# Homework 9 Hints

**2**

$f$ measurable so $\{x : f(x) > a\}$ measurable for any $a$. Since $\{x : g(x) > a\}$ differs from $\{x : f(x) > a\}$ by a set of measure zero and the measure space is complete, $\{x : g(x) > a\}$ is also measurable. For incomplete measure space, it is not necessarily true.

**9**

We note by the definition we will consider sets differ by a zero measure set as equivalent. Since some of the properties are seen immediately, we just prove the following property to show $(\mathscr{S}, \Sigma, \mu)$ is a metric space.
$(E_1 \Delta E_3) = (E_1 - E_3) \cup (E_3 - E_1) \subset (E_1 - E_2) \cup (E_2 - E_3) \cup (E_3 - E_2) \cup (E_2 - E_1) = (E_1 \Delta E_2) \cup (E_2 \Delta E_3)$.
So $d(E_1, E_3) \leq d(E_1, E_2) + d(E_2, E_3)$.
$L^p \Rightarrow \Sigma$ is not hard to show once we identify a $\Sigma$ measurable set with its characteristic function.
$\Sigma \Rightarrow L^p$ can be shown by considering functions in $L^p$ which are rational linear combination of characteristic functions of a countable set of measurable sets which is dense in $\Sigma$. We can show the set of such functions is dense in simple functions, then in $L^p$.

**24**

Consider the set function $\phi(E) = \int_E f d\mu$, $E \in \Sigma_0$, since $f$ is integrable, so $\phi(E)$ is absolutely continuous w.r.t. $\mu$ defined on $\Sigma_0$. Applying the Radon-Nikodym theorem, we get the existence and uniqueness of $f_0$ with the property $\int_E f d\mu = \int_E f d\mu$, $E \in \Sigma_0$. Then we get the result for $g$ simple functions and then for integrable measurable functions.