Local Well-posedness for the $2+1$ Dimensional Monopole Equation

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University of New Mexico
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Special Session on Harmonic Analysis Applied to PDE

Space-Time Monopole Equation

$$\text{(ME)} \qquad F_A = *D_A\phi,$$

The unknowns: (A, ϕ).

- A connection
  $A = A_0 dt + A_1 dx + A_2 dy, \quad A_\alpha : \mathbb{R}^{2,1} \to su(2)$.
- Higgs field $\phi : \mathbb{R}^{2,1} \to su(2)$.
- F_A curvature of A
  $F_A = \sum \frac{1}{2}(\partial_\alpha A_\beta - \partial_\beta A_\alpha + [A_\alpha, A_\beta])dx^\alpha \wedge dx^\beta$.
- $D_A = d + A$ the covariant exterior derivative associated to A
  $D_A\phi = d\phi + [A, \phi] = D_\alpha\phi dx^\alpha = (\partial_\alpha\phi + [A_\alpha, \phi])dx^\alpha$.
- $*$ Hodge star operator w.r.t $\mathbb{R}^{2+1}$ metric;

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- Paul Dirac (1931) -showed quantization of electric charge by introducing magnetic monopoles: isolated point-source of a magnetic charge;
- Magnetic (Euclidean, BPS) Monopoles extensively studied both in physics and in mathematics: See Jaffe-Taubes, Atiyah-Hitchin;
- Richard Ward - introduced (ME)-Wick Rotation of the Bogomolny Equation-Twistor theory point of view-(1989);
- (ME) - Derived from Anti-Self-Dual Yang Mills Equations on $R^{2,2}$ using dimensional reduction;
- (ME) - Dai, Terng & Uhlenbeck- survey as an integrable system (2006); Scattering transform → global existence and uniqueness up to a gauge transformation for small initial data in $W^{2,1}(\mathbb{R}^2)$.

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- (ME) is invariant under gauge transformations.
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    $\to$ if (A, ϕ) solves (ME), so does (A_g, ϕ_g).
    We work in the Coulomb Gauge: $\partial^i A_i = 0$.
- Scaling: what is the critical exponent?
  - If (A, ϕ) solve (ME), then so do
    $\lambda A(\lambda t, \lambda x) \quad and \quad \lambda\phi(\lambda t, \lambda x)$.
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    Expect local well-posedness for initial data in H^s, $s > 0$;

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Main Theorem

The space-time monopole equation (ME) in a Coulomb gauge is locally well-posed for initial data sufficiently small in $H^s(\mathbb{R}^2)$ for $s > \frac{1}{4}$.

- Need small data: global Coulomb gauge & to solve for A_0;
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(ME) in a Coulomb Gauge as a System of Wave & Elliptic Equations.

$$(ME) \quad F_A = *D_A\phi$$

can be written as

$$(ME) \qquad \begin{cases} \partial_t A + [A_0, A] - dA_0 = *d\phi + [*A, \phi], \\ dA + [A, A] = \partial_t \phi + [A_0, \phi], \end{cases}$$

where d acts only w.r.t. the spatial variables.

$A = (A_1, A_2)$.

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$$(aME) \qquad \begin{cases} \triangle A_0 = d^*[A_0, df] + d^*[df, \phi], \\ \Box u = \mathcal{B}_+(\phi, df, A_0), \\ \Box v = \mathcal{B}_-(\phi, df, A_0), \end{cases}$$

Solutions of (aME) produce solutions to (ME) in the Coulomb gauge.

$$\hat{\phi} = \frac{(\partial_t + i|\xi|)\hat{u} + (\partial_t - i|\xi|)\hat{v}}{2}, \quad i|\xi|\hat{f} = \frac{(\partial_t + i|\xi|)\hat{u} - (\partial_t - i|\xi|)\hat{v}}{2}.$$

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The Nonlinearities

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$\mathcal{B}_\pm$ is a linear combination of:

$$\begin{aligned} \mathcal{B}_1 &= [\partial_1 f, \partial_2 f], \\ \mathcal{B}_2 &= R_2[\partial_1 f, \phi] - R_1[\partial_2 f, \phi], \\ \mathcal{B}_3 &= [A_0, \phi], \\ \mathcal{B}_4 &= R_j[A_0, \partial^j f]. \end{aligned}$$

R_j denotes a Riesz Transform, $(-\triangle)^{-\frac{1}{2}}\partial_j$.

