We used Taylor series to evaluate π in class but since I didn't know that question was coming, I hadn't prepared good notes. Here are the details.

The key idea is that since $\tan(\pi/4) = 1$, we can compute π as $4\arctan(1)$. This in turn is computationally feasible because we have a Taylor series

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

and so for example we may write

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \ldots$$

But this is not so useful: the Alternating Series Test applies and tells us that this series not only converges but what it converges *to* is between any two consecutive partial sums. That sounds great but even after adding 100 terms, the partial sums will differ by $4/201 \approx .02$, so we would still not know even the second digit of π after the decimal point!

On the other hand, we can use some trigonometry to our advantage. Write the angle-addition formulas that you know for sine and cosine and divide them to get one for tangent:

$$\tan(u + v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)}$$

or, writing $x = \tan(u)$ and $y = \tan(v)$, we see $\tan(u + v) = (x + y)/(1 - xy)$, i.e.

$$\arctan((x + y)/(1 - xy)) = u + v = \arctan(x) + \arctan(y)$$

In particular, taking $y = x$ shows $2\arctan(x) = \arctan(2x/(1-x^2))$. Try this with $x = 1/5$ to discover $2\arctan(1/5) = \arctan(5/12)$, and then try it again with $x = 5/12$ to discover $4\arctan(1/5) = \arctan(120/119)$ which as you might imagine is very close to $\arctan(1)$. Indeed, using the angle-addition formula above, we can deduce

$$\arctan(1) - \arctan(y) = \arctan((1 - y)/(1 + y))$$

and when y=120/119 this gives us $\arctan(1) - \arctan(120/119) = -\arctan(1/239)$.

Combining all these parts then leaves us with the conclusion

$$\pi = 16\arctan(1/5) - 4\arctan(1/239)$$

Armed as well with the Taylor series for computing arctangents, we can add just a few terms by hand:

$$\pi = 16(1/5 - 1/375 + 1/15625 - \ldots) - 4(1/239 - \ldots)$$

But now the errors involved are no larger than the first omitted term in each sum, namely $16/(7 \cdot 5^7)$ and $4/(3 \cdot 239^3)$ respectively. These sum to less than .00003, meaning that we surely already have 4 correct decimal digits for π when we write

$$\pi \approx 16(1/5 - 1/375 + 1/15625) - 4(1/239) \approx 3.141621$$

(There are simpler fractions that approximate π even better, such as 22/7 and the amazing 355/113, but it's more difficult to discover these.)

Anyway, now you can use a combination of trigonometry and calculus to compute as many digits of π as you like, and to do so fairly quickly! After just 70 terms of the one series and 42 of the other, you would already know 100 decimal digits of π — a huge improvement over the Taylor series giving $\arctan(1)$.