Research statement

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May 2013

My research interests are in applied probability, and in particular the study of many-body systems. I have a broad training in applied mathematics and enjoy working on problems that interface between disciplines, requiring insight from a variety of perspectives. This is reflected in my past work, which focuses on mean-field models arising from fluid dynamics, physical chemistry, and agent-based systems. More recently I have become interested in dynamical processes on networks and corresponding applications in inference and optimization. As I will try to illustrate in the following discussion, a unifying thread in these problems is the emergence of self-organized structure in random phenomena.

1 Areas of current interest

In the past year, my interests have shifted towards network science and related statistical problems. To this effect, I developed and am currently teaching a topics course on complex networks for graduate students and advanced undergraduates. The course has not only been attended by students in mathematics and statistics, but by many in computer science, engineering, and physics. Central themes are random graphs (branching processes, Erdős-Rényi model, small-worlds, preferential attachment and power laws), dynamical processes on networks (routing, epidemics, contagion and submodular optimization), and statistical methods for analyzing network structure (node ranking, graph partitioning, spectral clustering, modularity maximization, blockmodels). A detailed course description, along with lecture notes, is available at: `http://www.ma.utexas.edu/users/rav/ComplexNetworks/`.

2 Previous work

2.1 Burgers turbulence and stochastic coalescence

My initial work is in the area of *Burgers turbulence*: a model originating in fluid mechanics which describes the dynamics of a large number of "sticky" particles with random initial velocities. This research aims to provide a deeper understanding of aggregation phenomena with applications in physical chemistry and materials science. It also happens to be intimately connected to problems in nonparametric statistics and epidemics at criticality.

Problem. One way to interpret the long-standing problem of hydrodynamic turbulence is as follows:

> *How does a random field evolve under a deterministic flow?*

In the setting of turbulence, the randomness of the velocity field describes uncertainty in the initial conditions and the deterministic flow is given by the nonlinear equations of fluid mechanics. To solve this problem, one must construct a time-varying random field which respects the flow. In fact, this question lies at the heart of many fundamental problems in mechanics: interfacial dynamics in phase separation, directed polymers, and surface growth by random deposition [6, 27, 35]. It also has roots in probability theory. When studying a stochastic process such as Brownian motion (a one-dimensional random field) it

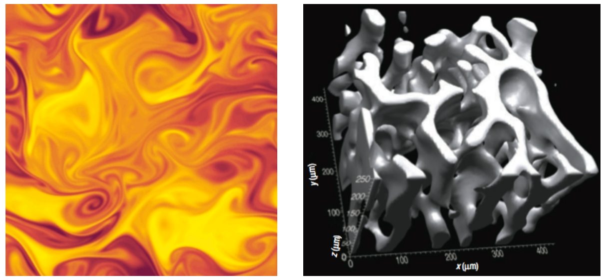

Figure 2.1: (a) Coherent structures in hydrodynamic turbulence; (b) Interfacial dynamics in a binary mixture [27].

is natural to ask how a distribution over an ensemble of paths "deforms" as a result of some deterministic transformation of the paths. As long as this transformation preserves the original ensemble—that is, satisfies a closure property—the problem is well-posed. A classical result of this nature is the Cameron-Martin-Girsanov theorem for semimartingales.

Applications and known results. 1-D Burgers turbulence is the study of Burgers equation

$$\partial_t u + \partial_x \left(\frac{1}{2} u^2 \right) = 0 \tag{2.1}$$

with random initial data or forcing. It has a remarkably rich structure despite its simplicity as a nonlinear model, and exact solutions have played an important role in its analysis [7, 9, 10, 11, 12, 18]. In particular, the solution to (2.1) with white noise initial data can be expressed in terms of the Markov process

$$Z(r) = \arg\max_{s \in \mathbb{R}} \left\{ W(s) - (r-s)^2 \right\} \tag{2.2}$$

where $W(s)$ is standard (two-sided) Brownian motion. The statistics of Z were explicitly derived by Groeneboom in his study of isotonic estimators in nonparametric statistics [21]. Here, the distribution of (2.2) is the analogue of the normal distribution for many problems with "cube-root" (as opposed to the typical square-root) asymptotics. These include maximum-likelihood estimation of a decreasing density on the half-line, and estimating the mode of a unimodal density by binning [13, 20].

