ALBERT A. BENNETT CALCULUS PRIZE EXAM	May 4 2013

Name:____________________________ **UT EID:**_________________________
Present Calculus Course:___________ **Instructor:**___________________________
Permanent Mailing Address:____

E-mail address:____
School (Natural Sciences, Engineering, etc.)__________________________________

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. For each real number a, evaluate $\int_0^\infty e^{ax}\cos(x)\,dx$ or explain why the integral diverges.

2. Evaluate $\sum_{n=2}^\infty \frac{1}{(n-1)(n+2)}$ or explain why the series diverges.

3. The polar equation of the curve shown in the attached figure is $r = e^{-\theta/10}$. Assume that the pattern of shaded and unshaded sections continues ad infinitum. What is the area of the shaded region? Simplify your answer as much as possible.

4. Here are four lines in space:

$$L_1 : \{x = 1, y = 0\} \qquad L_2 : \{y = 1, z = 0\} \qquad L_3 : \{z = 1, x = 0\} \qquad L_4 : \{x = y = -6z\}$$

For partial credit, find a line that intersects both L_1 and L_2. For full credit, find a line that intersects all three of L_1, L_2, and L_3. For extra credit, find a line that meets all four of the lines L_i.

5. Find a differentiable function $f(x, y)$ defined in the first quadrant of the plane which has this property: at each point (x, y) the gradient $\nabla f(x, y)$ is perpendicular to the vector $\langle x, y\rangle$ pointing directly away from the origin.

For extra credit: Is there such a function f defined on all of the plane?