# MATH 361K – REVIEW 2

Will not be graded, but will be discussed on Tue, April 14, in class.

## 1. Problem

(a) For which values of $p$ does the series $\sum(\frac{1}{\sqrt{1+n}})^p$ converge/diverge ?

(b) For which values of $p$ does the series $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}$ converge/diverge ?

## 2. Problems

Let $A \subseteq \mathbb{R}$, and $c \in A$ a cluster point. Prove that the epsilon-delta definition of continuity and the sequential definition of continuity of $f$ at $c$ are equivalent.

## 3. Problems

Let $I = [a, b] \subset \mathbb{R}$ be a closed, bounded interval. Assume $f : [a, b] \to \mathbb{R}$ is increasing and bounded, and let $C \subset (a, b)$ denote the set of locations where $f$ has a jump discontinuity.
(i) Prove that one can write $C = \{c_1, c_2, \ldots, c_j, \ldots\}$ where $j_f(c_i) \geq j_f(c_j)$ if $i < j$.
(ii) Prove that on every open interval $(c_j, c_{j'})$ between neighboring jump points, $f$ is uniformly continuous.
(ii) Is it possible that $f$ is uniformly continuous on all closed intervals $[c_j, c_{j'}]$ between neighboring jump points ?

## 4. Problem

Let $I = [a, b] \subset \mathbb{R}$ be a closed, bounded interval. Assume $f : [a, b] \to \mathbb{R}$ is a function. Moreover, assume that for every Cauchy sequence $(x_n)$ in $I$, it follows that $(f(x_n))$ is also a Cauchy sequence. Prove that $f$ is uniformly continuous on $I$.

## 5. Problem

Consider the function $g(x) = \sin\frac{1}{\sqrt{x}}$ for $x \in (0, 1)$. Prove that $g$ is not uniformly continuous on $(0, 1)$.