ALBERT A. BENNETT CALCULUS PRIZE EXAM 12/8/07

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. (20 pts.) Compute the following limits

(i) $\lim_{n\to\infty} \left(1 - \frac{2}{n}\right)^{3n}$

(ii) $\lim_{x\to 0} x^{-1} \int_3^{3+x} \cos(\pi y^2)\, dy$

(iii) $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{3^k}{k!}$

(iv) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k\pi}{n^2} \sin\left(\frac{k\pi}{n}\right)$

(v) $\lim_{x\to\infty} x\left(1 - e^{-(1/x)}\right)$

2. (10 pts.) A perfectly spherical apple of radius 3 centimeters is centered at the origin. A worm crawls along the x-axis, eating every bit of the apple whose distance from the x-axis is less than 1 centimeter. Find the volume of the remaining uneaten portion of the apple.

3. (10 pts.) Compute $\int_0^{\infty} \frac{1}{(1+x^2)^3}\, dx$.

4. (10 pts.) Line L is the intersection of the planes $2x + 2y + z = 4$ and $x - y - z = 1$. There are two spheres of radius 3 which pass through the origin and whose centers lie on L. Find the equations of the spheres.