Anton Kapustin : loop operators

Note Title 1/17/2007

Montonen Olive duality $\Longrightarrow$

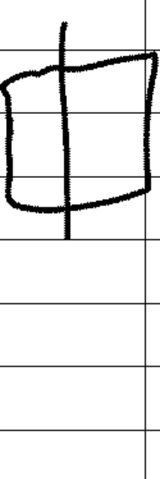
1. A-model of $(\mathcal{M}_H(G,C), \omega_K)$
   is
   B-model of $\mathcal{M}_{Flat}({}^LG_{\mathbb{C}}, C)_J$

2. B-model of $\mathcal{M}_{Higgs}(G,C)_I$
   is
   B-model of $\mathcal{M}_{Higgs}({}^LG, C)_J$

(comes from mixed topological/holomorphic twistings)

This has interesting generalizations to other gauge theories!

Loop & Line Operators



Line operators: change fields for all time $\Rightarrow$ not really operators in traditional way, but change the Hilbert space of the theory!

Suppose $M_4 = \Sigma \times C$, C compact
Can look at closed loop γ of forms

1. $p_\Sigma \times \gamma_C$ point × loop
2. $\gamma_\Sigma \times p_C$ loop × point

Type 1 $\Rightarrow$ normal local operators, act on Hilbert space of Σ.

Example: Wilson line in representation R

$W_R(p_\Sigma \times \gamma_C)$: TFT is topological, so independent of C

$\rightarrow$ can think of this as function on $\mathcal{M}_{flat}({}^L G, C)$ (holomorphic)

..... B-model state space is generated (in degree 0) by such functions ...

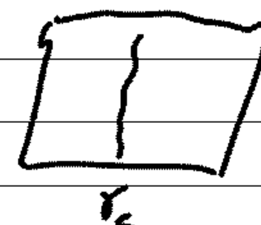
Harder to say for 't Hooft operators in A-model....

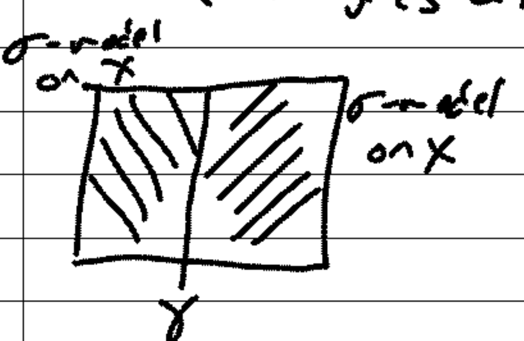
Type 2 : let $\Sigma = \mathbb{R}^2$, $\gamma \simeq \mathbb{R}^1 \hookrightarrow \Sigma$

σ-model with target X

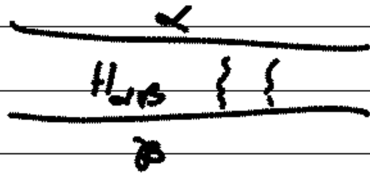
$\Rightarrow \gamma$ gives brane on $X \times X$

$\Leftrightarrow$ gives a functor branes$(X) \circlearrowleft$

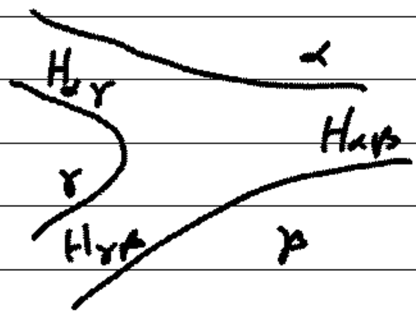




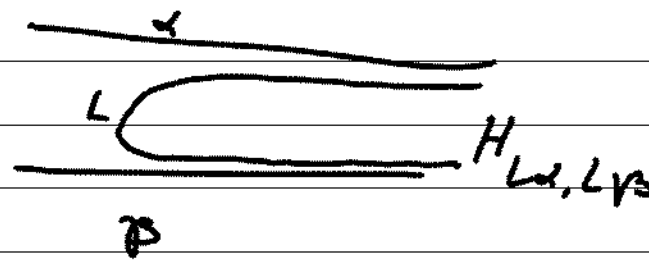
γ gives gluing condition $\Leftrightarrow$ map from half-plane to $X \times X$ $\Leftrightarrow$ fold to get a condition on $X \times X$



