Math 408C (Rusin): Exam III, Nov 22 2011. Put your NAME on each sheet you turn in.

1. Compute $\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{1}{\ln(x)} \right)$.

2. Suppose f is the function defined by

$$f(x) = \begin{cases} 2, & \text{if } 0 < x < 1 \\ 1, & \text{if } 1 < x < 2 \\ -1, & \text{if } 2 < x < 3 \end{cases}$$

Sketch the graph of f. Then sketch the graph of a continuous function F which is an antiderivative of f on the interval $(0, 3)$.

3. Compute $\int_{-2}^{2} (3+\sqrt{4-x^2})\, dx$. (Hint: you may wish to sketch the graph of this function first.)

4. Estimate the value of $\int_1^3 \frac{1}{x^3+1}\, dx$ by computing a Riemann sum for this integral. Your Riemann sum must have at least 4 summands. Your final answer may be in the form of unsimplified fractions, e.g. "$\frac{2}{3} + \frac{15}{16} + \frac{1}{2}$" would be a suitable form for an answer.

5. Use the properties of integrals to explain why $\int_1^{\pi} \frac{\sin(x^2)}{x}\, dx \le \ln(\pi)$.

6. If $G(x) = \int_{2x}^{x^2} \tan(\sqrt{t})\, dt$, then compute $G'(x)$.

7. Evaluate $\int_0^4 (4-t)\sqrt{t}\, dt$.

8. Evaluate $\int_{-\pi/2}^{\pi/2} x \sin(x^2)\, dx$.

9. Find an antiderivative of $\frac{e^t}{e^t+3}$

10. What is the volume of the portion of the unit sphere $x^2 + y^2 + z^2 \le 1$ where $z \ge \frac{1}{2}$? (You could call it the "top half of the northern hemisphere", although it clearly has less than half the volume even though it has half the height!)