PDE I – HOMEWORK ASSIGNMENT 11

For Monday, November 22, 2010. **Please write clearly, and staple your work !**

1. Problem

Reading assignment: Read chapter 7.4 of Evans' book on Semigroup theory.

2. Problem

Prove the following version of the Kato smoothing estimate. Let $n \geq 2$, and $f \in L^2(\mathbb{R}^n)$. Prove that

$$\|(1+|x|)^{-s}(-\Delta)^{\frac{1}{4}} e^{it\Delta} f\|_{L^2_{t,x}(\mathbb{R}\times\mathbb{R}^n)} \leq C(s)\, \|f\|_{L^2_x(\mathbb{R}^n)}$$

for $s > \frac{1}{2}$.

3. Problem

Reading assignment: Refresh your memory on the spectral theory of selfadjoint operators on Hilbert spaces, for instance based on the lecture notes by Arbogast and Bona (Methods of Applied Mathematics I).