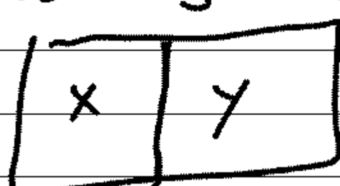
space of strings from brane α to brane β



Line operators give functors from category of branes to itself:



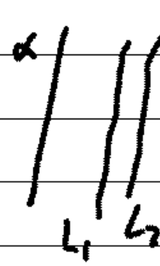
Every isomorphism of field theories can be considered as a brane on their product: give fields along a line using this isomorphism

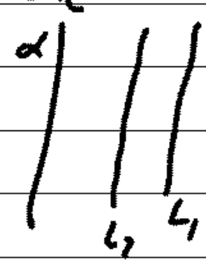


Wilson operator $W_R(\rho)$ in B-model on $\mathcal{M}_{flat}({}^L G, \mathbb{C})_J$ $\rightsquigarrow$ assign vector bundle on the moduli of flat connections:

On $\mathcal{M}_{flat}(G, C) \times C$ have a universal G-bundle & associated vector bundle in representation R. Can restrict it to $\mathcal{M}_{flat}(G, C) \times p_c$

$\Rightarrow$ holomorphic vector bundle (in each complex structure). Tensoring with this vector bundle is the functor corresponding to $W_R(p_c)$.

In our setting line operators all commute: to get from picture 

to 

just use freedom of choice of point in C to move L_1 around L_2..

$\Rightarrow$ can look for eigenbranes:

$$W_R \cdot \mathcal{E} \cong V_R \otimes \mathcal{E}$$ for some vector space V_R, & all W_R.

e.g. point sheaves give eigenbranes for all W_R simultaneously.

(1) MO duality predicts that Lagrangian A-branes which are smooth fibers of the Hitchin fibration of $\mathcal{M}_H(G,C)$ are eigenbranes of all 't Hooft operators $T_{^LR}(P_c)$ $\forall\, {}^LR, P_c$.

--- can check this directly using spectral cover construction

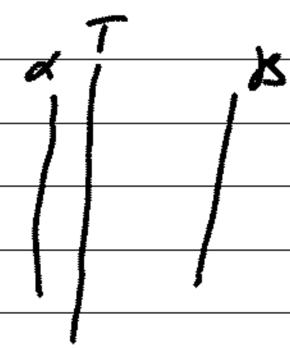
(2) MO duality predicts that 't Hooft functors satisfy

$$T_{^LR_1} \cdot T_{^LR_2} = \sum_{^LR} T_{^LR} \cdot \hom({}^LR, {}^LR_1 \otimes {}^LR_2)$$

i.e. 't Hooft operators satisfy representation ring of ^LG.

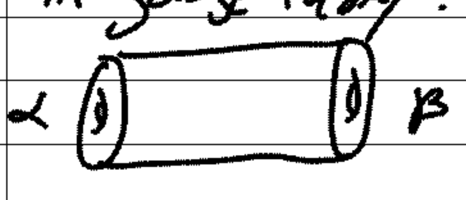
How to calculate compositions of the 't Hooft operators?

$$\mathrm{Hom}(T_{^LR}(\alpha), \beta) =: H_{^LR}$$



= space of states on strip $I \times \mathbb{R}$

in gauge theory: $M_4 = \underset{\text{time}}{\mathbb{R}} \times \underbrace{I \times C}_{W = \text{spatial slice}}$



Can eg have LHS fixed, RHS free ∂

Classically if don't allow singularities the bundle must be constant : space of classical vacua with given boundary conditions is a point if fix one end & leave other free ($Hom(\mathcal{O}_\lambda, \mathcal{O}_{Higgs})$)

.... so easy to quantize this classical space (BPS states). : SUSY quantum mechanics with target the space of vacua.

In this case get deRham / L^2 cohomology

Next allow singularities: get a Schubert cell in a Grassmannian as space of solns of BPS equations $\longrightarrow$ take its L^2 cohomology = IC of its closure!

Calculate this has the dimension of a representation of $^L G$.