ALBERT A. BENNETT CALCULUS PRIZE EXAM DATE 12/03/06

Name:______________________________ **UT EID**___________________________
Present Calculus Course:____________ **Instructor**_____________________________
Permanent Mailing Address:____
School (Nat'l Sciences, Engineering, *etc*)**:**______________________________________

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

1. Find all the relative maxima and relative minima for the function defined by

$$f(x) = \int_0^x t\,\sin(t)\,dt$$

2. In all that follows we assume that $f(x)$ is a function defined for all real numbers x. Then f is said to be an *odd function* if $f(-x) = -f(x)$ for all x, and f is said to be an *even function* if $f(-x) = f(x)$ for all x. Show the following

(i) If f is an odd function, then $f(0) = 0$.

(ii) if f is an odd function that is differentiable for all x, then the derivative $f'(x)$ is an even function.

(iii) if f is an even function that is differentiable for all x, then the derivative $f'(x)$ is an odd function.

Extra credit: suppose that f is an odd function that has derivatives of all orders for all x. Show that the Maclaurin Series (Taylor Series in powers of x) for $f(x)$ has only odd powers of x. That is, the series has the form

$$a_1 x + a_3 x^3 + a_5 x^5 + \ldots + a_{2n-1} x^{2n-1} + \ldots$$

3. Compute $\displaystyle\lim_{n\to\infty} \sum_{1\le k\le n} \left(\frac{1}{n}\right) e^{(k/n)}$.

4. Compute the volume of the solid ellipsoid given by

$$(x/a)^2 + (y/b)^2 + (z/c)^2 \le 1\ ,$$

where a, b, and c are positive constants.

5. The *Spiral of Archimedes* is the curve in the xy-plane given in polar coordinates by $r = \theta$. Find the arclength of the curve for $0 \le \theta \le 2\pi$. (Hint: Recall that $x = r\cos(\theta)$ and $y = r\sin(\theta)$.)