ALBERT A. BENNETT CALCULUS PRIZE EXAM **May 9 2015**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Let $g(x) = \dfrac{x}{(1-x^2)^2}$. Find $g^{(2015)}(0)$.

2. Evaluate the improper integral $\displaystyle\int_0^\infty \frac{4x}{x^4+4}\,dx$

3. Compute the first two coefficients a_0, a_1 of the Maclaurin series $a_0 + a_1x + a_2x^2 + \ldots$ for the function

$$f(x) = \begin{cases} e^{-\frac{1}{x}} & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

For Extra Credit, compute the next coefficient a_2.

4. Does the series $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ converge? Why or why not?

5. Does the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4y}{x^6+12y^2}$ exist? Why or why not?

Answers will be posted to `http://www.math.utexas.edu/users/rusin/Bennett/` shortly.