MATH 361K – HOMEWORK ASSIGNMENT 9

Due Thursday, April 9, 2009

Please write clearly, and staple your work !

1. Problem

Consider the functions $f(x) = \frac{x^2-1}{x-1}$, $g(x) = \frac{x+1}{x^2-1}$, and $h(x) = \sin\frac{1}{x-1}$ on the interval $I = (1,2)$. Which of these functions are uniformly continuous on I ? Present a proof for your argument.

2. Problem

Consider a monotonically increasing function $f(x)$ on $I = [0,10]$ with $f(0) = 0$ and $f(10) = 90$, which has a jump discontinuity at every integer $\{1, 2, \ldots, 9\}$.

(a) Assume that each of these jumps has the value 10. What does the function look like ? What if f were strictly monotonically increasing ?

(b) How many jumps of value 30 can the function f have so that the assumptions formulated above are still true ?

(c) Assume now that f is still monotonically increasing, but that it can have jump discontinuities anywhere in I, not only at the integers. Can f have countably infinitely many jumps, of value $\frac{1}{k}$, where all $k \in \mathbb{N}$ occur ?

3. Problem

Assume that f, $g : I \subset \mathbb{R}$ are differentiable at $c \in I$. Prove the product rule $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.