COMPLEX ANALYSIS – PRACTICE SET 2 – APRIL 7, 2014

1. PROBLEM

Assume that $f \in \mathcal{H}(\mathbb{C})$ is entire. Prove that f has finite order if and only if f' has finite order, and that if so, the orders are the same.

2. PROBLEM

(i) Determine the zeros z_j of $f(z) = e^{e^z} - 1$. For which $p > 0$ does $\sum_j \frac{1}{|z_j|^p}$ converge?

(ii) Does the following product converge to an entire function ?

$$g(z) := \prod_{j=1}^{\infty} \left(1 - \frac{z}{2^j} \right).$$

If yes, what is its order ?

3. PROBLEM

Assume that $\Omega \subset \mathbb{C}$ is a simply connected, proper subdomain. Let $f : \Omega \to \Omega$ be a biholomorphism which has two distinct fixed points $f(z_i) = z_i$, where $z_i \in \Omega$, and $i = 1, 2$. Prove that f must be the identity function.