Homework 3 – due Thursday, Sept 23 2010

1. Prove that if $P(x) = a + bx + cx^2$ with integer coefficients a, b, c and n is an integer, then

$$r \equiv s \pmod{n} \text{ implies } P(r) \equiv P(s) \pmod{n}.$$

Bonus: prove that the same is true for every integer polynomial P.

2. Show that for every integer a not divisible by 11 there is another integer b with $a \cdot b \equiv 1 \pmod{11}$. Show also that this b is unique modulo 11, that is, show that if c is another integer with $a \cdot c \equiv 1 \pmod{11}$ then $b \equiv c \pmod{11}$. Then solve the congruence $3x \equiv 7 \pmod{11}$ (presumably, but not necessarily, using the other ideas in this paragraph).

3. (a) Find a solution in integers x, y to the equation $13x + 21y = 4$.
(b) Find another solution.
(c) Find another.
(d) Stop me from continuing this question *ad infinitum* by describing all the solution pairs (x, y). (Hint: finding one solution is the hard part and you already did that; then if (x', y') were another solution, you'd have two equations to play with, one with x, y and one with x', y'. Subtract, rearrange terms, and see what you can conclude...)

4. Use the Fundamental Theorem of Arithmetic to prove the following: If a and b are positive integers and $a^3|b^2$, then $a|b$. Bonus: can you say anything similar if you are told that $a^m|b^n$ for some other pair of integers m, n?

Reminder: I believe we have agreed to the following schedule:
Tuesday 9/21 and Thursday 9/23 are regular class days.
Friday 9/24 at noon I will hold extra office hours in or near my office (RLM 9.140) as a review session.
Tuesday 9/28 will be our first exam, in the regular room at the regular time.