ALBERT A. BENNETT CALCULUS PRIZE EXAM **Dec 6 2015**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Find the 10^{th} derivative of $\dfrac{6}{x^3 + x^2 - 2x}$

2. Sasha Student has prepared poorly for the Calculus test and thinks that for all differentiable functions f and g it is true that

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)\,g'(x)$$

Amazingly, Sasha used this false result on a particular such product and nonetheless obtained the correct derivative of $f(x)g(x)$! Find a pair $\{f(x), g(x)\}$ of non-constant functions for which this is possible. (A few extra points will be awarded for finding additional, substantially different, such pairs.)

3. The equation $x = 2y + 3y^2 + 4y^3$ defines y implicitly as a function of x. (That is, the graph of this equation is the graph of some function $y = f(x)$.) Compute the 0th through 3rd terms of the Taylor series of this function at the origin.

4. For what values of x does this series converge?

$$\sum_{n=1}^{\infty} \frac{n^n\, x^{(n^2)}}{n!} = x + 2x^4 + \frac{9}{2}x^9 + \frac{32}{3}x^{16} + \dots$$

5. For what values of k does $f(x,y) = \dfrac{x^k y}{x^6 + y^2}$ have a (finite) limit as $(x,y) \to (0,0)$?

Answers will be posted to `http://www.math.utexas.edu/users/rusin/Bennett/` shortly.