M341 (92150), Homework #4

Due: 10:00am, Thursday, Jul. 25

*Instructions: Questions are from the book "Elementary Linear Algebra, 4th ed." by Andrilli & Hecker. Please show all your work, not only your final answer, to receive credit. Keep answers organized in the same order the problems have been assigned.*

**Linear systems and Gaussian elimination (2.1)**

p. 96-98, #1(b,c,f), 2, 5, 10

**Reduced row echelon form (2.2)**

p. 107-110, #1, 4(a), 11, 12

In addition:

A) Suppose $A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 3 \\ -4 \\ c \end{bmatrix}$. For what values of $c \in \mathbb{R}$ does the system $A\boldsymbol{x} = \boldsymbol{b}$ have solutions (that is, for what values of $c$ is the system consistent)? Find the complete solution set in this case.

B) What is the rank$(A)$ in the previous problem? Verify that the rank of $A$ plus the number of non-pivot columns of $A$ equals the number of variables in the system.

C) True or false (justify your answers):

   i. If the matrix $A$ for a linear system with $n$ variables satisfies rank$(A) < n$, then the system must have a nontrivial (i.e., nonzero) solution.

   ii. If the matrix $A$ for a linear system with $m$ equations satisfies rank$(A) = m$, then the system must have a nontrivial (i.e., nonzero) solution.