COMPLEX ANALYSIS – HOMEWORK ASSIGNMENT 6

Due Friday, March 8, 2013, at the beginning of class.

Please write clearly, and staple your work !

1. Problem

What can you say about an entire function whose real part is always less than its imaginary part? Justify your answer.

2. Problem

Evaluate the integrals

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$$

and

$$\int_0^{\infty} \frac{x^2}{x^2 + a^2} dx \quad , \quad a \in \mathbb{R}$$

using the method of residues.

3. Problem

Determine the integral

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$$

by integrating the function $(e^{2iz} - 1)/z^2$ along a suitable contour.

4. Problem

Assume that f is a holomorphic function on $\mathbb{C} \setminus \{z_j\}_{j=1}^J$. Moreover, for an arbitrary $j \in \{1, \ldots, J\}$, assume that around z_j, the Laurent series is given by

$$f(z) = \sum_{n=-\infty}^{\infty} a_n^{(j)} (z - z_j)^n \, .$$

Prove that the principal part,

$$P^{(j)}(z) := \sum_{n=-\infty}^{-1} a_n^{(j)} (z - z_j)^n \, ,$$

has convergence radius ∞, while the part

$$T^{(j)}(z) := \sum_{n=0}^{\infty} a_n^{(j)} (z - z_j)^n$$

has convergence radius $R_j := \min\{|z_j - z_\ell| \, | \, \ell \in \{1, \ldots, J\} \, , \, \ell \neq j\}$.