Toronto 10/04

M. Hopkins - The Work of Jack Morava

Blue jeans wind up being the cleanest thing you can wear because it's just in their nature to be washed a lot
— Andy Warhol

(comment on Jack's work: more it gets cleaned up, better it gets...)

Homotopy theory pre 1970s:

Adams spectral sequence
- Hopf invariant one problem
- many calculations of π_*(spheres)

K-theory Adams S-S analogue
- vector field problem
- image of J

} Adams

[manifestation of Bott periodicity in homotopy grps of spheres]
- self map of mod p Moore space

$$\Sigma^{2p-2} S^0 \cup_p e^1 \longrightarrow S^0 \cup_p e^1 \text{ iso in } K$$

Novikov on complex cobordism Adams S-seq.
Very elaborate computational apparati, lots of mistakes.

Quillen $MU \longleftrightarrow$ formal groups

Morava: secret documents
"Structure theorems for cobordism comodules"
"On Novikov's $Ext^{**}(U,U)$" . . . .

Quote from late mid 80s: Avant-propos

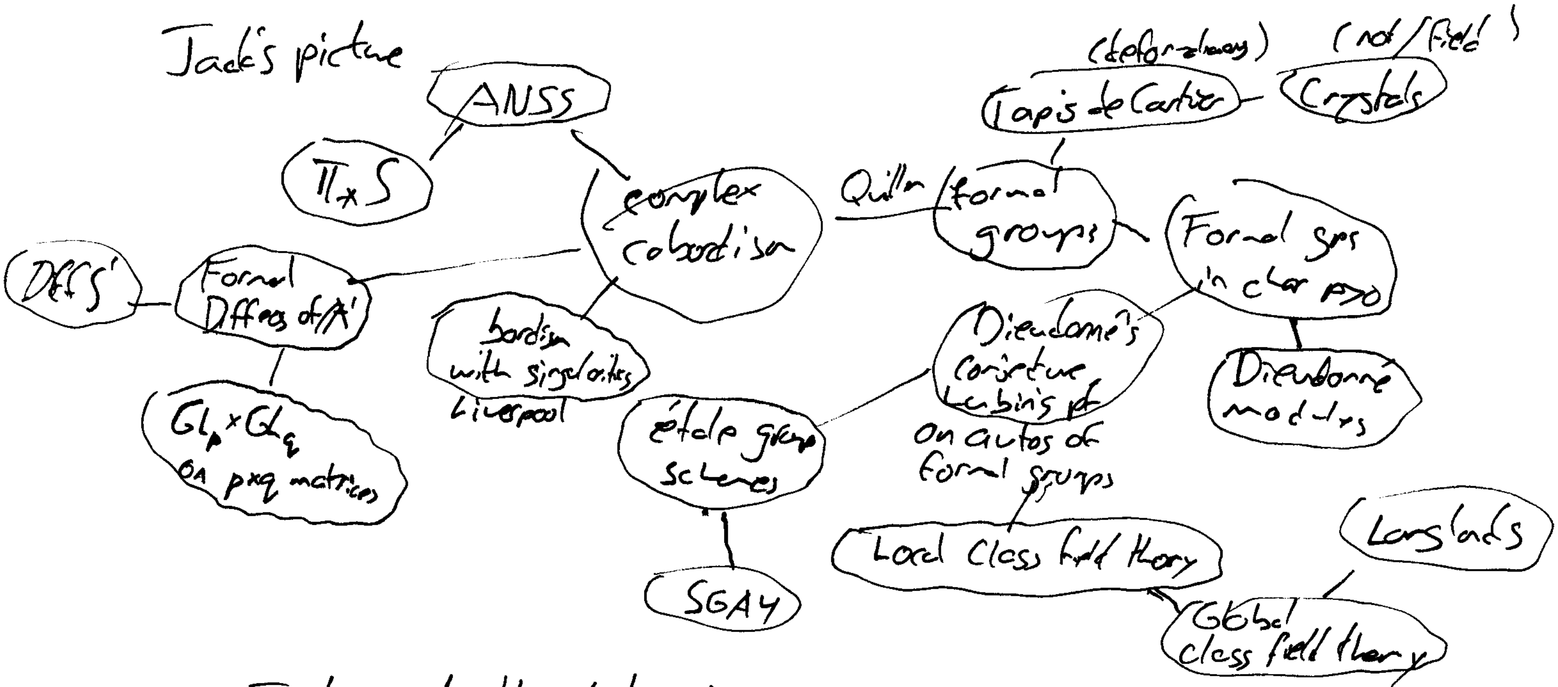
bordism with singularities

K-theory of highly ramified local fields

stable homotopy theory

fill in simplex

Jack's picture



First people through here!
Miller - Ravenel - Wilson, many others

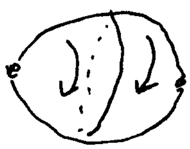
E^2-term of Adams-Novikov spectral sequence:
cohomology of $MU_* \rightrightarrows MU_*MU \;\substack{\rightarrow\\\rightarrow\\\rightarrow}\; MU_*MU \otimes_{MU_*} MU_*MU \ldots$
co-ordinates

$\Downarrow$ Spec

$\left\{ \begin{array}{l} \text{Formal groups} \\ \text{with coordinate} \end{array} \right\} \leftleftarrows \left\{ \begin{array}{c} \text{FGLs} \\ \times \\ \text{Group of} \\ \text{changes of coord's} \end{array} \right\} \;\substack{\leftarrow\\\leftarrow\\\leftarrow}\; \ldots$ bar construction for group acting on formal group laws

So ANSS is studying quotient $\left\{ \begin{array}{l} \text{formal} \\ \text{group laws} \end{array} \right\} \Big/ \begin{array}{l} \text{coord} \\ \text{changes} \end{array} \simeq \mathcal{M}_{FG}$

study $X \circlearrowleft G$ group acting on space cohomologically:



study equivariant cohomology - break up into orbits, which breaks up to cohomology of stabilizers

$\partial/\!/$ free action $\amalg \circ 2$: assemble $\cancel{H}$ $H^*_G X$ from cohomology of stabilizer groups $H^*_{G_x}(-)$

Case $X = \mathcal{M}_{FGL}$:

∴ localize at a prime, orbits indexed by height – stratify by height

Contribution of a single orbit: $H^*(\mathrm{Aut}\,\Gamma)$

Γ f.g. of height n. assemble everything from these.

… $\mathrm{Aut}\,\Gamma$ étale group scheme, so after étale base change just ordinary group cohomology of a p-adic Lie group

– first qualitative mechanism in calculation!

These groups have Poincaré duality $\Longrightarrow$ gives sectors of $\pi_*(S)$ which are understandable conceptually

– break up into wavelengths labelled by n.

– chromatic filtration

Try to make this filtration topological, not just algebraic

$$
\begin{tikzcd}
\bigoplus H^*(\mathrm{Aut}\,\Gamma) \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \bigoplus \pi_* L_{K(n)} S^0 \arrow[d, Rightarrow] \\
\text{Adams-Novikov } E^2 \text{ term} \arrow[r, Rightarrow] & \pi_* S^0
\end{tikzcd}
$$

$K(n)$ - Morava K-theory. $K(n)$-local spheres $\longleftrightarrow H^*(\mathrm{Aut}\,\Gamma)$

(Ravenel et al)

Try now to make algebraic topology depend functorially on formal groups

tmf – parts of topology derived on elliptic curves

Lurie: derived schemes.

Other directions of Morava:

"Forms of K-theory" $\Longrightarrow$ definition of elliptic cohomology (early 70s)

Knot & physics ~1990

· Gromov-Witten invariants with values in MU_*

· Generalized quantum cohomology ~1996

$\rightsquigarrow$ Givental

Rational Contexts for elliptic cohomology ·

-- higher genus partition functions · 10 years later: Alvarez-Singer 90 "Beyond elliptic genus"

Ginzburg's cobordism of symplectic manifolds $\leftrightarrow MU_* BT$

Rudimentary theory of topological 4-dim gravity

TQFT: specify a Lagrangian $\rightsquigarrow$ field theory

Lagrangian topologically: $\mathbb{Z}$-valued cobordism invariants

$$\begin{pmatrix}\text{Cobordism} \\ \text{theory}\end{pmatrix} \longrightarrow I \quad \text{where } [X, I] = \mathrm{Rhom}(\pi_* X, \mathbb{Z})$$

Problem: what does integration mean?

e.g. add up a bunch of complex lines into a vector space.

Morava: vector spaces ~~sheaves~~ (result of path integral) as element in K-theory or other generalized cohomology group

Σ pts in $U(1)$ $\in \mathbb{C}$ $\rightsquigarrow$ Σ sections of line bundles

Σ line bundles $\in$ Vector spaces $\leftrightarrow$ pt in K-theory

Σ K-module spectra $\in$ pt in $A(K)$ Waldhausen A of K-theory

- look for bordism invariants valued in Waldhausen K-groups