Homework 5 Hints

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From L^p convergence and **5.9** in the last homework, we see there is a subsequence f_{k_j} of f_k converges to f a.e. in E. Then using Fatou's Lemma, we can show that $\int_E |f|^p \leq \liminf \int_E |f_{k_j}|^p \leq M$.

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For $0 < p < 1$, $1/x \in L^p(0,1)$, for $p > 1$, $1/x \in L^p(1,\infty)$, there is no p such that $1/x \in L^p(0,\infty)$.

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Suppose otherwise then $m(\{x : |f(x)| > 0\}) > 0$, wlog we can suppose that $m(\{x : f(x) > 0\}) = \eta > 0$. But since $\{x : f(x) > 0\} = \cup_n \{f(x) > \frac{1}{n}\}$, so there is N such that $m(\{x : f(x) > \frac{1}{N}\}) > \frac{\eta}{2}$. Then $\int_{\{x:f(x)>\frac{1}{N}\}} f > \frac{\eta}{N} > 0$.