ALBERT A. BENNETT CALCULUS PRIZE EXAM **Dec 8 2012**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Evaluate the following limit (or explain why the limit does not exist):

$$\lim_{x\to 0} \frac{1}{x^4} \int_0^{x^2} \sin(t^2)\, dt$$

2. Determine whether this series converges or diverges. (Be sure to explain your reasoning.)

$$\sum_{n=2}^{\infty} \ln(n \sin(\frac{1}{n}))$$

3. Evaluate the following limit (or explain why the limit does not exist):

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x) + \frac{1}{2}x^2 - 1}{x^4 + y^4}$$

4. Find all functions $f(x, y)$ for which $\nabla f(x, y) = \langle y, -x \rangle$.

5. Consider the surface

$$S = \{(x, y, z) \,|\, xyz = 27,\ x > 0,\ y > 0,\ z > 0\}$$

Show that all pyramids formed by the three coordinate planes and a plane tangent to the surface S have the same volume.