ALBERT A. BENNETT CALCULUS PRIZE EXAM May 5 2012

Name:______________________________ **UT EID:**____________________________
Present Calculus Course:____________ **Instructor:**______________________________
Permanent Mailing Address:____

E-mail address:____
School (Natural Sciences, Engineering, etc.)____________________________________

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Determine whether series (a) converges or diverges, and give the radius of convergence for series (b). (Be sure to justify your answer.)

$$(a)\ \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{\pi n}\right) \qquad\qquad (b)\ \sum_{n=1}^{\infty} \frac{n! x^n}{n^n}$$

2. Compute the following limit, or show that it does not exist:

$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\ln(x)} \right)$$

3. Compute the first four terms $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ of the Maclaurin series (i.e. the Taylor series at 0) for

$$f(x) = \ln(1 - x + x^2)$$

4. Find the equation of a plane that contains the points $P_1 = (1, 3, 4)$ and $P_2 = (1, 2, 3)$ and also forms a 60° angle with the plane $x + y - 2z = 6$. (There are two correct answers; you need find only one.)

5. Find the point (x, y) on the ellipse $x^2 + 4y^2 = 74$ where the function $F(x, y) = (x + 12y) + (x + 12y)^3$ is largest.