Function Spaces

$$\|u\|_{H^{s,\theta}} = \|\Lambda^s \Lambda_-^\theta u\|_{L^2(\mathbb{R}^{2+1})},$$

$$\|u\|_{\mathcal{H}^{s,\theta}} = \|u\|_{H^{s,\theta}} + \|\partial_t u\|_{H^{s-1,\theta}} \approx \|\Lambda^{s-1}\Lambda_+\Lambda_-^\theta u\|_{L^2(\mathbb{R}^{2+1})}.$$

$$\widehat{\Lambda^\alpha f}(\xi) = (1+|\xi|^2)^{\frac{\alpha}{2}}\hat{f}(\xi),$$

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Selberg (1999) if $\theta > \frac{1}{2}$

$$H^{s,\theta} \hookrightarrow C_b(\mathbb{R}, H^s),$$

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Inverting the Wave Operator

- For the purposes of local in time estimates replace $\Box^{-1}$ by $\Lambda_+^{-1}\Lambda_-^{-1}$.
- Bourgain (1993) for the Schrödinger and KdV equations;
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- Selberg (1999 & 2002) $\Lambda_+^{-1}\Lambda_-^{-1+\epsilon}$: use initial data as large as you wish;general framework for local well-posedness of wave equations: enough to show

$$\|\Lambda_+^{-1}\Lambda_-^{-1+\epsilon}\mathcal{B}_\pm(\partial u, \partial v, A_0)\|_{\mathcal{H}^{s+1,\theta}} \lesssim \|u\|_{\mathcal{H}^{s+1,\theta}} + \|v\|_{\mathcal{H}^{s+1,\theta}}.$$

  Need to show similar estimates for differences–for bilinear forms follows from the above;
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We have small data: no ϵ needed;

Inverting the Wave Operator

- For the purposes of local in time estimates replace $\Box^{-1}$ by $\Lambda_+^{-1}\Lambda_-^{-1}$.
- Bourgain (1993) for the Schrödinger and KdV equations;
- Kenig-Ponce-Vega (1993) for the KdV;
- Klainerman-Machedon (1995) first estimates for the wave equation: small initial data needed;
- Selberg (1999 & 2002) $\Lambda_+^{-1}\Lambda_-^{-1+\epsilon}$: use initial data as large as you wish;general framework for local well-posedness of wave equations: enough to show

  $$\|\Lambda_+^{-1}\Lambda_-^{-1+\epsilon}\mathcal{B}_\pm(\partial u, \partial v, A_0)\|_{\mathcal{H}^{s+1,\theta}} \lesssim \|u\|_{\mathcal{H}^{s+1,\theta}} + \|v\|_{\mathcal{H}^{s+1,\theta}}.$$

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Main Estimates Needed

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Observe $\Lambda_+\Lambda_-\mathcal{H}^{s+1,\theta} = H^{s,\theta-1}$, as well as that

$$\|df\|_{H^{s,\theta}}, \|\phi\|_{H^{s,\theta}} \lesssim \|u\|_{\mathcal{H}^{s+1,\theta}} + \|v\|_{\mathcal{H}^{s+1,\theta}}.$$

Enough to show:

$$\begin{aligned}
[\partial_1 f, \partial_2 f] &\rightsquigarrow \|[\partial_1 f, \partial_2 f]\|_{H^{s,\theta-1}} \lesssim \|df\|_{H^{s,\theta}}^2 \\
R_2[\partial_1 f, \phi] - R_1[\partial_2 f, \phi] &\rightsquigarrow \|[df, \phi]\|_{H^{s,\theta-1}} \lesssim \|df\|_{H^{s,\theta}}\|\phi\|_{H^{s,\theta}} \\
[A_0, \phi] &\rightsquigarrow \|A_0\phi\|_{H^{s,\theta-1}} \lesssim \|A_0\|\|\phi\|_{H^{s,\theta}} \\
R_j[A_0, \partial^j f] &\rightsquigarrow \|A_0 df\|_{H^{s,\theta-1}} \lesssim \|A_0\|\|df\|_{H^{s,\theta}},
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Enough to show:

