BENNETT DIFFERENTIAL EQUATION PRIZE EXAM May 10 2022

Name:_____ **UT EID:**_____

Differential Equations Course:_____ **When?**_____ **Instructor:**_____

Permanent Mailing Address:_____

_____

E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

(Suggestion: Do preliminary work on scratch paper that you don't submit; write up final solutions neatly and in order)

---

1. Austin entrepreneur Melon Tusk plans to build an amusement park ride built on an extremely tall tower – its height will be several times the diameter of Earth! Customers will rocket to the top of the tower and then descend inside a spherical capsule in free-fall until retrorockets brake the capsule shortly before returning to land. Until those braking rockets fire, the capsule will accelerate according to Newton's Law ($F = ma$); the only forces acting on the sphere will be gravitational force (whose strength is inversely proportional to the square of the distance to the center of the earth) and air resistance (which is proportional to the square of the velocity). With those assumptions, will the falling sphere achieve a terminal velocity?

2. Find the general solution of the differential equation $(y')^3 - 27xy' + 27y = 0$. Is it possible to find two different solutions that have the same initial conditions $y(x_0) = y_0$ and $y'(x_0) = v_0$?

3. Find the general solution of the system of differential equations

$$\frac{dy}{dx} = y - z - 4, \qquad \frac{dz}{dx} = y + 3z + 4x$$

4. Which solutions of the equation $f'(x) = x - f(x)^3$ stay bounded as $x \to +\infty$?

5. Find a non-constant function $u = u(x, y)$ which satisfies

$$y\, u_{xy} + u = 0$$

at all points in the first quadrant, and vanishes along the curve $y = e^{-x/2}$.

(This version of the exam corrects typos.)