MATH 361K – HOMEWORK ASSIGNMENT 2

Due Thursday, Feb 5, 2009

Please write clearly, and staple your work !

1. Problems

Assume that $S \subset \mathbb{R}$ is a bounded set. Then (please answer with proof):

(a) Assume that S is open. Is $\sup(S)$ an element of S or not ?
(b) Assume that S is closed. Is $\sup(S)$ an element of S or not ?
(c) Assume that $\sup(S) = \inf(S)$.
  – Is this possible if S is open ?
  – Is this possible if S is closed ?

2. Problem

Deduce from the Archimedean property of $\mathbb{N}$ that $u := \inf\{\frac{1}{n^3} | n \in \mathbb{N}\} = 0$. (Hint: Prove that $0 \leq u < \epsilon$ for any $\epsilon > 0$.)

3. Problem

Find

(a) $\sup\{1 - \frac{1}{n} | 1 \leq n \leq 10^4\}$.
(b) Supremum and infimum of $\{1 - \frac{1}{n} | n \in \mathbb{N}\}$.
(c) $\inf\{a \in \mathbb{R} | \sup\{n^a | n \in \mathbb{N}\} = \infty\}$

4. Problem

Let $X = (x_n)$ denote a sequence in $\mathbb{R}$ which converges to $x \in \mathbb{R}$.

(a) State this fact using $V_\epsilon(x)$, the ϵ-neighborhood of x.
(b) Prove that the sequence (x_n^2) converges to x^2.

5. Problem

Determine

(a) $\lim_{n\to\infty} ne^{-n}$
(b) $\lim_{n\to\infty} n \ln n$
(c) $\lim_{n\to\infty} \frac{1}{n} \ln n$