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\|[\partial_1 f, \partial_2 f]\|_{H^{s,\theta-1}} &\lesssim \|df\|^2_{H^{s,\theta}} \\
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\|A_0 \phi\|_{H^{s,\theta-1}} &\lesssim \|A_0\| \|\phi\|_{H^{s,\theta}} \\
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\end{aligned}$$

These last two estimates are equivalent.

Main Estimates Needed

Enough to show:

$$\begin{aligned} \|[\partial_1 f, \partial_2 f]\|_{H^{s,\theta-1}} &\lesssim \|df\|^2_{H^{s,\theta}} && \text{Null Forms} \\ \|[df, \phi]\|_{H^{s,\theta-1}} &\lesssim \|df\|_{H^{s,\theta}} \|\phi\|_{H^{s,\theta}} && \text{Null Forms} \\ \|A_0 w\|_{H^{s,\theta-1}} &\lesssim \|A_0\| \|w\|_{H^{s,\theta}} && \text{Elliptic: Sobolev, } H^{s,\theta} \text{ analog of} \\ & && \text{Sobolev \& Klainerman-Tataru Thm (1999)} \end{aligned}$$

We take a closer look only at the first two estimates.

Null Forms

$$\text{Show: } \|[\partial_1 f, \partial_2 f]\|_{H^{s,\theta-1}} \lesssim \|df\|^2_{H^{s,\theta}}$$

$[\partial_1 f, \partial_2 f]$ has a structure of a null form Q_{ij} :

$$[\partial_1 f, \partial_2 f] = \partial_1 f \partial_2 f - \partial_2 f \partial_1 f = Q_{12}(f, f).$$

- Null Forms: much better regularity than for the general products.
- Unfortunately, results for $Q_{\alpha,\beta}$ in $2d$ are not as optimal as in higher dimensions or for Q_0.
- Using iteration methods we are $\frac{1}{4}$ away from the critical level;

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Null Form $Q_{\alpha,\beta}$ in 2d

- Why are we stuck with the $\frac{1}{4}$???
  - Conditions in the estimates and counterexamples in Foschi and Klainerman (2000);
  - Examine the first iterate: follow the discussion in Klainerman-Selberg (2002);
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We have: $df \in H^{s,\theta} \Rightarrow \|\Lambda^s\Lambda_-^{\theta}Df\|_{L^2} < \infty$. Inspection of Zhou's proof shows we can still handle $Q_{12}(f,f)$.

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Null Forms

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$$\|[df,\phi]\|_{H^{s,\theta-1}} \lesssim \|df\|_{H^{s,\theta}}\|\phi\|_{H^{s,\theta}}.$$

Analysis of the first iterate → $s > \frac{3}{4}$ so we have to work harder:

$$[\partial_i f,\phi] = \frac{1}{4}[(R_i\partial_t+\partial_i)u-(R_i\partial_t-\partial_i)v,(\partial_t+iD)u+(\partial_t-iD)v].$$

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New Null Form

Use the bilinearity of the bracket. It is enough to show we can handle

$$[(R_i\partial_t + \partial_i)u, (\partial_t + iD)u] = [R_i\partial_t u, \partial_t u + iDu] + [\partial_i u, \partial_t u] + [\partial_i u, iDu].$$

The second term → Q_{tj}, and the third term → Q_{tj} in the first iterate.
Proposition: Let $s > \frac{1}{4}, \theta = s + \frac{1}{2}$.
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$$Q(\varphi, \psi) = \partial_t R_i \varphi(\partial_t + iD)\psi - (\partial_t + iD)\varphi \partial_t R_i \psi,$$

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Summary

- We took (ME) in a Coulomb gauge and associated to it a new system of equations (aME);
- (aME) is locally well-posed $\rightsquigarrow$ (ME) locally well-posed;
- 2 dimensions are hard–with this formulation $\frac{1}{4}$ away is the best we can do;

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