings $\{\mathbb{Z}/n\}$ where the structure maps are given by projection $\mathbb{Z}/n \to \mathbb{Z}/m$ when $m \mid n$. Describe the inverse limit of this system, which is called $\widehat{\mathbb{Z}}$.
(5) Prove that $\widehat{\mathbb{Z}}$ is compact and totally disconnected, with $\mathbb{Z}$ as a dense subset.
(6) Relate $\mathbb{Z}_p$ and $\widehat{\mathbb{Z}}$.