Interestingly, related functionals of Brownian motion also appear in SIR epidemics (Reed-Frost model) at the critical threshold. Understanding fluctuations at criticality is especially important since a mean-field description cannot capture the stochastic effects that determine whether an epidemic takes off. As it turns out, first passage times of Brownian motion with a parabolic drift can be used to determine the total size of an outbreak, among other properties, in the scaling limit. We refer the interested reader to [2, 24, 26].

Our contribution and future directions. In collaboration with Govind Menon (Brown University), we have developed a methodology for the study of 1-D scalar conservation laws with random initial data [28, 30, 33]. Our work uses tools from probability and stochastic processes, kinetic theory, and integrable systems. Precisely, we have proven an interesting closure property: if the initial data to the conservation law $\partial_t u + \partial_x f(u) = 0$ with convex flux f is a Markov process in x with only downward jumps, the entropy solution $u(x,t)$ retains this property in x for each $t \geq 0$. Furthermore, its generator $\mathcal{A}(x,t)$ in x satisfies

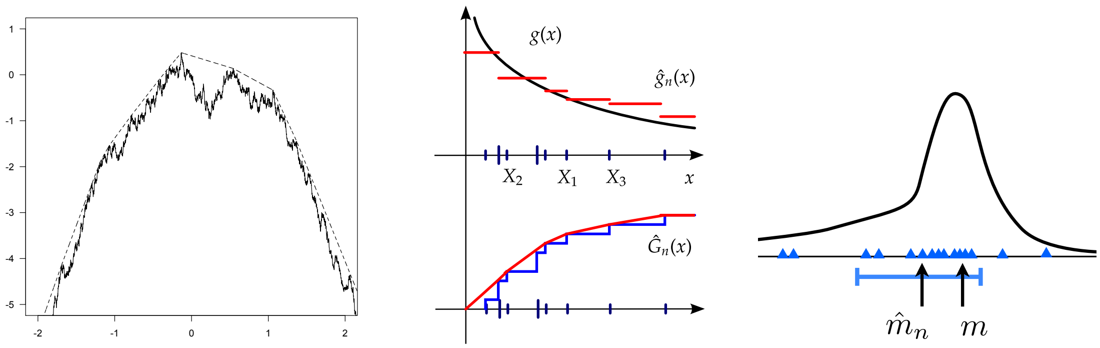

Figure 2.2: (a) A Brownian path with parabolic drift [22]; (b) The MLE $\hat{g}_n(x)$ of a decreasing density $g(x)$ is the derivative of the concave hull of the empirical cdf $\hat{G}_n(x)$, and satisfies $cn^{1/3}\left(\hat{g}_n(x) - g(x)\right) \xrightarrow{\text{law}} Z(x)$ as $n \to \infty$ [20]; (c) The binning estimator $\hat{m}_n$ of the mode m of a unimodal density satisfies $cn^{1/3}\left(\hat{m}_n - m\right) \xrightarrow{\text{law}} Z(0)$ as $n \to \infty$ [13].

the zero-curvature equation

$$\partial_t \mathcal{A} - \partial_x \mathcal{B} = [\mathcal{A}, \mathcal{B}], \tag{2.3}$$

where $\mathcal{B}(x,t)$ is explicitly given in terms of $\mathcal{A}$ and f and serves as a "generator" in t. This describes a natural flow on the space of transition probabilities and has an elegant geometric interpretation [34].

Broadly, our results imply that the evolution of the simplest example of a random field under the simplest nonlinear equation yields an exactly solvable system. Equation (2.3) admits Groeneboom's solution for white noise initial data, among others, as special cases. Our work unifies these under a common framework, and hints at undeveloped links to random matrix theory. We have also shown that (2.3) is equivalent to a kinetic equation, which in the most basic case reduces to a fundamental mean-field model: Smoluchowski's coagulation equation with additive kernel [3]. Some related work delves into the asymptotic behavior of such equations, including self-similarity and rates of convergence [29, 32].

