ALBERT A. BENNETT CALCULUS PRIZE EXAM **Dec 7 2014**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Evaluate the integrals:

$$(a)\int_0^1 \frac{x^3-x^2}{x^2-3x+2}\,dx \qquad (b)\int_0^{\pi/6} \sin(3x)\,\sin(5x)\,dx$$

2. Find the integer part of $\displaystyle\sum_{n=1}^{40000}\frac{1}{\sqrt{n}}$. (That is, if the sum is evaluated numerically, what are the digits to the left of the decimal point?)

3. For $t>0$ let $F(t)=\dfrac{1}{t}\displaystyle\int_0^{\frac{\pi}{2}t}|\cos(2x)|\,dx$. Compute $\lim_{t\to 0}F(t)$.

4. Find all the critical points of the function $f(x,y)=x^2+y^2(1-x)^3$, and classify them as local minima, absolute (global) maxima, saddle points, etc.

5. Points P and Q move together around the parabola $y=x^2$ in such a way that the area cut off from the parabola by the line segment PQ always has area $\frac{4}{3}$. Let M be the midpoint of PQ. What curve does M trace out as P and Q vary around the parabola?

Answers will soon be posted to `http://www.math.utexas.edu/users/rusin/Bennett/` .