

QUATERNIONIC GEOMETRY AND DUALITIES IN MATHEMATICS AND PHYSICS

PROJECT 1. Mirror symmetry and Langlands duality in the non-Abelian Hodge theory of a curve
(with Thaddeus and Rodriguez-Villegas)

PROJECT 2. L^2 cohomology of manifolds with fibered boundary (with Hunsicker and Mazzeo)

PROJECT 3. Yang-Mills instantons on ALF gravitational instantons (with Etesi)

PROJECT 4. Toric hyperkähler varieties (with Sturmfels and Swartz)

PROJECT 5. Integration theory on circle compact hyperkähler manifolds (with Szenes and Proudfoot)

ZOO

DIMENSION	d=4	d=3	d=2
PDE	$F_A = *F_A$ self-dual Yang-Mills equations	$F_A = *d_A\phi$ Bogomolny equations	$F_A = -[\Phi, \Phi^*]$, $d_A''\Phi = 0$ Hitchin's self-duality equations
MODULI SPACES	$\mathcal{M}(\mathbf{v}, \mathbf{w})$ moduli space of Y-M instantons on ALE gravitational instantons	\mathcal{M}_k^0 (universal \mathbb{Z}_k -cover of) moduli spaces of centered magnetic monopoles on \mathbb{R}^3 of charge k	\mathcal{M}_G^g moduli space of Higgs G - bundles on a genus g surface
HYPERKÄHLER METRIC	Kronheimer-Nakajima (1990) Nakajima (1994)	Atiyah-Hitchin (1988)	Hitchin (1987)
S-DUALITY TEST IN $N = 4$ SUSY YM	Vafa-Witten (1994) (PROJECT 2)	Sen (1994) (PROJECT 2)	Bershadsky-Johansen- -Sadov-Vafa (1995) Hausel-Thaddeus (2001) (PROJECT 1)
TOPOLOGICAL L^2 COHOMOLOGY	Nakajima (1994)	Segal-Selby (1996)	Hausel (1998) (PROJECT 2)
COHOMOLOGY OF MODULI SPACES	Nakajima (2000) Lusztig (2000); Hausel (2001) Bialewski-Dancer (2000) Konno (2000) Hausel-Sturmfels (2001) (PROJECT 4,5)	Segal (1979)	Hitchin (1987) Gothen (1994) Hausel-Thaddeus (2000) Markman (2000) Hausel-Thaddeus (2001) (PROJECT 1)
YM INSTANTONS ON GRAVITATIONAL INSTANTONS	Kronheimer-Nakajima (1990)	Etesi-Hausel (2000,2001) Hausel-Hunsicker-Mazzeo (2001); (PROJECT 2,3)	Hausel-Hunsicker-Mazzeo (2001) (PROJECT 2)
GRAVITATIONAL INSTANTONS	A_k, D_k, E Gibbons-Hawking (1978) Kronheimer (1989)	A_k, D_k Gibbons-Hawking (1978) Cherkis-Kapustin (1999)	$D_{\leq 4}, E$ Cherkis-Kapustin (2001)
EXAMPLES (TOPOLOGY)	A_1 , Eguchi-Hanson, (T^*S^2)	D_0 double Atiyah-Hitchin $\tilde{\mathcal{M}}_2^0$, (2-bundle over S^2) A_1 , Taub-NUT (\mathbb{R}^4)	D_4 , toy-model (elliptic fibration, type \tilde{D}_4)
ASYMPTOTICS LOCALLY	\mathbb{R}^4 ALE	$\mathbb{R}^3 \times S^1$ ALF	$\mathbb{R}^2 \times T^2$ ALG