ALBERT A. BENNETT CALCULUS PRIZE EXAM   May 8 2018

Name:________________________ **UT EID:**____________________
Present Calculus Course:__________ **Instructor:**________________________
Permanent Mailing Address:____________________________________

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E-mail address:____
College (Natural Sciences, Engineering, etc.)________________________

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Work first on scrap paper that you don't submit; write up final solutions neatly and in order, with your name on all pages submitted.)

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1. Compute (with explanation) the following limit, or show that it does not exist:

$$\lim_{x\to 0} \frac{x^2 \sin(\frac{1}{x})}{\sin(x)}$$

2. Compute the derivative of $f(x) = x^{x^x}$.

3. Compute $\displaystyle\int \frac{\sin(t)+\cos(t)}{\sqrt{2\sin(t)\cos(t)}}\,dt$. (*Hint*: if $u = \sin(t) - \cos(t)$, what is u^2?)

Extra Credit: Use this idea to evaluate $\displaystyle\int \sqrt{\tan(t)}\,dt$ by first computing

$$\int \sqrt{\tan(t)} + \sqrt{\cot(t)}\,dt \quad \text{and} \quad \int \sqrt{\tan(t)} - \sqrt{\cot(t)}\,dt$$

4. Do these series converge or diverge? Explain.

$$(A)\ \sum_{n=1}^{\infty}(-1)^n\left(1+\frac{1}{n}\right)^{-n} \qquad\qquad (B)\ \sum_{n=1}^{\infty}(-1)^n\frac{2+\cos(\pi n)}{n}$$

5. Find the volume of the intersection of the solid bounded by the cylinders $x^2+z^2=R^2$ and $y^2+z^2=R^2$

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .