Notes by D. Ben-Zvi

2D Langlands meeting 10/25/04

D. Gaitsgory : 2D Langlands ?

$X \ni x$ point on curve, K_x 1-dim local field

	local	global
Classical	{ Category of reps of $G(K_x)$ }	{ Vector space of automorphic functions }
Geometric	{ · Representations of $\hat{\mathfrak{g}}(K_x)$ · Sheaves on flags $G(K_x)/I$ }	{ Category of $\mathcal{D}$-modules/sheaves on $\mathrm{Bun}_G(X)$ }

Categories on which the group acts : analog of studying subreps of $\mathrm{Fun}(G/I)$

.... if rep appears on G/I, appears almost uniquely

$\mathcal{D}$-mod on $G(K_x)/I$ generalization of group algebra

--- category with group action : analog of individual representation

$\Rightarrow$ 2-category of categories with G-action ("1-representations")

So local $\Longleftrightarrow$ global, classical $\to$ geometric are categorification

Dimension 2 S global surface $\supset X$ curve $\supset x$ point

$\hat{X}$ punctured tubular neighborhood $\mathbb{C}((s))((t))$ where $X = \{t=0\}$

	$x \in X \subseteq S$	$\hat{X}$	S
Classical	2-category	Category	Vector space
Naive classical	1-category	Vector space	number?
Geometrically	?? 3-category ??	2-Category	category (D-modules on something?)

Dual side   2dim CFT :

K_2 K_1 K_0 classifies abelian covers by points

abelian Galois side $\longleftrightarrow$ K-theory side

(Kazhdan) Galois side has operations telling us deep things about reps:
e.g. $\oplus \implies$ parabolic induction

(Beilinson) π_1(curves) ~ free essentially
π_1(surfaces) far from this, even locally
— higher relations — do they appear on Langlands side?

$Bun_G(S)$ typically singular stack, & needs compactification
Components labelled by 2nd Chern class.

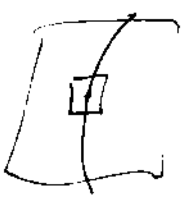
Uhlenbeck $\quad Uhl_G^a = Bun_G^a(S) \cup \bigcup_b Bun_G^{a-b}(S) \times Sym^b(S)$

bundles with singularities at fin many points of curve, with positive integer measuring singularity
Worked out in case of $\mathbb{P}^2$, singularities have local nature. (

here sings. of moduli space all come from sings. of bundles

Fibers of IC of this are expressed in terms of dual affine Lie algebra $(\hat{\mathfrak{g}})^\vee$.

Given a G-bundle P_G on S natural to modify along a curve X : $P'_G \sim P_G|_{S \setminus X}$



Locally $S = X \times C$, trivialize P_G
$\Rightarrow$ data of P'_G is same as map $X \longrightarrow Gr_G(C, c)$
affine Grassmannian of C at c.
... compactify space of maps by quasimaps

Really should ~~compare~~ have maps $X \to$ BD Grassmannian
many curves moving where have singularities.

Hecke correspondences $\longleftrightarrow$ sections of stack

$$X \longrightarrow G(\mathcal{O}_c) \backslash G(K_c) / G(\mathcal{O}_c)$$

along curve C:

fibers of
$$\begin{tikzcd} & \mathcal{H}_X \arrow[dl] \arrow[dr] & \\ Bun_G(S) & & Bun_G(S) \end{tikzcd}$$

are sections of bundle of affine Grassmannians

$\longleftrightarrow$ maps to $G(\mathcal{O}) \backslash G(K) / G(\mathcal{O})$

... functions on maps as such are our Hecke algebra.