

BENNETT DIFFERENTIAL EQUATION PRIZE EXAM May 13 2019

Name: _____ UT EID: _____

Differential Equations Course: _____ When? _____ Instructor: _____

Permanent Mailing Address: _____

E-mail address: _____

College (Natural Sciences, Engineering, etc.) _____

Submit your solutions on the sheets provided, with your name on each sheet.
No calculators allowed. You must justify your claims.

1. If $f(x)$ is the function defined by

$$f'(x) = \frac{f(x)}{4f(x) + 3x - 3} \quad \text{and} \quad f(0) = 1 ,$$

what is the value of $f(3)$? (Partial credit will be given for a numerical estimate of this value, with more credit for a closer approximation.)

2. For some functions $A(x)$ and $B(x)$, the set of solutions of the differential equation

$$y' = A(x)y + B(x)$$

includes both the tangent function $y = \tan(x)$ and the cosine function $y = \cos(x)$.
What is the solution to the initial-value problem

$$y' = A(x)y + B(x), \quad y(0) = \pi ?$$

3. Find a solution to the partial differential equation

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = z$$

which is not a polynomial in x and y . For extra credit give the general solution.

4. Find a (nonzero) solution of the linear differential equation

$$5x^2 y'' + x(1+x)y' - y = 0$$

5. Does every solution of the differential equation $y'' + e^x y = 0$ stay bounded as $x \rightarrow \infty$?

Answers will soon appear at <http://www.math.utexas.edu/users/rusin/Bennett/> .