COMPLEX ANALYSIS – HOMEWORK ASSIGNMENT 3

Due Friday, February 7, 2014, at the beginning of class.

Please write clearly, and staple your work !

1. Problem

Expand $\frac{2z+3}{z+1}$ in powers of $z-1$. What is the radius of convergence?

2. Problem

Assume that γ denotes the contour given by the triangle with vertices at 0, 1, and i, oriented in counter-clockwise direction. Determine the contour integrals

$$\oint_\gamma \frac{z^3+1}{z^2-4}dz \quad , \quad \oint_\gamma \overline{z}\, dz\,.$$

3. Problem

Let $P(z)$ be a polynomial of degree n. Prove that $P(z)=0$ has n solutions in $\mathbb{C}$.

4. Problem

(a) Determine the conformal maps $\mathbb{C}_\infty \to \mathbb{C}$.
(b) Determine the conformal maps $\mathbb{C} \to \mathbb{H}$.

5. Problem

A map g is called open (closed) if the image under g of any open (closed) set is open (closed). Recall that a function g is continuous if the pre-image of any open set under g is open.
(a) Give an example of a function $g : \mathbb{R} \to \mathbb{R}$ that is continuous but not open.
(b) Show that the angle function $\theta : S^1 \to [0, 2\pi)$ is bijective, open and closed, but not continuous.
Hint: Note that this is in the topology of $[0, 2\pi)$ as the full space and not as a subset of $\mathbb{R}$. Thus, the open subsets are generated by intervals of the form $[0, a)$, $(a, 2\pi)$, and (a, b) with $0 < a < b < 2\pi$.

6. Problem

Assume that $f : \Omega \to f(\Omega)$ is holomorphic and injective on $\Omega \subset \mathbb{C}$.
(a) Prove that $f' \neq 0$ everywhere in Ω.
(b) Prove that f is an open map.
(c) Prove that $f^{-1} : f(\Omega) \to \Omega$ is holomorphic.