MATH 380, PROBLEM SET 2

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1. Problems

(1) From the text: 3.4.8, 3.5.10, 4.4.18.

(2) Let $H \subseteq G$ and consider the action of G on G/H. Prove that a subgroup $K \subseteq G$ has a fixed point if and only if K is subconjugate to H (i.e., K is contained in a conjugate of H).

(3) Let Σ_n act on $\mathbb{R}^n$ by

$$\sigma, (x_1, \ldots, x_n) \mapsto (x_{\sigma(1)}, \ldots, x_{\sigma(n)}).$$

Explain why this is not a group action. How do you fix the formula to make a group action in this vein?

(4) Let G be a group with order p^n for a prime p. Suppose that G acts on a finite set X. Show that $|X|$ is equal to the number of fixed points of the action modulo p.

(5) Describe the coproduct in sets. Then give an explicit description of the coproduct in groups (this is usually called the free product); justify why your description is correct. Is the coproduct of two finite groups finite? Explain. (Extra credit: describe the pushout in groups explicitly.)