ALBERT A. BENNETT CALCULUS PRIZE EXAM   12/7/08

Name:_____   UT EID:_____

Present Calculus Course:_____   Instructor:_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided.
(Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final
solutions neatly and in order; write your name on all pages turned in.)

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1. Let T be the triangle in the xy-plane whose vertices are (1,2), (3,3), and (2,5). Find
   the volume of the solid object obtained by rotating T about the y-axis.

2. Find the sum of each of the following series for $|x| < 1$.

   (a) $\displaystyle\sum_{n=1}^{\infty} nx^{n+1}$

   (b) $\displaystyle\sum_{n=2}^{\infty} n(n-1)x^{2n}$

3. Suppose that $f(x)$ and $g(x)$ are 3-times differentiable functions for all x. Let $h(x) = g(f(x))$. Suppose that $f'(0) = f''(0) = f'''(0) = 0$. Show that $h'(0) = h''(0) = h'''(0) = 0$.

4. Find the equation of the plane which contains the points $(1, 0, 0)$ and $(0, 0, 1)$ and is
   tangent to the curve $(x, y, z) = (2, t, 2t^2)$ at some point. (There are actually two such
   planes. Find them both.)

5. Let $f(x) = x^2 \sin(\pi/x)$. Show that there are infinitely many values of x between 0
   and 1 such that $f'(x) = 0$.