2.2 *Mean-field games and kinetic modeling*

The study of social phenomena has become increasingly prevalent in the mathematical physics community. This has resulted in models that, at least on a phenomenological level, display statistical characteristics supported by data (e.g., Pareto tails for wealth distribution or fragmentation in opinion dynamics). While complex networks and interacting particle systems underlie such models, continuum approaches based on partial differential equations can still be of great utility in understanding large-scale properties. My work in this area has been in the analysis of some models of information aggregation.

Problem. A common element to many problems in the social sciences is:

> *How can we describe the collective behavior of a large population of decision-making agents?*

Both the structure of the interaction network and the detailed nature of interactions are crucial. A contemporary discussion of this matter from the perspective of interacting particle systems is given by Aldous [1]. In order to develop even the most basic theory, we must focus on specialized cases under restrictive assumptions. For the remainder of our discussion, we will assume that agents are indistinguishable. This implies that they interact through the mean-field and yields kinetic models as in gas dynamics. It also leads to the relatively new area of *mean-field games* (MFG), wherein an infinite number of interacting players optimize their individual strategies by anticipating the future actions of others.

Applications and recent results An example arising from financial economics is the flow of information in so-called "dark markets" [14]. These over-the-counter markets, in which derivatives and collateralized debt obligations are traded, are characterized by a lack of transparency among participants. As demonstrated by the financial crises of the previous decade, it is important to understand how asset prices are established in decentralized settings and how information sharing can be appropriately incentivized. One family of phenomenological models proposed by Duffie and co-authors uses the following simple framework [15, 16]. Agents are initially endowed with heterogenous information about the true value of an asset. Upon interaction with others, information sets are merged and each performs a Bayesian updating of her belief, indexed by $\theta \in \mathbb{R}$. Larger θ correspond to a higher confidence that the asset is valuable. Randomized meetings between agents implies that the distribution of beliefs in the market satisfies a kinetic equation of Smoluchowski- or Boltzmann-type:

$$\partial_t \mu = \mu^C \star \mu^C - \left(\int_{\mathbb{R}} \mu^C(d\theta) \right) \mu^C + \eta(\pi - \mu), \qquad \mu^C(t, d\theta) = C(t,\theta)\mu(t,d\theta) \tag{2.4}$$

Here, $\mu(t, d\theta)$ is the distribution of agents with belief θ at time $t \geq 0$, $C(t,\theta)$ modulates interaction rates, and $\star$ denotes convolution. The first two terms on the right-hand side describe the aggregation of agents' information, while the last is a flux of new agents at rate $\eta \geq 0$ with beliefs drawn from a given law $\pi(d\theta)$.

A nonlocal mean-field game arises from (2.4) if $C(t,\theta)$ is taken to be a control parameter. This yields the coupling of a forward-in-time Fokker-Planck equation for μ with a backward-in-time Hamilton-Jacobi-Bellman equation for C, describing agents' ability to optimize their matching rates. Solutions to the coupled system are Nash equilibria. The forward-backward structure is a source of many analytical difficulties, and lies at the heart of MFG theory developed by Lasry and Lions [25]. Similar models have been used to describe a diverse set of socio-economic problems including urban planning, global oil production, creation of price volatility, and even the propagation of "Mexican waves" in stadiums [23].

Our contribution and future directions. The kinetic equation (2.4) is akin to a Kac equation, for which heavy-tailed (non-Gaussian) distributions are known to serve as limits under rescaling. This is in analogy to the appearance of stable laws in classical probability theory [17]. Previous work on similar models has yielded self-similar solutions using probabilistic and Fourier methods [5, 8, 29]. In collaboration with Irene Gamba (UT Austin), I have applied this theory to characterize the behavior of (2.4) with constant C and more complex interaction laws corresponding to "imperfect" information aggregation [19].

In other ongoing work with Mihai Sirbu (UT Austin), we have obtained partial analytical results for the nonlocal mean-field game described the the previous section [31]. This work mainly utilizes tools from stochastic control and partial differential equations. In particular, we considered a related optimal stopping problem in a stationary (time-independent) regime. Since previous research in this area has focused almost exclusively on local equations, our study has required the development of some new techniques. It also points to many promising avenues of research. For example, how does one incorporate network structure and agent heterogeneity into mean-field games? Can interacting particle systems such as the gossip process of [4] be studied in a continuum setting?

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